Auger Enhancement of the $M\alpha$ X-Ray Satellite Lines

F. R. HIRSH, JR.
Pasadena, California
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IN 1931 the writer published visual measurements[1] on the $M\alpha$- and $M\beta$-satellite lines. Much later, in 1950, the writer plotted up[2] the sharp maxima noted on the $M\beta$-density plots[3] and discovered that there were six distinct satellites on these plates.[1] In the hope of finding similar sharp maxima for the $M\alpha$-satellites, these plates of my 1931 thesis[1] were very kindly photometered by Professor Jesse Greenstein of the California Institute of Technology at the suggestion of Dr. Robert F. Bacher. The results are shown in Fig. 1. Each $M\alpha$-envelope line is the arithmetical average of three runs made on the top, middle, and bottom of each $M\alpha$-spectral line. The fact to be noted is the complete absence of sharp satellite maxima; this is caused by Auger broadening of the satellites, while in the case of the $M\beta$-satellites the Auger effect has been shown to be impossible.[3]

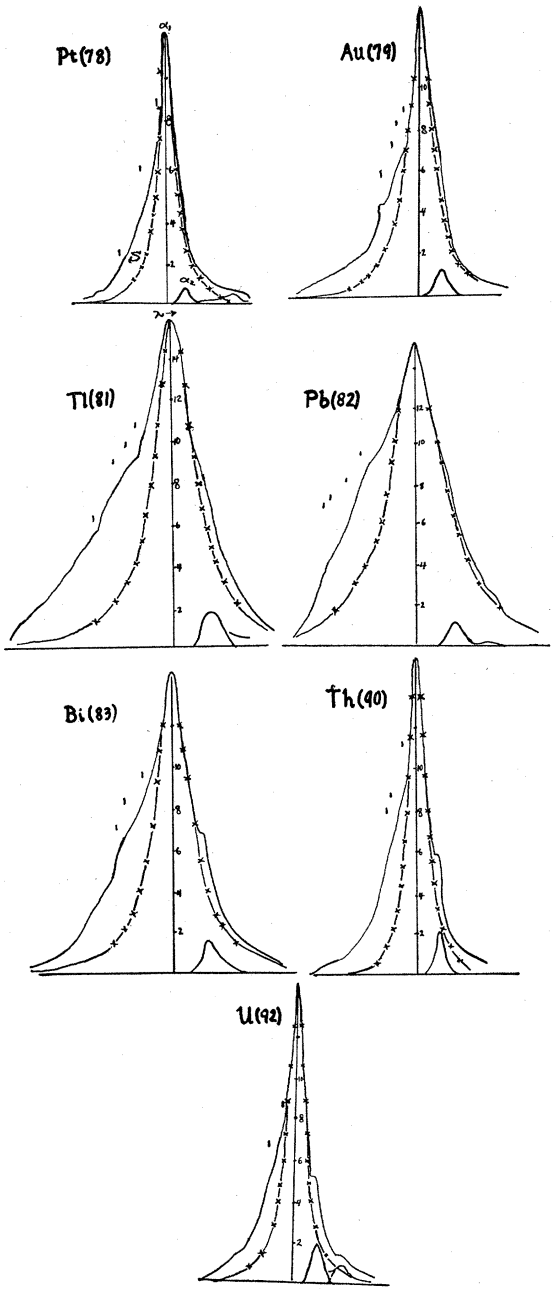

FIG. 1. Microphotometric records of plates of reference 1 fitted with "classical" $M\alpha_1$-lines, delineating satellite area S and the $M\alpha_2$-line which is subtracted off. Original ordinate and abscissa scale: one unit = $\frac{1}{2}$ inch. Vertical dashes on short wavelength side of lines (left) are visual measurements of reference 1.

TABLE I. The pertinent line data.

