Total Cross Sections of 60-Mev Mesons in Hydrogen and Deuterium*

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(Received November 16, 1951)

The total cross sections of positive and negative mesons of average energy 58 Mev have been measured in hydrogen and deuterium. The results are

$$\sigma(\pi^+ + p) = 27.8 \pm 2.5 \text{ mb}$$
$$\sigma(\pi^- + p) = 17.6 \pm 2.2 \text{ mb}$$
$$\sigma(\pi^+ + d) = 38.3 \pm 3.1 \text{ mb}$$
$$\sigma(\pi^- + d) = 32.8 \pm 3.1 \text{ mb.}$$

The deuteron cross sections are related to the elementary scattering amplitudes in the impulse approximation, and the observed interference is compared to that predicted in weak coupling meson theory.

TOTAL interaction cross sections of π^- and π^+ mesons with average energy of 58 Mev in hydrogen and deuterium have been measured, using the scintillation counter arrangement and analysis of the beams described in a previous report on total cross sections of 85-Mev π^- mesons.[1] In the present experiment, the final $4\frac{1}{2}$-inch diameter counter was placed $5\frac{1}{2}$ inches behind the center of the absorber, so that "total cross section" applies to all interactions except those in which a meson is scattered into a forward angle of less than 22 degrees.

The hydrogen cross sections were found from $CH_2 - C$ subtractions, and the results were combined with those of $D_2O - H_2O$ subtractions to yield the deuterium cross sections. The mesons enter the hydrogenous materials with an energy of 72 Mev, and leave with an energy of 40 Mev. The carbon and polyethylene absorbers contain the same weight of carbon. A strong energy dependence found experimentally for the total cross section of π^- mesons in carbon[2] leads to a substantial correction in the results of the $CH_2 - C$ subtraction.[3] The results, corrected also for a measured 6 percent μ-meson beam component which is assumed non-interacting, are as follows:

$\sigma(\pi^+ + p)$	$\sigma(\pi^+ + d)$
27.8±2.5 mb	38.3±3.1 mb
$\sigma(\pi^- + p)$	$\sigma(\pi^- + d)$
17.6±2.2 mb	32.8±3.1 mb.

The errors combine statistical standard deviation and an estimated 20 percent maximum error in the energy dependence correction. The cross section for positive pions in hydrogen is greater than that for negative pions, outside of possible experimental error. This is perhaps surprising, since the charge exchange scattering is possible only for negative pions. It is not in conflict, however, with a possible charge symmetry: $\pi^+ + p \leftrightarrow \pi^- + n$, etc. In fact, the deuterium cross sections for positive and negative mesons are approximately equal within experimental error, and we therefore assume charge symmetry.

The following reactions may contribute appreciably to the deuterium cross section: 1. Scattering $\pi^+ + d \to n + p + \pi^+$ (inelastic), $\pi^+ + d \to d + \pi^+$ (elastic). 2. Charge exchange $\pi^+ + d \to \pi^0 + p + p$. 3. Radiative absorption $\pi^+ + d \to \gamma + p + p$. 4. Absorption $\pi^+ + d \to p + p$.

The last reaction has been measured independently,[4] and has a total cross section of 7±1 mb for mesons of this energy. The first three reactions therefore contribute $36 - 7 = 29 \pm 3$ mb. These reactions occur also with single nucleons, and except for interference effects, we would have

$$\sigma_d - \sigma_d^{\text{absorption}} = \sigma_p + \sigma_n = 45 \pm 3.5 \text{ mb.}$$

There exists therefore an interference in the deuteron of 16±5 mb, or 35 percent of the combined cross section of neutron and proton.

The analysis of this interference is possible from a phenomenological point of view, if it is assumed that the interaction amplitudes of the neutron and proton are linearly superposable and that the several possible interactions do not interfere with each other. The last assumption is probably justifiable, provided the cross sections in question are small compared to the deuteron, and this is actually the case. The problem is similar to that of photomeson production in deuterium. This has been treated by Chew and Lewis,[5] and we follow their

* Research supported by joint program of the ONR and AEC.

