Masses of Lead and Bismuth*

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The masses of Pb^{208} and Bi^{209} have been determined by measuring the time of flight of their ions in a magnetic field. The mass values are: $Pb^{208}=208.0416\pm0.0015$, $Bi^{209}=209.0466\pm0.0015$, $Bi^{209}-Pb^{208}=1.0050\pm0.0015$.

THE helical-orbit time-of-flight mass-spectrometer previously described[1] has been used to measure the masses of Pb^{208} and Bi^{209}. A magnetic field regulator has been added which holds the field constant to about one part in 10^5 during a run. This feature has greatly improved the consistency of the runs and has enabled us to see small disturbances previously masked by scatter in the data. The resulting precision at mass 200 is thus very close to the theoretical value of ±1 mMU.

In all measurements reported here, ion fragments of cyclo-$C_6F_{11}-CF_3$ were used as standards. All values reported below are based on[2] $C=12.003895$ and $F=19.00445$, the latter being obtained from the $F^{19}(p,\alpha)O^{16}$ reaction data.[3] Lead and/or bismuth vapor obtained from a bead of metal fused on a heated molybdenum wire was fed directly to the electron impact ion source into which C_7F_{14} vapor was also flowing.

The energy of these heavy ions is quite low in our apparatus, namely between 10 and 15 ev. They are therefore easily affected by disturbing fields. In this case interpolation using three standards, instead of the usual two, was required to yield results which were consistent within the accuracy aimed at. This means essentially that not only a small quadratic correction but also a small cubic term was used in the relation between mass and time of flight. The standards used were the fragments $C_3F_5^+$ (mass approximately 131), $C_4F_7^+(181)$, and $C_5F_9^+(231)$. Although the total correction in the case of Pb arranged from 2 to 20 mMU (presumably depending on surface potentials within the vacuum can) the largest deviation from the mean of 13 separate runs (each involving about eight consecutive readings of each of the time intervals) was only -2.6 to $+2.3$ mMU; the probable error of the mean was ±0.4 mMU. From this series of measurements the mass $Pb^{208}=208.0410\pm0.0015$ was derived. The more conservative value given for the limits of error allows for possible systematic effects and is a better representation both of the design precision of the instrument and of the actual accuracy of individual readings.

TABLE I. Masses[a] of Pb and Bi.

$Pb^{208}=208.0416\pm0.0015$
$Bi^{209}=209.0466\pm0.0015$
$Bi^{209}-Pb^{208}=1.0050\pm0.0015$

[a] Based on C = 12.003895, F = 19.00445 (see text).

For the measurements of Bi^{209} the total correction in nine runs varied from 15 to 31 mMU, but the cubic term turned out to be zero within our experimental error. Nevertheless, using the three standards, the largest deviations from the mean value for the mass were -2.6 to $+3.1$ mMU; the probable error was again ±0.4 mMU and the derived mass $Bi^{209}=209.0472\pm0.0015$.

An independent check was made by measuring Bi^{209} with respect to Pb^{208} and one standard $C_4F_7(181)$. Because of the proximity of Bi and Pb, knowing the mass of Pb to ±10 mMU was sufficient to evaluate the quadratic correction (only 3 mMU at most) from the time of flight of C_4F_7 and Pb; the mass difference $Bi^{209}-Pb^{208}$ was then derived. The largest deviations from the mean of twelve runs were ±4 mMU; the probable error was 0.4 mMU. The value obtained for the difference was $Bi^{209}-Pb^{208}=1.0050\pm0.0015$. It will be seen that this is in agreement with the individual values given previously.

Since these three measurements are independent, they may be combined to produce presumably more accurate values of Pb and Bi individually. This has been done in the final values displayed in Table I.

If the masses of the standards change, the mass (M) of lead or bismuth must be changed by amount δM given by

$$\frac{\delta M}{M}=\frac{\delta A}{A}\left(\frac{M-B}{A-B}\right)\left(\frac{M-C}{A-C}\right)+\frac{\delta B}{B}\left(\frac{M-A}{B-A}\right)\left(\frac{M-C}{B-C}\right)+\frac{\delta C}{C}\left[\left(\frac{M-A}{B-A}\right)\left(\frac{M-B}{C-B}\right)+\left(\frac{M-B}{A-B}\right)\left(\frac{M-A}{C-A}\right)\right],$$

where A, B, C represent the masses of C_3F_5, C_4F_7, and C_5F_9, respectively, and δA, δB, δC represent possible future changes in their total masses.

