One finds the interesting result

$$I_{+1}(\tau,\sigma)=\sum_{n=0}^{\infty}\frac{(-)^n\tau^n}{2^n n!}\sum_{\kappa=0}^{\infty}(-)^\kappa\binom{n/2}{\kappa}\delta(\sigma-\sigma_0-4\kappa). \quad (3.36)$$

This solution, which should be more exact than those obtained with the Taylor series assumption, gives a discrete spectrum in contrast to the solutions presented in previous sections of this paper.

One interprets this result as follows. The method of Radau replaced an integral over μ' by two terms only; *viz.*, $\mu'=\pm 1$, i.e., $\vartheta'=0$, π. The above solution is for $\mu=1$ so the result should contain only those contributions of gammas which are unscattered, twice scattered through π, four times scattered through π, etc. Since each scattering through π involves a wavelength increase of two Compton units, the spectral distribution should contain the following wavelengths only (along the direction $\mu=1$), $\lambda=\lambda_0$, $\lambda=\lambda_0+4\gamma$, $\lambda=\lambda_0+8\gamma$, etc. where $\gamma=h/mc$.

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Angular Correlation of Electrons in Double Beta-Decay*

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Angular correlation functions for the emitted electrons with corresponding electron energy spectra are obtained, assuming the Majorana theory of the neutrino, for allowed and first-forbidden nuclear double beta-transitions. The associated decay mean lives are estimated $\approx 10^{12}$ yr, and $\approx 10^{16}$ yr, respectively, for $Z\approx 50$, and an atomic mass difference ≈ 1.5 Mev. A brief discussion is given of the reported experimental observations of double beta-decay in the light of the deduced angular correlation functions, electron energy spectra, decay mean lives, and selection rules; the improbability of actual occurrence of the allowed 10^{12}-yr transitions is pointed out.

INTRODUCTION

THE phenomenon of nuclear double beta-decay has been investigated theoretically by Goeppert-Mayer[1] on the basis of the original Fermi description, i.e., with Dirac neutrinos and antineutrinos, while Furry[2] has studied the problem on the assumption of Majorana neutrinos. The Furry treatment, which entails the simultaneous emission of two electrons without any accompanying neutrinos, predicts half-lives 10^9–10^{11} times shorter (for comparable degree of forbiddenness and energy release) than that of Goeppert-Mayer, which necessitates the simultaneous emission of two electrons and two neutrinos. Experimental observations of nuclear double beta-decay have been reported by Fireman[3] and Inghram and Reynolds;[4] these yield half-lives $\approx 10^{16}$ yr and $\approx 10^{21}$ yr for the double beta-process in $_{50}Sn^{124}$ and in $_{52}Te^{130}$, respectively. It is, however, not yet known whether the sum of the energies of the different pairs of electrons, emitted by the different decaying nuclei of a double beta-active substance, is or is not the same in all individual decays; experimental decision between these two possibilities would at once indicate either the Majorana or the Dirac theory.

In the present note we have calculated, assuming the Majorana theory,[5] the transition probabilities—selection rules, mean lives, electron energy spectra, and angular correlation functions—for allowed and for first-forbidden double beta-transitions. The selection rules, mean lives, and electron energy spectra for the allowed and for some of the first-forbidden transitions have previously been obtained by Furry.[2] Our expressions differ from his in yielding more explicit formulas for the various transition nuclear matrix elements as a

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[1] M. Goeppert-Mayer, Phys. Rev. **48**, 512 (1935).

[2] W. H. Furry, Phys. Rev. **56**, 1184 (1939); see also, B. Touschek, Z. Physik **125**, 108 (1948–1949).

[3] E. L. Fireman, Phys. Rev. **75**, 323 (1949).

[4] M. G. Inghram and J. H. Reynolds, Phys. Rev. **76**, 1265 (1949); **78**, 822 (1950); M. G. Inghram, Chicago Conference, September, 1951.