[1] Chedester, Isaacs, Sachs, and Steinberger, Phys. Rev. 82, 958 (1951).

[2] The total π^- cross section in carbon (not including elastic scattering into smaller angles than 50°) is 314±10 mb at 85 Mev, 237±10 mb at 60 Mev, and 195±10 mb at 40 Mev. The energy variation has also been measured in the geometry of the subtraction experiments for π^- mesons. The π^+ variation is assumed to be the same for the purpose of the corrections.

[3] This correction must also be applied to the previously reported result at 85 Mev (see reference 1). This raises the cross section of π^- mesons on hydrogen from 13 mb to approximately 21 mb. This value is somewhat in doubt because the variation of the carbon cross section has not been mesured as well in this energy region.

[4] Durbin, Loar, and Steinberger, Phys. Rev. 84, 581 (1951).

[5] G. Chew and H. W. Lewis, Phys. Rev. 84, 779 (1951). The method has also been applied to the meson scattering in deuterium by Fernbach, Green, and Watson, University of California Radiation Laboratory Report No. 1443 and to charge exchange scattering by B. Segall, Phys. Rev. 83, 1247 (1951).

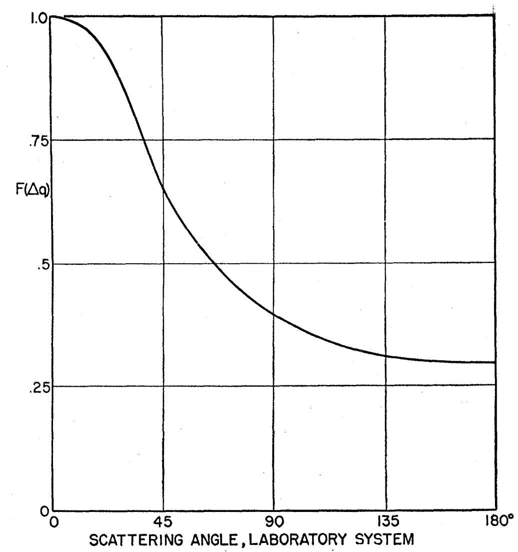

FIG. 1. The form factor $F(\Delta q)$.

treatment entirely, stating only the result and referring to their article for the justification of the method.

We write the elementary interactions of the positive pion and nucleon

$$I_s = s^p + \mathbf{S}^p\cdot\boldsymbol{\sigma}^p + s^n + \mathbf{S}^n\cdot\boldsymbol{\sigma}^n \text{ (scattering)},$$

$$I_c = c^n + \mathbf{C}^n\cdot\boldsymbol{\sigma}^n \text{ (charge exchange scattering)},$$

$$I_r = r^n + \mathbf{R}^n\cdot\boldsymbol{\sigma}^n \text{ (radiative absorption)},$$

where $\boldsymbol{\sigma}^n$ and $\boldsymbol{\sigma}^p$ are the Pauli spin operators for neutron and proton, **S**, **C**, **R** are the spin interaction amplitudes, and s, c, r are the spin independent amplitudes. s, c, r, **S**, **C**, and **R** are functions of meson energy and reaction product angle. They are normalized so that the free nucleon cross sections are simply

$$d\sigma^p/d\omega = |s^p|^2 + |\mathbf{S}^p|^2, \quad d\sigma_s^n/d\omega = |s^n|^2 + |\mathbf{S}^n|^2,$$

$$d\sigma_c^n/d\omega = |c^n|^2 + |\mathbf{C}^n|^2, \quad d\sigma_r^n/d\omega = |r^n|^2 + |\mathbf{R}^n|^2.$$