The mass of Pb^{208} agrees with the one given by Duckworth and Preston,[4] namely $Pb^{208}=208.0422\pm0.0015$.

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[1] S. A. Goudsmit, Phys. Rev. **74**, 622 (1948); Hays, Richards, and Goudsmit, Phys. Rev. **84**, 824 (1951).
[2] K. T. Bainbridge, Phys. Rev. **81**, 146 (1951).
[3] Strait, Van Patter, Buechner, and Sperduto, Phys. Rev. **81**, 747 (1951).
[4] H. E. Duckworth and R. S. Preston, Phys. Rev. **82**, 468 (1951).

The indicated limit of error in this latter value is the standard deviation derived from seven measurements.

The fact that the difference $_{83}Bi^{209}-{}_{82}Pb^{208}$ is 5 mMU larger than unity indicates a sharp increase in the slope of the packing fraction curve. This agrees with the expectation since Bi^{209} has one proton more than the magic number 82. The addition of this single proton adds, in this case, only 3 Mev to the binding energy of the nucleus. This result is in reasonable agreement with the difference of 1.004 mass units derived from the disintegration data of Harvey.[5]

[5] J. A. Harvey, Phys. Rev. **81**, 353 (1951).

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Divergence of Perturbation Theory in Quantum Electrodynamics

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An argument is presented which leads tentatively to the conclusion that all the power-series expansions currently in use in quantum electrodynamics are divergent after the renormalization of mass and charge. The divergence in no way restricts the accuracy of practical calculations that can be made with the theory, but raises important questions of principle concerning the nature of the physical concepts upon which the theory is built.

ALL existing methods of handling problems in quantum electrodynamics give results in the form of power-series in e^2. The individual coefficients in these series are finite after mass and charge renormalization. The technique of renormalization can at present be applied only to the separate coefficients, and not to the series as a whole. If the series converges, its sum is a calculable physical quantity. But if the series diverges, we have no method of calculating or even of defining the quantity which is supposed to be represented by the series.

Several authors have remarked[1] that the series after renormalization will be divergent in a trivial way, if the series represents a scattering amplitude of a free particle, in circumstances where the particle has a possibility of being captured into a permanently bound system. In this situation a perturbation expansion of the scattering amplitude will diverge, even in nonrelativistic quantum mechanics,[2] and in the relativistic theory the series will diverge for the same reason. It is to be expected that such trivial divergences will not impose any fundamental limitations on the use of the renormalization method. In fact, a new method of carrying through the renormalization program has been developed,[3] a method which is applicable to problems involving bound systems and in which divergences of this elementary nature cannot occur. In the new method the series expansion arises from a formal integration of the equations of motion over a finite interval of time, and in an elementary nonrelativistic theory such a perturbation expansion would necessarily be convergent. For this reason it was claimed as probable[4] that the power series arising from the application of the new method in quantum electrodynamics would always converge. If the claim had been accompanied by a proof of convergence, then the theoretical framework of quantum electrodynamics could have been considered closed, being within its limits a complete and consistent theory.

The purpose of this note is to present a simple argument which indicates that the power-series expansions obtained by integrating the equations of motion in quantum electrodynamics will be divergent after renormalization. The divergence is of a basic character, different from the trivial divergences mentioned above, and is present equally in the results obtained from the new and the older methods of calculation. The argument here presented is lacking in mathematical rigor and in physical precision. It is intended only to be suggestive, to serve as a basis for further discussions. To me it seems convincing enough to merit publication in its present incomplete form; also I am glad to have this opportunity to withdraw the erroneous argument previously put forward[5] to support the claim that the power series should converge.

The argument for divergence is as follows. According to Feynman,[6] quantum electrodynamics is equivalent to a theory of the motion of charges acting on each other by a direct action at a distance, the interaction between two like charges being given by the formula

$$e^2\delta_+(s_{12}^2), \tag{1}$$

where e is the electron charge. The action-at-a-distance formulation is precisely equivalent to the usual formulation of the theory, in circumstances where all emitted radiation is ultimately absorbed. We shall suppose that

[1] B. Ferretti, Nuovo cimento **8**, 108 (1951); K. Nishijima, Prog. Theor. Phys. **6**, 37 (1951).

[2] R. Jost and A. Pais, Phys. Rev. **82**, 840 (1951).

[3] F. J. Dyson, Proc. Roy. Soc. (London) **A207**, 395 (1951). Phys. