mined on temperature of heat treatment; all the data are in substantial agreement. These equilibrium concentrations are, however, some 10 times larger than the values found for A_0. The solid line gives an activation energy of 27,000 cal or 1.2 ev and an extrapolated concentration for infinite temperature of 2.4×10^{21} cm^{-3}. The dashed line gives 32,000 cal or 1.4 ev and 4.55×10^{22} cm^{-3}, the concentration of germanium atoms which, Shockley has pointed out, the extrapolated concentration should approximately equal, whatever be the nature of the acceptors. This line fits the data fairly well, particularly in view of a comparatively slow quench of the ingot bar.

[1] H. C. Torrey and C. A. Whitmer, *Crystal Rectifiers* (McGraw-Hill Book Company, Inc., New York, 1948), pp. 365ff.; J. H. Scaff and H. C. Theuerer, Trans. Am. Inst. Mining. Met. Engrs. **189**, 59, J. Metals, TP2996E (1951).
[2] It is an unnecessary refinement, since the actual resistivity is in general appreciably less than the intrinsic resistivity, to deal with the exact expressions for the carrier concentrations in terms of these two resistivities.
[3] The electron and the hole mobilities are taken as 3600 and 1700 cm^2 volt^{-1} sec^{-1}; J. R. Haynes and W. Shockley, Phys. Rev. **81**, 835 (1951).
[4] The 5.0-ohm-cm sample was 1.8 cm in diameter, 0.4 cm thick, and polycrystalline, while the 18.5-ohm-cm one was a single-crystal bar 0.5 $\times1.0\times2.0$ cm.
[5] This sample was a polycrystalline prism about 3 cm long cut from an ingot.
[6] W. Jost, *Diffusion und chemische Reaktion in festen Stoffen* (T. Steinkopff, Leipzig, 1937).
[7] One sample was the polycrystalline prism used in the junction measurements, the other, a single-crystal bar.
[8] This polycrystalline bar was about 3 mm in cross section and 2 cm long.

The Lateral Spread of Cosmic-Ray Showers in Air and Lead

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PREVIOUS theoretical work[1-3] on the lateral spread of extensive air showers has been devoted almost exclusively to the electron-photon component. This has been partly the result of an early misconception of the nature of the primary component, but mainly to ignorance of the differential cross sections for nuclear collisions at ultrarelativistic energies. We recently tried to remedy this situation by deriving semiempirical cross sections suggested by analysis of data on the energy spectrum of nucleons in cosmic radiation. We found in this way[4] that the mean square angle of scatter of particles with energy U resulting from a nucleon-nucleon collision at ultrarelativistic energies should be $(U+U')/UU'$, where U' is the energy of the primary measured in proton-mass units. By tracing the development of a nucleon cascade within the nucleus, we have also obtained[5] the mean square angle of scatter in a nucleon-nucleus collision.

We have now applied these results to investigate the lateral development of the nucleon cascade in air. If $f(\mathbf{p},\mathbf{r})dp_1dp_2dp_3dr_1dr_2$ is the differential probability of finding a particle with momentum $\mathbf{p}$ at depth r_3 in the atmosphere and at a displacement (r_1, r_2) from the shower axis, one has

$$\frac{1}{k\delta(r_3)}\frac{\mathbf{p}}{p}\cdot\frac{\partial f(\mathbf{p})}{\partial \mathbf{r}}+f(\mathbf{p})=\int f(\mathbf{p}')n(\mathbf{p}',\mathbf{p})d\mathbf{p}', \tag{1}$$

where $n(\mathbf{p}',\mathbf{p})d\mathbf{p}$ is the differential probability of finding a particle with momentum $\mathbf{p}(d\mathbf{p})$ as a result of the collision of a nucleon of momentum $\mathbf{p}'$ with an air nucleus; $\delta(r_3)$ is the density of air, which in this application we have supposed to vary vertically as in an isothermal atmosphere; and $k=(1/75)$ cm^2/g.

We have solved Eq. (1) as far as required to compute the mean square lateral displacement $\langle r^2\rangle$, as a function of depth

$$\langle r^2\rangle=\frac{\int\int\int\int\int(r_1^2+r_2^2)fdp_1dp_2dp_3dr_1dr_2}{\int\int\int\int\int fdp_1dp_2dp_3dr_1dr_2} \tag{2}$$

for nucleons with energy exceeding any given value U. The only approximation which we found necessary was to neglect the mean fourth power of the angular deviation of the particles from the shower axis. The validity of this approximation we confirmed by calculating the mean square angular deviation. For particles with energies greater than U (which is assumed to exceed the geomagnetic cutoff) and assuming an integral power law with exponent 1.1 for the primary spectrum, the mean square angle is

$$\langle\chi^2\rangle=1.34U^{-1}(1-e^{-0.2\theta})(\text{radians})^2 \tag{3}$$

where θ is the depth measured in cascade units of 75 g/cm^2. This is obviously small for the ultrarelativistic energies which we consider.