If, in the scattering on deuterium, we are not interested in the state of the final nucleon system, then it is valid[5] to apply a closure approximation in summing over the final nucleon states. Neglecting the deuteron D states, the result is

$$d\sigma_s^d/d\omega = d\sigma_s^p/d\omega + d\sigma_s^n/d\omega + 2F(\Delta q)[\mathrm{Re}(s^{p*}s^n) + \tfrac{1}{3}\,\mathrm{Re}(\mathbf{S}^{p*}\cdot\mathbf{S}^n)], \quad (1)$$

$$d\sigma_c^d/d\omega = d\sigma_c^n/d\omega - F(\Delta q)[|c^n|^2 + \tfrac{1}{3}|\mathbf{C}^n|^2], \quad (2)$$

$$d\sigma_r^d/d\omega = d\sigma_r^n/d\omega - F(\Delta q)[|r^n|^2 + \tfrac{1}{3}|\mathbf{R}^n|^2], \quad (3)$$

where

$$F(\Delta q) = \int \mu_d^2(r) e^{i\Delta\mathbf{q}\cdot\mathbf{r}} d\mathbf{r},$$

$\Delta q = |$momentum of incoming pion—momentum of outgoing pion or γ-ray$|$,

$\mu_d =$ deuteron wave function (in the calculations we use $\mu_d(r) \propto (e^{-\alpha r} - e^{-\beta r})$, $\alpha = (ME_b/\hbar)^{\frac{1}{2}}$, $\beta = 7\alpha$),

$E_b =$ deuteron binding energy.

$F(\Delta q)$ is the form factor which measures the interference of the nucleons, either because of the Pauli principle (reactions 2 and 3), or because the scattering takes place on two centers (reaction 1). $F(\Delta q)$ has been evaluated as a function of the angle of the outgoing meson (Fig. 1). For reaction 3 the angular dependence of $F(\Delta q)$ is slightly different, but, as will be seen, this reaction contributes very little.

Because the angular dependence of the interference terms is not known, the angular integration can only be carried out approximately. We note, however, that

$$(1/4\pi)\int F(\Delta q) d\omega = 0.46;$$

$$(3/4\pi)\int F(\Delta q)\cos^2\theta d\omega = 0.51;$$

$$(3/8\pi)\int F(\Delta q)\sin^2\theta d\omega = 0.44.$$

No great error is introduced if these angular distributions are replaced by their average. We therefore write tentatively

$$\sigma^d = \sigma^p + \sigma^n + \sigma(\text{absorption}) + 0.46\times 4\pi[\overline{2\,\mathrm{Re}(s^{p*}s^n)} + \overline{\tfrac{2}{3}\,\mathrm{Re}(\mathbf{S}^{p*}\cdot\mathbf{S}^n)} - \overline{|c^n|^2} - \overline{\tfrac{1}{3}|\mathbf{C}^n|^2} - \overline{|r^n|^2} - \overline{\tfrac{1}{3}|\mathbf{R}^n|^2}].$$

The only interference terms for which independent evidence exists are r and R. In this case the inverse reaction has been measured[6] and gives the total cross section by detailed balance, as well as evidence that $r^n \ll |R^n|$. We therefore set

$$4\pi\overline{|\mathbf{R}^n|^2} = \sigma_r^n = 0.8 \pm 0.2 \text{ mb}.$$

Then

$$4\pi\times 0.46[\overline{2\,\mathrm{Re}(s^{p*}s^n)} + \overline{\tfrac{2}{3}\,\mathrm{Re}(\mathbf{S}^{p*}\cdot\mathbf{S}^n)} - \overline{|c^n|^2} - \overline{\tfrac{1}{3}|\mathbf{C}^n|^2}] = -16 \pm 5 \text{ mb}.$$

The charge exchange term contributes always destructively to the interference, from between 15 to 45 percent of the elementary charge exchange cross section σ^c, depending only on the relative importance of the spin

[6] J. Steinberger and A. S. Bishop, Phys. Rev. (to be published).

dependent and independent terms.[7] The scattering terms may be either destructive or constructive. We point out only that in the pseudoscalar perturbation theory with pseudoscalar coupling,

$$4\pi\,\mathrm{Re}(s^{p*}s^{n})=\sigma^{p}=\sigma_{s}^{n};\quad \mathrm{Re}(\mathbf{S}^{p*}\cdot\mathbf{S}^{n})=0.$$