[5] The quantized amplitude of the Dirac neutrino-antineutrino field ψ (which together with the quantized amplitude of the Dirac electron-positron field enters the β-decay nucleon-lepton interaction Hamiltonian, $\mathcal{H}_{\text{inter}}$) involves both destruction operators for positive energy neutrinos and creation operators for positive energy antineutrinos of various spin orientations and linear momenta; each of these destruction, creation operators is multiplied by the corresponding positive energy, spin orientation, linear momentum eigenspinor ϕ, and $(C\phi)^*$, respectively (C=charge conjugation operator). For the purposes of a theory of β-decay, the quantized amplitude of the Majorana neutrino field can then be obtained from ψ merely by replacing the various creation operators for the positive energy antineutrinos by the corresponding creation operators for positive energy neutrinos. Such a formulation enables the usual techniques employed in calculations with Dirac particles, e.g., the projection operator procedure for summing over the two possible spin orientations associated with a given linear momentum, to be immediately used with Majorana neutrinos as well.

consequence of the use of closure in the sum over the intermediate (virtual) nuclear states; this results in several cases in an assignment of a degree of forbiddenness one unit larger than Furry's. We have, in addition, also obtained the double beta angular correlation function of the two emitted electrons; this quantity may conceivably become susceptible to experimental observation. Comparison of any such experiments with the various forms for the double beta angular correlation function would then serve as a test of the assumptions regarding (1) the fundamental coupling between nucleons and leptons and (2) the nuclear spin and parity change in the double beta-transitions. Studies of this type in double beta-decay would obviously supplement analogous theoretical[6] and experimental[7] investigations into electron-neutrino angular correlation in single beta-decay.

RESULTS

As regards our quantitative results, they may be summarized as follows: Let $P(E, E', \theta)dEdE'2\pi\sin\theta d\theta$ represent the double beta-decay transition probability per unit time for emission of the two electrons with momenta making an angle one with the other between θ and $\theta+d\theta$, and with energies between mc^2E and $mc^2(E+dE)$, and between mc^2E' and $mc^2(E'+dE')$, respectively; also let $F(\theta)\cdot\frac{1}{2}\sin\theta d\theta$, $S(E)dE$, and τ, denote the angular correlation function, the electron energy spectrum, and the decay mean life. We then find as a result of a second-order perturbation calculation with $\mathcal{H}_{\text{inter}}$,[5] performed in accordance with the general scheme of Furry:[2] initial nucleus→intermediate (virtual) nucleus+(virtual) neutrino+electron→final nucleus+two electrons,[8]

$$P(E, E', \theta)dEdE'2\pi\sin\theta d\theta$$
$$=2^{-6}\pi^{-5}G^4(mc^2/\hbar)\left|\int\Psi_{\text{fin}}^*\sum_{k,j}(Q_kQ_j/x_{kj}^2)A_{kj}\Psi_{\text{ini}}\right|^2$$
$$\times\left(\frac{2\pi Z}{137}\frac{E}{p}\right)\left(\frac{2\pi Z}{137}\frac{E'}{p'}\right)$$
$$\times pEp'E'f_1(E, E')\delta(E_0-E-E')$$
$$\times\{1+[f_2(E,E')/f_1(E,E')]\cos\theta\}dEdE'\tfrac{1}{2}\sin\theta d\theta \quad (1)$$

$$F(\theta)=1+\{f_2(E, E')/f_1(E, E')\}\cos\theta$$
$$S(E)dE=dE\int\int P(E, E', \theta)dE'2\pi\sin\theta d\theta$$
$$=\text{const}E^2(E_0-E)^2f_1(E, E_0-E)dE$$
$$\tau^{-1}=\int_1^{E_0-1}S(E)dE,$$

with

$$A_{kj}=(\boldsymbol{\sigma}_k\times\boldsymbol{\sigma}_j)\cdot\mathbf{x}_{kj}/x_{kj};\quad f_1(E, E')=(1+1/EE');$$
$$f_2(E, E')=(p/E)(p'/E');$$

allowed; $\Delta J=0$, "yes"; T, A. (2)

$$A_{kj}=\tfrac{1}{2}(\boldsymbol{\sigma}_k\cdot\boldsymbol{\sigma}_j)^n x_{kj};\quad f_1(E, E')=(E-E')^2(1-1/EE')$$
$$+(2/n')(E-E')(p^2/E-p'^2/E')+(1/n')^2$$
$$\times([p^2+p'^2](1+1/EE')-2(p^2/E)(p'^2/E));$$
$$f_2(E, E')=(1+2/n')(E-E')^2(p/E)(p'/E')$$
$$+(1/n')^2\{[p^2+p'^2](p/E)(p'/E')$$
$$-2pp'(1+1/EE')\};$$

first-forbidden; $\Delta J=0$, "no";
$T(n=1, n'=9)$, $A(n=1, n'=-9)$,
$V(n=0, n'=3)$, $S(n=0, n'=-3)$ (3)

