values as shown in the plot of p. 194, reference 4, secured from density plots. These microphotometer plots are thus essentially lines in absorption.

[1] F. R. Hirsh, Jr., Phys. Rev. **38**, 914 (1931).
[2] F. R. Hirsh, Jr., Physica **XVI**, 377 (1950).
[3] F. R. Hirsh, Jr., Phys. Rev. **62**, 137 (1942).
[4] F. R. Hirsh, Jr., Phys. Rev. **50**, 191 (1936).
[5] D. Coster and A. Bril, Physica **9**, 84 (1942).
[6] D. Coster and R. DeL. Kronig, Physica **2**, 13 (1935).
[7] A similar curve for the Auger effect has been presented by J. N. Cooper, Phys. Rev. **61**, 284 (1942).

Comment on the "Impulse Approximation"

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The expansion in powers of U' mentioned just above Eq. (33) of the paper "Impulse Approximation"[1] by Chew and Wick can be achieved by the following, perhaps not very rigorous, procedure.

First define a generalized Møller operator Ψ by the equation,

$$\Psi\phi_a=\psi_a. \tag{1}$$

Because of the fact that the states ϕ_a form a complete system, Eq. (1) defines Ψ completely. One can then write Eq. (12) of I.A. in the more symmetrical form,

$$T_{ba}=(\phi_b,\,V\Psi\phi_a) \tag{2}$$

or

$$T=V\Psi. \tag{3}$$

We now intend to expand the operator T in powers of U. Needless to say, the matrix elements T_{ba} of T cannot be so expanded, since the wave functions ϕ_a and ϕ_b contain as a factor quantized states of the bound system with the potential U. It turns out, however, that Eq. (2) separates out successfully the features in T_{ba} which prevent the expansion.

A simple expression for the operator Ψ is obtained from the remark that ψ_a is the wave function that evolves, under the action of the full Hamiltonian $H=K+U+V$, from an initial state $e^{-iH_0t_1}\phi_a$ at a remote time t_1 in the past. That is

$$\psi_a(t)=\lim_{t_1\to-\infty} e^{-iH(t-t_1)}e^{-iH_0t_1}\phi_a, \tag{4}$$

where $H_0=K+U$. It will be understood that if (4) has to be applied to a state ϕ_a of exactly defined energy, the following sequence has to be observed: first apply (4) to a wave packet state ϕ_a and take the limit to $t_1\to-\infty$, then let ϕ_a tend to the desired state. In the following the "lim" sign will often be omitted.

In Eq. (1) ψ_a was written for the space part of the wave function, i.e., $\psi_a(t)=e^{-iHt}\psi_a$. Hence ψ_a is obtained from (4) simply by setting $t=0$. Hence the operator Ψ of Eq. (1) is

$$\Psi=e^{iHt_1}e^{-iH_0t_1} \tag{5}$$

(in the limit $t_1\to-\infty$).

This can now be expanded in powers of U (although this may seem at first questionable since U is multiplied by a large t_1!) by the customary formulas of perturbation theory.[2] For instance, to the first order in U;

$$\Psi=e^{i(K+V)t_1}e^{-iKt_1}+ie^{i(K+V)t_1}\int_{t_1}^0 e^{-iKt}Ue^{iK(t-t_1)}dt$$
$$-i\int_{t_1}^0 e^{i(K+V)t}Ue^{-i(K+V)(t-t_1)}dte^{-iKt_1}+\cdots. \tag{6}$$

Now the first term $e^{i(K+V)t_1}e^{-iKt_1}$ is the analog of (5) for the problem without binding, that is the operator Ω of I.A. The last term is also seen to contain Ω. The middle term can be written

$$ie^{i(K+V)t_1}e^{-iKt_1}\int_{t_1}^0 dte^{iK(t_1-t)}Ue^{\ K(t-t_1)}, \tag{7}$$

and then transformed by $t_1-t=s$ into

$$ie^{i(K+V)t_1}e^{-iKt}\int_{t_1}^0 e^{+iKs}Ue^{-iKs}ds. \tag{8}$$

Here again one recognizes to the left, the operator which tends to Ω when $t_1\to-\infty$. Although the operator is here applied after the integral term, which also depends on t_1, it may be shown that the limit can be taken on the two factors independently.[3] Then,

$$\Psi=\Omega+i\Omega\int_{-\infty}^0 e^{iKt}Ue^{-iKt}dt$$
$$-i\int_{-\infty}^0 e^{i(K+V)t}Ue^{-i(K+V)t}\Omega dt+\cdots.$$