The interference is expected to be strongly constructive, in disagreement with the experimental result. The pseudovector coupling results, neglecting nuclear recoil, are

$$4\pi\,\mathrm{Re}(s^{p*}s^{n})=-\sigma^{n}\cos^{2}\theta=-\sigma^{n}\cos^{2}\theta,$$

$$4\pi\,\mathrm{Re}(\mathbf{S}^{p}\cdot\mathbf{S}^{n})=\sigma^{p}\sin^{2}\theta=\sigma^{n}\sin^{2}\theta,$$

$$2\int d\omega F(\Delta q)[\mathrm{Re}(s^{p*}s^{n})+\tfrac{1}{3}\mathrm{Re}(\mathbf{S}^{n*}\cdot\mathbf{S}^{n})] =-0.1\sigma^{p}=-0.1\sigma^{n}\approx-2.5\text{ mb}.$$

The net interference is destructive, but smaller than the experimentally observed interference.

[7] The charge exchange cross section is necessarily less than the cross section of π^{-} on p; the interference due to the charge exchange reaction is therefore at most 8 mb, and this only if all scattering of negative pions or protons were charge exchange.

PHYSICAL REVIEW VOLUME 85, NUMBER 5 MARCH 1, 1952

The Beta-Spectrum of RaD

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(Received October 15, 1951)

The electronic radiations from a very thin source of RaD are analyzed by means of a proportional counter, operating as a 2π spectrometer. The β-rays of the continuum which coincide with the photoelectrons of highest energy form the tail on the pulse distribution which is analyzed to isolate the β-particles and to compare their spectrum with Fermi theory. The disintegration energy is found to be 64.5 ± 2.5 kev and the limiting energy of the β-rays$=18\pm2.5$ kev. The form of the spectrum agrees with the theoretical shape for an allowed transition and $E_0=18$ kev. The presence of a complex system of soft photoelectron peaks, probably emitted mostly in cascade, is deduced from the remarkable shape of the pulse spectrum at low energies (5 to 25 kev).

INTRODUCTION

THE unsatisfactory situation existing as late as 1949 in our understanding of the decay of RaD has been well summarized by Feather.[1] Isolation of the continuous β-spectrum had not been achieved and the rather large number of low intensity γ-rays detected (energies 7.3, 16.1, 23.2, 31.3, 37, 42.6, 46.7 kev) did not readily suggest a unique system of levels. Feather, however, points out two significant groupings of quantum energies. Very recent work by Cranberg,[2] Frilley *et al.*,[3] and Butt and Brodie[4] has extended knowledge of the decay, but they have not been concerned with the main objective of the present work, which was the examination of the β-spectrum. Most observers[5, 6] agree that the β-rays are very soft, but even the limiting energy has not been well defined, estimates ranging from $\sim$15 to $\sim$40 kev.

The proportional tube spectrometer is clearly suited to the study of the soft radiations of RaD and has been employed in examining the L x-rays and soft γ-rays.[7] Early work on the particles proved somewhat difficult, mainly on account of difficulties in preparing gaseous or solid sources effectively free of RaE and RaF, both of which contributed very large disturbing pulses. More recently, however, relatively clean sources ($\sim$97 percent) have been separated following the method used by Cranberg[2] and this has greatly assisted in the successful application of the instrument. It has been found possible, by direct examination of the particles emitted from a thin source ($\sim$20 μg of chloride) mounted on thin foils (Nylon $\sim$50 μg/cm^2 and aluminium $\sim$1.6 mg/cm^2), to separate the β-rays from the relatively very intense and complex system of photoelectron lines of low energy. The separation was sufficiently good to permit comparison of the shape of the spectrum with Fermi theory. The tube with source at an aperture in the cathode constitutes a spectrometer of 2π acceptance angle.

APPARATUS

The proportional counter, 1.4 inches in diameter and 8 inches in fully operating length[8] was enclosed in a cylindrical vessel C, Fig. 1, into which the thin source could be slipped. The vessel was filled to a pressure of one atmosphere and the mixture (ethylene+16 cm of argon) was such that the efficiency of the L x-rays of RaE from the source was <5 percent. The spectrometer was thus rendered responsive chiefly to photoelectrons and β-rays, but it was, of course, sensitive to the very soft M x-radiations. It integrated the ionizing events

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