The mean square distance from the shower axis was found to be

$$\langle r^2\rangle=162U^{-1}e^{-0.2\theta}\left\{\frac{0.2\theta}{1^2\cdot1!}+\frac{(0.2\theta)^2}{2^2\cdot2!}+\cdots\right\}\text{km}^2. \tag{4}$$

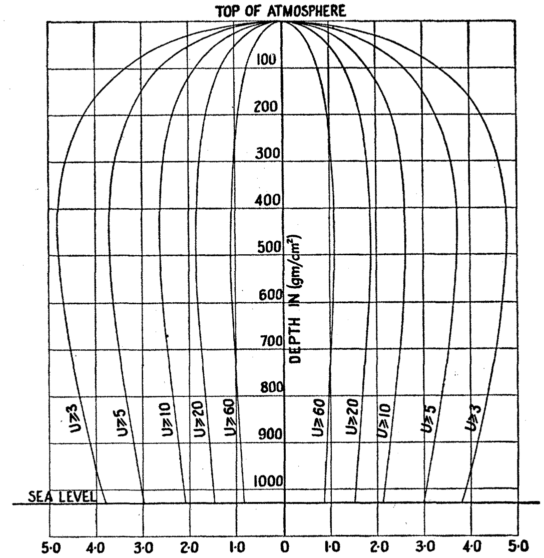

FIG. 1. A plot of the root-mean-square lateral spread of nucleons with energies greater than U, against atmospheric depth measured in g/cm^2. The middle line corresponds to the axis of the shower. The horizontal scale is in kilometers, and U is measured in proton mass units. The curves are for an incident integral primary power law spectrum with exponent 1.1.

This result is exhibited in Fig. 1. It will be noticed that the lateral spread of particles with energy above a given value is very rapid at great heights; this is because of the long mean free path, which allows particles to travel freely in a lateral direction. At 6500 meters the lateral spread attains its maximum value, and below this level there is a gradual decrease, owing to the increasing degradation of the energies of the nucleons concerned. The mean radius of the shower is inversely proportional to the square root of the minimum energy of the particles observed, and proportional to the mean absolute temperature, assumed to be 273°K in (4). The shape of the curve depends rather sensitively on the power law of the primary spectrum at the very high energies which are almost exclusively responsible for the result.

The root-mean-square distance at sea level of a particle with energy greater than 10 Bev from the axis of the shower is 2.1 km; this value is in qualitative agreement with recent experimental indications[6-8] and supports the evidence for an integral power law of 0.1.

It is now widely accepted that the electron-photon component arises from the decay of neutral mesons produced in nuclear collisions; it is therefore heavily dependent on the lateral spread of the nucleon component for its own lateral development. High energy electrons and gamma-rays produced early in the cascade may be expected to multiply and form a density populated core to the shower. However, any account of the development of the soft component outside this core, which left out of account the nucleon component, would be meaningless.

We have made similar calculations for the lateral spread of high energy nucleons in lead and find, for example, that under a block of 56-cm thickness the root-mean-square distance from the shower axis of particles with energy above 10 Bev is 6.4 cm. Owing to the constant density of the medium, the maximum spread is not attained within several meters of lead.

It is hoped to present a more detailed account of this work shortly.

[1] G. Molière, *Cosmic Radiation*, edited by W. Heisenberg (Dover Publications, New York, 1946).
[2] J. Roberg and L. W. Nordheim, Phys. Rev. **75**, 444 (1949).
[3] A. Borsellino, Nuovo cimento **7**, 4 (1950).
[4] H. S. Green and H. Messel, Phys. Rev. **83**, 842 (1951); Proc. Phys. Soc. (London) (to be published).
[5] H. Messel and H. S. Green, Phys. Rev. **83**, 1279 (1951).
[6] D. V. Skobeltsyn *et al.*, Dokl. Akad. Nauk. S.S.S.R. **73**, 1157 (1950).
[7] G. T. Zatsepin *et al.*, Dokl. Akad. Nauk. S.S.S.R. **74**, 29 (1950).
[8] L. K. Eidus *et al.*, Dokl. Akad. Nauk. S.S.S.R. **75**, 669 (1950).

The Drift Mobility of Electrons in Silicon

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THE mobility of electrons injected into *p*-type silicon has been determined by measuring transit times between the emitter and collector points with single crystal rods.

The techniques used to measure the drift mobility of injected carriers in germanium[1] were altered in these measurements not only because the shape of the advancing wave front of injected carriers in silicon is affected by a temporary trapping but also because of the much higher impedance of voltage probe points.