With $f(K+V)\Omega=\Omega f(K)$, this transforms easily to

$$\Psi=\Omega+i\int_{-\infty}^0 e^{i(K+V)t}[\Omega,\,U]e^{-iKt}dt+\cdots. \tag{9}$$

By inserting (9) into (3) and writing $V\Omega=\mathbf{t}$, we have finally,

$$T=\mathbf{t}+i\int_{-\infty}^0 Ve^{i(K+V)t}[\Omega,\,U]e^{-iKt}dt+\cdots. \tag{10}$$

The second-order term has also been evaluated and simplified. It can be written, for instance,

$$V\int_{-\infty}^0 dt\int_{-\infty}^0 dse^{i(K+V)s}[U,\,e^{i(K+V)t}[\Omega,\,U]e^{-iKt}]e^{-iKs}. \tag{11}$$

The first-order term in Eq. (10) is not exactly the same as that given in I.A. but the structure of the terms is extremely similar, and it does not seem that any of the estimates previously made have to be modified.

These formulas also allow one to clarify the connection with the time-symmetrical treatment, see I.A., Eq. (31) and following lines. One can of course define an operator Ψ^- such that $\Psi^-\phi_a=\psi_a^-$ and a $T^-=\Omega^{-\dagger}V$ which is equivalent to T on the $K+U$ energy shell, i.e., $T_{ba}^-=T_{ba}$ if $E_a=E_b$. One finds

$$T^-=\mathbf{t}^--i\int_0^{+\infty} dte^{iKt}[\Omega^{-\dagger},\,U]e^{-i(K+V)t}V+\cdots. \tag{10'}$$

The equivalence between T and T^- can then be verified as follows. First show that $\mathbf{t}-\mathbf{t}^-=[\Omega^0,\,K]$ where $\Omega^0=\Omega+\Omega^{-\dagger}$. This identity shows that Ω^0, unlike Ω and Ω^-, is finite on the kinetic-energy shell (because $\mathbf{t}=\mathbf{t}^-$ thereon). Hence on the $K+U$ energy shell, $\mathbf{t}-\mathbf{t}^-=[\Omega^0,\,K+U]-[\Omega^0,\,U]=[U,\,\Omega^0]$, which vanishes when U goes to zero because Ω^0 remains finite. Thus in taking the difference between the expansions (10) and (10′), the zero-order terms nearly cancel leaving only a first-order residuum, which may be shown to be nearly canceled after some simple manipulations by the first-order terms in the expansions, etc.

[1] G. F. Chew and G. C. Wick, Phys. Rev. **85**, 636 (1952), quoted as I.A. in the following.
[2] The advantage of the time-dependent formulation lies just in the ease with which such expansions can be made (Schwinger, Feynman, etc.). See especially R. P. Feynman, Phys. Rev. **84**, 108 (1951). G. F. Chew and M. L. Goldberger, however, have kindly communicated to us a stationary method which gives results essentially equivalent to ours.
[3] The effect of the integral operator on a given finite wave packet is easily examined in the momentum representation, where it can be seen that the transformed wave function is again (even in the limit $t_1\to-\infty$) an essentially finite wave packet for which the identification of $e^{i(K+V)t_1}e^{-iKt_1}$ with Ω is legitimate.

Nonlinearities Resulting from Vacuum Polarization in Meson-Nucleon Interactions

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Investigations of the pseudoscalar interaction of pseudoscalar mesons with nucleons show that in addition to the usual divergences associated with mass and charge, there is a distinct ϕ^4 divergence which is associated with the scattering of mesons by mesons.[1,2] We wish to point out that to understand the origin of this effect, it is sufficient to consider a simple vacuum polarization calculation which takes account of the creation of virtual nucleon-antinucleon pairs by a prescribed slowly varying pseudoscalar meson field.