Properties of Thermally Produced Acceptors in Germanium

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It was discovered by Scaff and Theuerer that when n-type germanium is heated to 800°C and rapidly cooled, it is converted to p-type; that this conversion may be reversed by prolonged heating at 500°C; and that n-type germanium of very low resistivity cannot be converted.[1]

Recent studies have indicated that (1) the process is characterized by the diffusion of a $p-n$ boundary from the surface of the germanium into the interior and that (2) the concentration of acceptor centers approaches an equilibrium value dependent on temperature in a conversion range of about 550°C to the melting point.

Measurements were made at various temperatures of the locations for given heating times of $p-n$ junctions in samples of different resistivities and also of the equilibrium n- or p-type resistivities in partially or completely converted samples. Assuming that all donor and acceptor centers are ionized, the concentration of added acceptors equals the decrease in electron concentration in partially converted material, equals the initial electron concentration at a $p-n$ junction, and equals this plus the final hole concentration if the conversion is complete. These electron and hole concentrations are[2] $1/(e\mu_n\rho_n) = 1.73\times10^{15}/\rho_n$ and $1/(e\mu_p\rho_p) = 3.7\times10^{15}/\rho_p$, where ρ_n and ρ_p are the respective resistivities in ohm-cm, with e the electronic charge and μ_n and μ_p the mobilities.[3]

In experiments of the first author, $p-n$ junction locations were determined to within 0.002 cm by probing electrically, employing n-type samples of known uniform initial resistivity, as well as a sample possessing a unidirectional resistivity gradient. These were heated in helium for various times at temperatures in the conversion range (with the conversions reversed by heating at 500°C for 24 hours) and rapidly cooled. Assuming the surface concentration A_0 of acceptors to be constant at a given temperature, the concentration A at a $p-n$ boundary is given by

$$A = A_0\,\mathrm{erfc}(x/2D^{\frac{1}{2}}t^{\frac{1}{2}}) = 1.73\times10^{15}/\rho_n, \tag{1}$$

where x is the depth of the boundary below the surface, t is heating time, D is the diffusion constant, and

$$\mathrm{erfc}\,u \equiv 1 - \frac{2}{\sqrt{\pi}}\int_0^u e^{-\beta^2}d\beta. \tag{2}$$

Measurements on two samples of differing ρ_n heated at the same temperature determine D and A_0. Values so determined are given in Table I. The first column applies to different samples,[4] the others to different locations on the variable resistivity sample.[5] The initial resistivities along this sample, computed from D and A_0 (and the values of x and t) are in fairly good agreement with the observed values, as Fig. 1 indicates. The values of D, among the largest observed for diffusion in solids,[6] are given by

$$D = 0.02\exp(-12{,}000/RT), \tag{3}$$

where R equals 1.98 calories and T is absolute temperature, corresponding to an activation energy of about 0.5 ev.

The equilibrium concentrations were determined both from $p-n$ junction locations, and, by the second author, from resistivity data. The junctions were observed in two variable resistivity samples[7] subjected to prolonged heatings at temperatures from 560° to 760°C. The resistivity data were obtained for the center, bottom, and top of an ingot from a bar[8] cut from along its axis, and heated for 24 hours at various temperatures both before and after treatment at the highest temperature, 900°C.

Figure 2 shows the dependence of the concentrations so deter-

Table I. The diffusion constant D and the surface concentration A_0 of acceptors, at several temperatures.

Temperature, °C	760	670	770	850
$(\rho_n)_1$, ohm-cm	5.0	3.2	3.0	1.5
$(\rho_n)_2$, ohm-cm	18.5	6.3	6.0	6.0
D, cm² sec⁻¹	5.9×10^{-5}	3.2×10^{-5}	5.5×10^{-5}	9.5×10^{-5}
A_0, cm⁻³	21×10^{14}	8.0×10^{14}	7.8×10^{14}	13.7×10^{14}

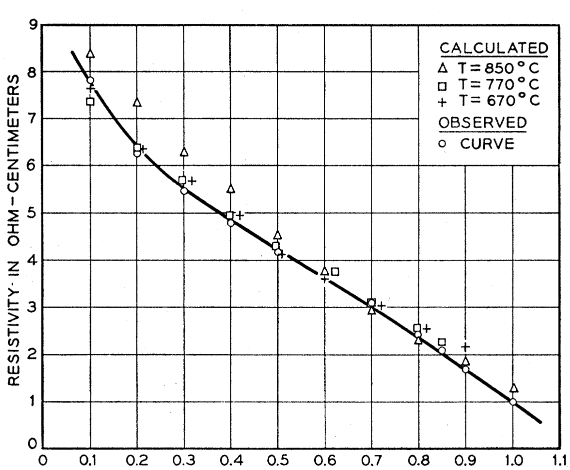

Fig. 1. Agreement between the calculated and observed values of resistivity.