$$[\mathbf{A}_{kj}]_T=-(\boldsymbol{\sigma}_k\times\boldsymbol{\sigma}_j)B_{kj}+(\boldsymbol{\sigma}_k\times\boldsymbol{\sigma}_j)\cdot(\mathbf{x}_{kj}/x_{kj})$$
$$\times\{(\mathbf{x}_k+\mathbf{x}_j)/|\mathbf{x}_k+\mathbf{x}_j|\}\left(\frac{1}{2}\frac{Z}{137}\right)$$
$$-\tfrac{1}{3}(\boldsymbol{\alpha}_k\cdot\boldsymbol{\sigma}_j-\boldsymbol{\alpha}_j\cdot\boldsymbol{\sigma}_k)(\mathbf{x}_{kj}/x_{kj}),$$
$$[\mathbf{A}_{kj}]_A=(\boldsymbol{\sigma}_k\times\boldsymbol{\sigma}_j)B_{kj}+(\boldsymbol{\sigma}_k\times\boldsymbol{\sigma}_j)\cdot(\mathbf{x}_{kj}/x_{kj})$$
$$\times\{(\mathbf{x}_k+\mathbf{x}_j)/|\mathbf{x}_k+\mathbf{x}_j|\}\left(\frac{1}{2}\frac{Z}{137}\right)$$
$$+(1/3i)(\boldsymbol{\alpha}_k\cdot\boldsymbol{\sigma}_k\boldsymbol{\sigma}_j-\boldsymbol{\alpha}_j\cdot\boldsymbol{\sigma}_j\boldsymbol{\sigma}_k)\times(\mathbf{x}_{kj}/x_{kj}),$$
$$[\mathbf{A}_{kj}]_V=(\boldsymbol{\alpha}_k-\boldsymbol{\alpha}_j)\times(\mathbf{x}_{kj}/x_{kj});$$
$$f_1(E, E')=(1-1/EE');$$
$$f_2(E, E')=-\tfrac{1}{3}(p/E)(p'/E');$$

first-forbidden; $\Delta J=0, \pm1$,
but not $0\to0$, "no"; T, A, V. (4)

In Eqs. (1)–(4), S, T, V, A denote scalar, vector, tensor, and axial vector couplings, respectively; $p=(E^2-1)^{\frac{1}{2}}$, $p'=[(E')^2-1]^{\frac{1}{2}}$; $mc^2(E_0-2)$ is the energy equivalent of the atomic mass difference available for the transition; G is the dimensionless nucleon-lepton coupling constant; we take $G=4\times10^{-12}$ on the basis of the theory of single beta-decay and the observed 12.8-min neutron half-life.[9] Within the square of the nuclear matrix element,[10] Ψ_{ini} and Ψ_{fin} are the wave functions

[6] E. Grueling and M. L. Meeks, Phys. Rev. **82**, 531 (1951); M. E. Rose, Phys. Rev. **75**, 1444 (1949); D. R. Hamilton **71**, 456 (1947).

[7] C. S. Sherwin, Phys. Rev. **82**, 52 (1951); Allen, Paneth, and Morrish, Phys. Rev. **75**, 570 (1949).

[8] Our double beta-decay transition probability amplitude, P.A., calculated in accordance with the above scheme, involves a sum over all linear momenta and spin orientations of the (virtual) neutrino and over all intermediate (virtual) nuclear states, n; the latter sum is performed by closure, after the energy eigenvalues of the various states n in the energy denominators of the second-order perturbation expression for the P.A. are replaced by an average effective energy (see reference 12). As a consequence, the operator connecting Ψ_{ini} and Ψ_{fin} in the ultimate formula for the P.A. involves a product of two sums in the individual nucleon position, spin, and charge operators, i.e., involves, as is seen in Eqs. (1)–(4), a sum of terms each of which refers to a pair of nucleons.

[9] J. M. Robson, Phys. Rev. **83**, 349 (1951). In our numerical estimate of G, the dominant nucleon-lepton interaction has been taken as tensor (or axial vector), see reference 14.