Plate No.	El. Z	a_2b_2/a_1b_1	$S/\pi a_1b_1$	$\alpha_2(\Delta\nu/R)\alpha_1$	Crystal
55	Pt(78)	0.04	0.17	0.4″	calcite
53	Au(79)	0.05	0.14	0.5	calcite
35	Tl(81)	0.05	0.28	0.8	calcite
27	Pb(82)	0.03	0.12	0.9	calcite
37	Bi(83)	0.04	0.21	0.8	calcite
41	Th(90)	0.05	0.33	0.5	quartz
47	U(92)	0.06	0.21	0.55	quartz

Each $M\alpha$-line envelope was fitted with a classical $M\alpha_1$-line so that $M\alpha_2$, subtracted off, had very closely the relative intensity $\alpha_2/\alpha_1=0.05$ (see Table I). The area at the left (short wavelength) side of each $M\alpha$-line has been assigned to energy of double ionization-satellites (denoted as S in the Pt $M\alpha$-envelope). The classical line formula is $y=a/[1+(x/b)^2]$, where a is the maximum ordinate and b the half-width at half maximum. The area under the curve is πab, and hence the relative intensity of two such lines is a_1b_1/a_2b_2. The relative energy of multiple to single ionization is $S/\pi a_1b_1$. In Table I are given the pertinent line data: plate No., element, atomic No., Z, a_2b_2/a_1b_1, $S/\pi a_1b_1$. In Fig. 2 is plotted $S/\pi a_1b_1$ *vs* Z; it will be noted there are two maxima of relative energy at $Z=81$ and near $Z=90$. The Auger crossovers I found[4] at $Z=88$ and 91 for the energy of the radiationless transition $M_{\mathrm{III}}\rightarrow M_{\mathrm{V}}$ and the ionization energy plots for the $N_{\mathrm{IV,V}}$ shells ($M_{\mathrm{V}}N_{\mathrm{IV,V}}\rightarrow N_{\mathrm{VII}}N_{\mathrm{IV,V}}$). Coster and Bril[5] find the upper crossover for ν/R, N_{V} with ν/R, $M_{\mathrm{III}}\rightarrow M_{\mathrm{V}}$ at $Z=93$ or 94. Thus quite confidently, one may assign the two intensity maxima in Fig. 2 to the crossovers of the ν/R values for $M_{\mathrm{III}}\rightarrow M_{\mathrm{V}}$ and the values of ν/R for the ionization energy of N_{IV} and N_{V}. (The intensity maximum should occur at 4 or 5 atomic numbers below the particular crossover.[6,7]) This is another bit of evidence in favor of the theory of Coster and Kronig.[6]

The use of microphotometer records to represent intensity may seem questionable, but it would seem to be not so naive. The function $y=a/[1+(x/b)^2]$ accurately represents the intensity plots of simple x-ray spectral lines. In this work, this function closely fits these average microphotometer curves (see Fig. 1). Moreover, the plot of Fig. 2 has exactly the same range of ordinate

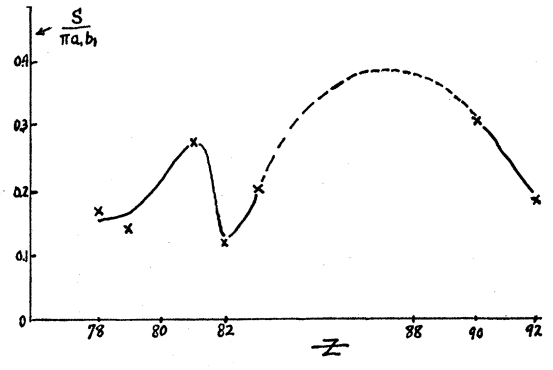

FIG. 2. Relative energy of double to single ionization plotted against atomic number, showing two maxima for the double Auger crossover.

values as shown in the plot of p. 194, reference 4, secured from density plots. These microphotometer plots are thus essentially lines in absorption.

[1] F. R. Hirsh, Jr., Phys. Rev. **38**, 914 (1931).
[2] F. R. Hirsh, Jr., Physica **XVI**, 377 (1950).
[3] F. R. Hirsh, Jr., Phys. Rev. **62**, 137 (1942).
[4] F. R. Hirsh, Jr., Phys. Rev. **50**, 191 (1936).
[5] D. Coster and A. Bril, Physica **9**, 84 (1942).
[6] D. Coster and R. DeL. Kronig, Physica **2**, 13 (1935).
[7] A similar curve for the Auger effect has been presented by J. N. Cooper, Phys. Rev. **61**, 284 (1942).