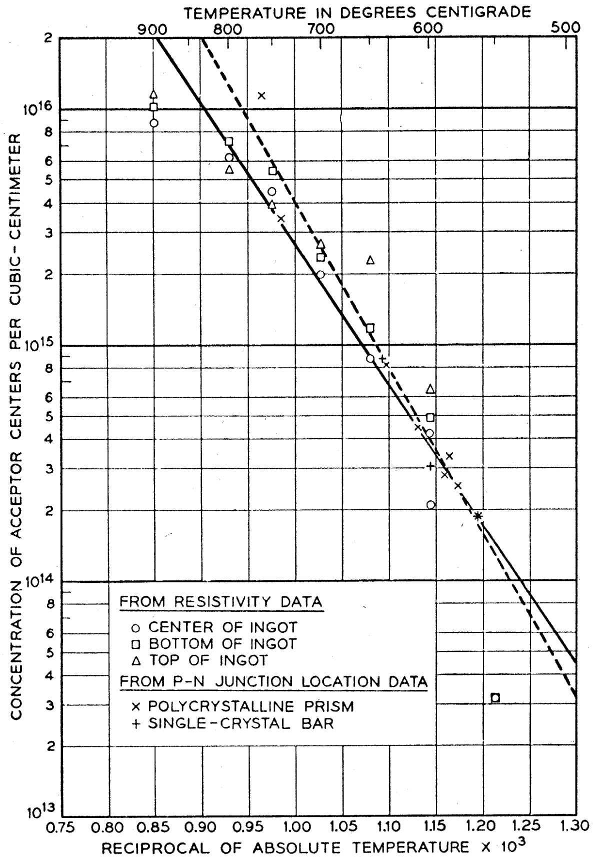

Fig. 2. Variation of acceptor concentration with temperature.

mined on temperature of heat treatment; all the data are in substantial agreement. These equilibrium concentrations are, however, some 10 times larger than the values found for A_0. The solid line gives an activation energy of 27,000 cal or 1.2 ev and an extrapolated concentration for infinite temperature of 2.4×10^{21} cm^{-3}. The dashed line gives 32,000 cal or 1.4 ev and 4.55×10^{22} cm^{-3}, the concentration of germanium atoms which, Shockley has pointed out, the extrapolated concentration should approximately equal, whatever be the nature of the acceptors. This line fits the data fairly well, particularly in view of a comparatively slow quench of the ingot bar.

[1] H. C. Torrey and C. A. Whitmer, *Crystal Rectifiers* (McGraw-Hill Book Company, Inc., New York, 1948), pp. 365ff.; J. H. Scaff and H. C. Theuerer, Trans. Am. Inst. Mining. Met. Engrs. **189**, 59, J. Metals, TP2996E (1951).
[2] It is an unnecessary refinement, since the actual resistivity is in general appreciably less than the intrinsic resistivity, to deal with the exact expressions for the carrier concentrations in terms of these two resistivities.
[3] The electron and the hole mobilities are taken as 3600 and 1700 cm^2 volt^{-1} sec^{-1}; J. R. Haynes and W. Shockley, Phys. Rev. **81**, 835 (1951).
[4] The 5.0-ohm-cm sample was 1.8 cm in diameter, 0.4 cm thick, and polycrystalline, while the 18.5-ohm-cm one was a single-crystal bar 0.5 ×1.0 ×2.0 cm.
[5] This sample was a polycrystalline prism about 3 cm long cut from an ingot.
[6] W. Jost, *Diffusion und chemische Reaktion in festen Stoffen* (T. Steinkopff, Leipzig, 1937).
[7] One sample was the polycrystalline prism used in the junction measurements, the other, a single-crystal bar.
[8] This polycrystalline bar was about 3 mm in cross section and 2 cm long.

The Lateral Spread of Cosmic-Ray Showers in Air and Lead

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PREVIOUS theoretical work[1–3] on the lateral spread of extensive air showers has been devoted almost exclusively to the electron-photon component. This has been partly the result of an early misconception of the nature of the primary component, but mainly to ignorance of the differential cross sections for nuclear collisions at ultrarelativistic energies. We recently tried to remedy this situation by deriving semiempirical cross sections suggested by analysis of data on the energy spectrum of nucleons in cosmic radiation. We found in this way[4] that the mean square angle of scatter of particles with energy U resulting from a nucleon-nucleon collision at ultrarelativistic energies should be $(U+U')/UU'$, where U' is the energy of the primary measured in proton-mass units. By tracing the development of a nucleon cascade within the nucleus, we have also obtained[5] the mean square angle of scatter in a nucleon-nucleus collision.