[10] Summed over all final nuclear spin orientation substates and averaged over all such initial substates.

of the initial and final nuclear states involved in the decay; Q_k, Q_j; $\boldsymbol{\sigma}_k$, $\boldsymbol{\sigma}_j$; $\boldsymbol{\alpha}_k$, $\boldsymbol{\alpha}_j$; $(\hbar/mc)x_{kj}=(\hbar/mc)|\mathbf{x}_k-\mathbf{x}_j|$ are the neutron-to-proton transformation operators, the Pauli spin operators, the Dirac velocity operators and the distance vectors, for the kth and jth nucleons;[11] $\langle B_{kj}\rangle_{\text{Av}}=-\langle B_{jk}\rangle_{\text{Av}}\approx[(m/2M)\langle x_{kj}\rangle_{\text{Av}}]$.[12] The effect of the nuclear Coulomb field on the spinor wave functions of the emitted electrons, and so on the energy spectrum and the angular correlation function, has been considered and is approximately described (for $Z/137\approx\frac{1}{4}-\frac{1}{2}$, and $E_0\approx 3-6$) by the Z-containing terms in Eqs. (1)–(4); in particular in Eq. (4) we take:

sum of last two terms of $\langle[A_{kj}]_T\rangle_{\text{Av}}$ or of $\langle[A_{kj}]_A\rangle_{\text{Av}}$

$$\gg\langle(\boldsymbol{\sigma}_k\times\boldsymbol{\sigma}_j)\cdot(\mathbf{x}_{kj}/x_{kj})(\mathbf{x}_k+\mathbf{x}_j)\cdot(\mathbf{p}+\mathbf{p}')\rangle_{\text{Av}},$$

which results in an $F(\theta)$ independent of Z.

In the case more or less relevant to the experiments: $Z\approx 50$, and $mc^2(E_0-2)\approx 1.5$ Mev, the Eqs. (1)–(4) yield $\tau\approx 10^{12}$ yr and $\tau\approx 10^{16}$ yr, for the allowed and first-forbidden transitions, respectively.[13] These values of τ are some hundred times shorter than those given by Furry,[2] in all cases where the corresponding matrix elements are similarly estimated; the discrepancy is largely due to his use of the too small value of G which was then current.

DISCUSSION

Recent work in single beta-decay indicates that the dominant nucleon-lepton coupling type is either axial vector or tensor, and in fact, more probably the latter.[14] Moreover, it is expected from considerations of nuclear stability that both the parent and the daughter nuclei in all double beta-processes are of the even-even type, and indeed, the two examples reported thus far conform to this rule. Now the ground states of all even-even nuclei have $J=0$ and, quite probably, even parity; the ground state→ground state double beta-transition is then not allowed but rather first forbidden, with $F(\theta)$ and $S(E)$ given by Eqs. (1), (3)—i.e., in rough approximation, $F(\theta)\approx 1+\cos\theta$. The reported 10^{16}-yr ${}_{50}\text{Sn}^{124}\rightarrow{}_{52}\text{Te}^{124}$ transition[3] may well be either of this first-forbidden type or of the first-forbidden type of Eq. (4); in the latter case the transition goes to an excited state of the daughter nucleus and has $F(\theta)=1-\frac{1}{3}(p/E)(p'/E')(1-1/EE')^{-1}\cos\theta\approx 1-\frac{1}{3}\cos\theta$; $S(E)=\text{const}E^2(E_0-E)^2[1-1/E(E_0-E)]$. The reported 10^{21}-yr ${}_{52}\text{Te}^{130}\rightarrow{}_{54}\text{Xe}^{130}$ transition[4] can be understood as a first-forbidden double beta-process with Majorana neutrinos, if we make the admittedly improbable assumption that the available energy is much less than 1.5 Mev; e.g., this transition may be of the ground state→ground state type with $mc^2(E_0-2)\approx 150$ kev [Eqs. (1), (3)]. In addition, we may remark that it is unlikely that it will be possible to observe the 10^{12}-yr allowed transition of Eq. (2) since low-lying states in even-even nuclei with $J=0$ and odd parity, in particular first-excited states of this kind, seem to be very rare.[15]

One further comment should be made. The 10^{21} yr-${}_{52}\text{Te}^{130}\rightarrow{}_{54}\text{Xe}^{130}$ decay fits the half-life expected from the Dirac neutrino theory[1] if $G=4\times 10^{-12}$ is used, if $mc^2(E_0-2)\approx 2$ Mev, and if a ground state→ground state transition (which is allowed on the Dirac theory) is assumed. Allowed double beta-decay half-lives$\approx 10^{21}$-yr would, of course, make double beta-angular correlation experiments quite impracticable for a long time to come, so that a decision on the feasibility of such experiments seems to rest on any future laboratory confirmation or refutation of Fireman's 10^{16}-yr result.†

[11] Strictly speaking, one should write $\beta_k\boldsymbol{\alpha}_k$, $\beta_j\boldsymbol{\alpha}_j$ instead of $\boldsymbol{\alpha}_k$, $\boldsymbol{\alpha}_j$ in the last term of $[\mathbf{A}_{kj}]_T$ since the nonrelativistic form of the former is different from that of the latter.