Comment on the "Impulse Approximation"

J. ASHKIN AND G. C. WICK
Carnegie Institute of Technology, Pittsburgh, Pennsylvania
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THE expansion in powers of U' mentioned just above Eq. (33) of the paper "Impulse Approximation"[1] by Chew and Wick can be achieved by the following, perhaps not very rigorous, procedure.

First define a generalized Møller operator Ψ by the equation,

$$\Psi\phi_a=\psi_a. \tag{1}$$

Because of the fact that the states ϕ_a form a complete system, Eq. (1) defines Ψ completely. One can then write Eq. (12) of I.A. in the more symmetrical form,

$$T_{ba}=(\phi_b, V\Psi\phi_a) \tag{2}$$

or

$$T=V\Psi. \tag{3}$$

We now intend to expand the operator T in powers of U. Needless to say, the matrix elements T_{ba} of T cannot be so expanded, since the wave functions ϕ_a and ϕ_b contain as a factor quantized states of the bound system with the potential U. It turns out, however, that Eq. (2) separates out successfully the features in T_{ba} which prevent the expansion.

A simple expression for the operator Ψ is obtained from the remark that ψ_a is the wave function that evolves, under the action of the full Hamiltonian $H=K+U+V$, from an initial state $e^{-iH_0t_1}\phi_a$ at a remote time t_1 in the past. That is

$$\psi_a(t)=\lim_{t_1\to-\infty} e^{-iH(t-t_1)}e^{-iH_0t_1}\phi_a, \tag{4}$$

where $H_0=K+U$. It will be understood that if (4) has to be applied to a state ϕ_a of exactly defined energy, the following sequence has to be observed: first apply (4) to a wave packet state ϕ_a and take the limit to $t_1\to-\infty$, then let ϕ_a tend to the desired state. In the following the "lim" sign will often be omitted.

In Eq. (1) ψ_a was written for the space part of the wave function, i.e., $\psi_a(t)=e^{-iHt}\psi_a$. Hence ψ_a is obtained from (4) simply by setting $t=0$. Hence the operator Ψ of Eq. (1) is

$$\Psi=e^{iHt_1}e^{-iH_0t_1} \tag{5}$$

(in the limit $t_1\to-\infty$).

This can now be expanded in powers of U (although this may seem at first questionable since U is multiplied by a large t_1!) by the customary formulas of perturbation theory.[2] For instance, to the first order in U;

$$\Psi=e^{i(K+V)t_1}e^{-iKt_1}+ie^{i(K+V)t_1}\int_{t_1}^{0}e^{-iKt}Ue^{iK(t-t_1)}dt$$
$$-i\int_{t_1}^{0}e^{i(K+V)t}Ue^{-i(K+V)(t-t_1)}dte^{-iKt_1}+\cdots. \tag{6}$$

Now the first term $e^{i(K+V)t_1}e^{-iKt_1}$ is the analog of (5) for the problem without binding, that is the operator Ω of I.A. The last term is also seen to contain Ω. The middle term can be written

$$ie^{i(K+V)t_1}e^{-iKt_1}\int_{t_1}^{0}dte^{iK(t_1-t)}Ue^{K(t-t_1)}, \tag{7}$$

and then transformed by $t_1-t=s$ into

$$ie^{i(K+V)t_1}e^{-iKt}\int_{t_1}^{0}e^{+iKs}Ue^{-iKs}ds. \tag{8}$$

Here again one recognizes to the left, the operator which tends to Ω when $t_1\to-\infty$. Although the operator is here applied after the integral term, which also depends on t_1, it may be shown that the limit can be taken on the two factors independently.[3] Then,

$$\Psi=\Omega+i\Omega\int_{-\infty}^{0}e^{iKt}Ue^{-iKt}dt$$
$$-i\int_{-\infty}^{0}e^{i(K+V)t}Ue^{-i(K+V)t}\Omega dt+\cdots.$$