We have now applied these results to investigate the lateral development of the nucleon cascade in air. If $f(\mathbf{p},\mathbf{r})dp_1dp_2dp_3dr_1dr_2$ is the differential probability of finding a particle with momentum $\mathbf{p}$ at depth r_3 in the atmosphere and at a displacement (r_1, r_2) from the shower axis, one has

$$\frac{1}{k\delta(r_3)}\frac{\mathbf{p}}{p}\cdot\frac{\partial f(\mathbf{p})}{\partial\mathbf{r}}+f(\mathbf{p})=\int f(\mathbf{p}')n(\mathbf{p}',\mathbf{p})d\mathbf{p}', \tag{1}$$

where $n(\mathbf{p}',\mathbf{p})d\mathbf{p}$ is the differential probability of finding a particle with momentum $\mathbf{p}(d\mathbf{p})$ as a result of the collision of a nucleon of momentum $\mathbf{p}'$ with an air nucleus; $\delta(r_3)$ is the density of air, which in this application we have supposed to vary vertically as in an isothermal atmosphere; and $k=(1/75)$ cm^2/g.

We have solved Eq. (1) as far as required to compute the mean square lateral displacement $\langle r^2\rangle$, as a function of depth

$$\langle r^2\rangle=\frac{\int\int\int\int\int(r_1^2+r_2^2)fdp_1dp_2dp_3dr_1dr_2}{\int\int\int\int\int fdp_1dp_2dp_3dr_1dr_2} \tag{2}$$

for nucleons with energy exceeding any given value U. The only approximation which we found necessary was to neglect the mean fourth power of the angular deviation of the particles from the shower axis. The validity of this approximation we confirmed by calculating the mean square angular deviation. For particles with energies greater than U (which is assumed to exceed the geomagnetic cutoff) and assuming an integral power law with exponent 1.1 for the primary spectrum, the mean square angle is

$$\langle\chi^2\rangle=1.34U^{-1}(1-e^{-0.2\theta})(\text{radians})^2 \tag{3}$$

where θ is the depth measured in cascade units of 75 g/cm^2. This is obviously small for the ultrarelativistic energies which we consider.

The mean square distance from the shower axis was found to be

$$\langle r^2\rangle=162U^{-1}e^{-0.2\theta}\left\{\frac{0.2\theta}{1^2\cdot1!}+\frac{(0.2\theta)^2}{2^2\cdot2!}+\cdots\right\}\text{km}^2. \tag{4}$$

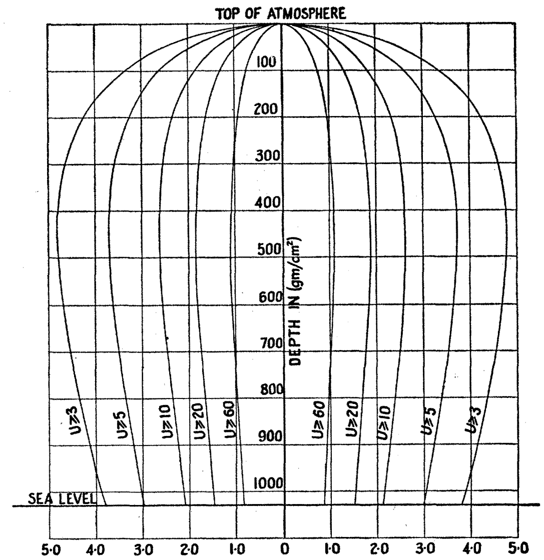

FIG. 1. A plot of the root-mean-square lateral spread of nucleons with energies greater than U, against atmospheric depth measured in g/cm^2. The middle line corresponds to the axis of the shower. The horizontal scale is in kilometers, and U is measured in proton mass units. The curves are for an incident integral primary power law spectrum with exponent 1.1.

This result is exhibited in Fig. 1. It will be noticed that the lateral spread of particles with energy above a given value is very rapid at great heights; this is because of the long mean free path, which allows particles to travel freely in a lateral direction. At 6500 meters the lateral spread attains its maximum value, and below this level there is a gradual decrease, owing to the increasing degradation of the energies of the nucleons concerned. The mean radius of the shower is inversely proportional to the square root of the minimum energy of the particles observed, and proportional to the mean absolute temperature, assumed to be 273°K in (4). The shape of the curve depends rather sensitively on the power law of the primary spectrum at the very high energies which are almost exclusively responsible for the result.

The root-mean-square distance at sea level of a particle with energy greater than 10 Bev from the axis of the shower is 2.1 km; this value is in qualitative agreement with recent experimental indications[6–8] and supports the evidence for an integral power law of 0.1.

It is now widely accepted that the electron-photon component arises from the decay of neutral mesons produced in nuclear collisions; it is therefore heavily dependent on the lateral spread of the nucleon component for its own lateral development. High energy electrons and gamma-rays produced early in the cascade may be expected to multiply and form a density populated core to the shower. However, any account of the development of the soft component outside this core, which left out of account the nucleon component, would be meaningless.