[12] The quantity $\langle B_{kj}\rangle_{\text{Av}}$ is, in order of magnitude, essentially the average effective energy of the intermediate (virtual) nuclear states, divided by $mc^2\epsilon_{\text{Av}}$, the average effective energy of the (virtually) emitted and reabsorbed neutrino associated with the neutron to proton transformation of the kth and jth nucleons. As is most easily seen from the uncertainty principle, one has $\epsilon_{\text{Av}}\approx\{\langle x_{kj}\rangle_{\text{Av}}\}^{-1}$. As indicated in footnote 8, B_{kj} enters into Eqs. (1), (4) as a result of our use of closure in the sum over the intermediate (virtual) nuclear states. To obtain a numerical estimate, we have: $\langle B_{kj}\rangle_{\text{Av}}\approx(mc\epsilon_{\text{Av}})^2/2Mmc^2\epsilon_{\text{Av}}\approx m/2M\langle x_{kj}\rangle_{\text{Av}}\approx 1/75$, with M the nucleon mass, and $\langle x_{kj}\rangle_{\text{Av}}$ taken $\approx$ {nuclear radius for $Z=50$} $\div(\hbar/mc)\approx 1/50$. Thus all of the individual terms in $\langle[A_{kj}]_T\rangle_{\text{Av}}$, $\langle[A_{kj}]_A\rangle_{\text{Av}}$, $\langle[A_{kj}]_V\rangle_{\text{Av}}$, of Eq. (4) are, to within a factor of perhaps 10, of the same order of magnitude, which, moreover, is about that of $\langle A_{kj}\rangle_{\text{Av}}$ in Eq. (3). In Furry's estimates (see reference 2), however, the quantity analogous to $\langle B_{kj}\rangle_{\text{Av}}$ is, in effect, taken ≈ 1; as a result the tensor or axial vector transition in Eq. (4) is classified by him as allowed rather than first forbidden.

[13] We are not writing out here our calculated $P(E, E', \theta)$ for first-forbidden transitions with $\Delta J=2$, 3, 4, "no." These also have $\tau\approx 10^{16}$ yr. We have not considered nucleon-lepton pseudoscalar coupling since its need for the interpretation of single beta-decay is as yet unsettled (see however, A. G. Petschek and R. E. Marshak, Phys. Rev. **85**, 698 (1952)). On the other hand, a dominant pseudoscalar coupling is completely ruled out since it would give a free neutron decay spectrum quite different in shape from the Fermi allowed, in contradiction to the experiment of reference 9. In the interest of simplicity, we have not considered various possible linear combinations of the couplings.

[14] L. M. Langer and R. J. D. Moffat, Phys. Rev. **82**, 635 (1951); H. W. Fulbright and J. C. D. Milton, Phys. Rev. **82**, 635 (1951). However, E. Feenberg and G. Trigg have stressed that the interpretation of the comparative half-lives in the single beta-decay allowed-favored transitions demands an appreciable admixture of vector and/or scalar coupling. Direct experimental evidence for such an admixture has recently been adduced by R. Sherr's discovery of single beta-transitions in C^{10} and O^{14}, most reasonably interpreted as $0\rightarrow 0$.

[15] This is indicated, at least for first-excited states, in M. Goldhaber and A. W. Sunyar, Phys. Rev. **83**, 906 (1951), Sec. V and Fig. 17.

† *Note added in proof.*—Very recent mass spectroscopic determinations of the mass of ${}_{52}\text{Te}^{130}$ [Duckworth, Kegley, Olson, and Stanford, Phys. Rev. **83**, 1114 (1951)] and of the mass of ${}_{54}\text{Xe}^{130}$ [Halsted, Bull. Am. Phys. Soc. **26**, No. 6 (1951), paper L13; Hays, Richards, and Goudsmit, Phys. Rev. **84**, 824 (1951)] yield $mc^2(E_0-2)=1.8$ Mev, thus providing evidence in favor of the Dirac theory. Further possible evidence in favor of the Dirac theory is provided by the work of W. Libby (reported by Inghram, reference 4) and by that of J. S. Lawson, Jr. [Phys. Rev. **81**, 299 (A)(1951)] who have obtained negative results in a study of the double beta-decay of ${}_{50}\text{Sn}^{124}$, with corresponding estimated lower limits on its half-life of 10^{17}-yr and 10^{16}-yr, respectively.