A schematic diagram of the circuit used successfully with silicon is shown in Fig. 1. Electrons are injected at the time of the current pulse and flow down the crystal under the influence of the electric field. When the pulse of electrons arrives at the collector point, a signal is produced.[2] A sketch of the oscilloscope pattern obtained is shown in Fig. 2. The dots represent 10-μsec marker intervals. The current pulse used to inject the electrons produces a corresponding voltage pulse at the start of the oscilloscope trace as a result of the resistance of the silicon rod between the collector point and ground. After this initial pulse the voltage remains constant for some 40 microseconds. During this time the injected electrons are moving down the silicon crystal from the emitter point which was placed a centimeter away from the collector. When the electron pulse arrives at the collector, the voltage pulse shown is observed. The transit time, t, for the injected electrons is the time represented by the distance from the center of the voltage pulse to the maximum signal produced by the arrival of the electron pulse.

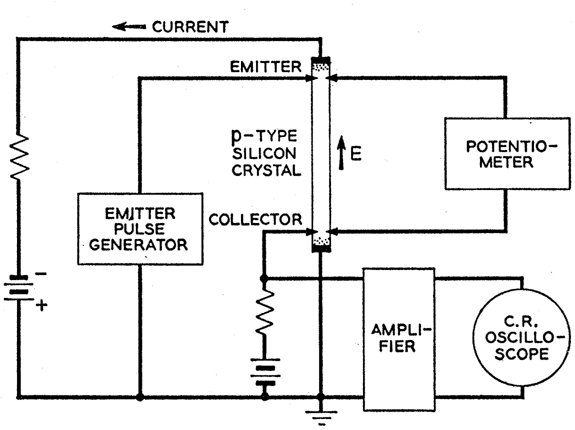

FIG. 1. Schematic of circuit used to measure the drift mobility of injected electrons in silicon.

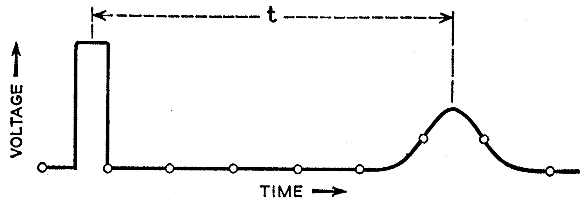

FIG. 2. Drawing of oscilloscope trace showing time of injection pulse and subsequent electron arrival.

It is found that the transit time measured in this way is a function of the amplitude of the current pulse as a result of conductivity modulation of the silicon rod. As the amplitude of the current pulse is continuously decreased, the signal arrival time becomes earlier, at first rapidly and then more slowly. The transit time corrected for conductivity modulation could therefore easily be obtained visually by extrapolating the locus of arrival time points to zero signal.

The drift mobility μ_D of electrons in silicon was calculated from the relation $\mu_D = L^2/Vt$, where t is the transit time, L is the distance from emitter to collector, and V is the voltage difference in silicon between emitter and collector.

Representative values of L and V used are shown in Table I together with the resultant measured transit time. The corresponding value of μ_D has been corrected for nonuniformity of electric field and temperature rise produced by the dc current.[1] The average value of mobility obtained for all data is 1210 cm^2/volt sec.

TABLE I. Representative values of electron drift mobility data.

Sample No.	L(cm)	V(volts)	t(μ sec)	μ_D(cm^2/volt sec)
I	0.537	5.70	43.9	1204
I	1.10	12.3	84.0	1248
I	1.12	29.5	41.8	1194
II	0.64	5.56	63.3	1190
II	0.64	11.8	32.6	1162
II	0.64	4.49	73.8	1283

This value is more than four times as large as the mobility of electrons in silicon reported by Pearson and Bardeen[3] (300 cm^2/volt sec for electrons, 100 cm^2/volt sec for holes). Their values, however, were obtained from Hall effect data in multicrystalline samples, and their extremely low values were most probably caused by inhomogeneities produced by crystal grain boundaries.

In view of the experience with germanium, it is felt that even though only two samples have been examined, the drift mobility of electrons in silicon is within 10 percent of 1200 cm^2/volt sec. Preliminary measurements with injected holes in *n*-type silicon indicate a drift mobility for holes in the neighborhood of 250 cm^2/volt sec giving a ratio of electron to hole mobility of 4.8.

The authors are indebted to W. Shockley for advice and to G. K. Teal and E. Buehler who provided the silicon ingot.

[1] J. R. Haynes and W. Shockley, Phys. Rev. **81**, 835 (1951).
[2] See reference 1 for a more complete description of the main features of the circuit.
[3] G. L. Pearson and J. Bardeen, Phys. Rev. **75**, 865 (1949).

Photomeson Production in Carbon and Hydrogen*

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CYLINDRICAL targets of paraffin, heavy paraffin, and graphite were exposed simultaneously to the x-ray spectrum produced by 330-Mev electrons in the M.I.T. synchrotron. The