With $f(K+V)\Omega=\Omega f(K)$, this transforms easily to

$$\Psi=\Omega+i\int_{-\infty}^{0}e^{i(K+V)t}[\Omega, U]e^{-iKt}dt+\cdots. \tag{9}$$

By inserting (9) into (3) and writing $V\Omega=\mathbf{t}$, we have finally,

$$T=\mathbf{t}+i\int_{-\infty}^{0}Ve^{i(K+V)t}[\Omega, U]e^{-iKt}dt+\cdots. \tag{10}$$

The second-order term has also been evaluated and simplified. It can be written, for instance,

$$V\int_{-\infty}^{0}dt\int_{-\infty}^{0}dse^{i(K+V)s}[U, e^{i(K+V)t}[\Omega, U]e^{-iKt}]e^{-iKs}. \tag{11}$$

The first-order term in Eq. (10) is not exactly the same as that given in I.A. but the structure of the terms is extremely similar, and it does not seem that any of the estimates previously made have to be modified.

These formulas also allow one to clarify the connection with the time-symmetrical treatment, see I.A., Eq. (31) and following lines. One can of course define an operator Ψ^- such that $\Psi^-\phi_a=\psi_a^-$ and a $T^-=\Omega^{-\dagger}V$ which is equivalent to T on the $K+U$ energy shell, i.e., $T_{ba}^-=T_{ba}$ if $E_a=E_b$. One finds

$$T^-=\mathbf{t}^--i\int_{0}^{+\infty}dte^{iKt}[\Omega^{-\dagger}, U]e^{-i(K+V)t}V+\cdots. \tag{10'}$$

The equivalence between T and T^- can then be verified as follows. First show that $\mathbf{t}-\mathbf{t}^-=[\Omega^0, K]$ where $\Omega^0=\Omega+\Omega^{-\dagger}$. This identity shows that Ω^0, unlike Ω and Ω^-, is finite on the kinetic-energy shell (because $\mathbf{t}=\mathbf{t}^-$ thereon). Hence on the $K+U$ energy shell, $\mathbf{t}-\mathbf{t}^-=[\Omega^0, K+U]-[\Omega^0, U]=[U, \Omega^0]$, which vanishes when U goes to zero because Ω^0 remains finite. Thus in taking the difference between the expansions (10) and (10'), the zero-order terms nearly cancel leaving only a first-order residuum, which may be shown to be nearly canceled after some simple manipulations by the first-order terms in the expansions, etc.

[1] G. F. Chew and G. C. Wick, Phys. Rev. **85**, 636 (1952), quoted as I.A. in the following.
[2] The advantage of the time-dependent formulation lies just in the ease with which such expansions can be made (Schwinger, Feynman, etc.). See especially R. P. Feynman, Phys. Rev. **84**, 108 (1951). G. F. Chew and M. L. Goldberger, however, have kindly communicated to us a stationary method which gives results essentially equivalent to ours.
[3] The effect of the integral operator on a given finite wave packet is easily examined in the momentum representation, where it can be seen that the transformed wave function is again (even in the limit $t_1\to-\infty$) an essentially finite wave packet for which the identification of $e^{i(K+V)t_1}e^{-iKt_1}$ with Ω is legitimate.

Nonlinearities Resulting from Vacuum Polarization in Meson-Nucleon Interactions

B. J. MALENKA
Department of Physics, Harvard University, Cambridge, Massachusetts
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INVESTIGATIONS of the pseudoscalar interaction of pseudoscalar mesons with nucleons show that in addition to the usual divergences associated with mass and charge, there is a distinct ϕ^4 divergence which is associated with the scattering of mesons by mesons.[1,2] We wish to point out that to understand the origin of this effect, it is sufficient to consider a simple vacuum polarization calculation which takes account of the creation of virtual nucleon-antinucleon pairs by a prescribed slowly varying pseudoscalar meson field.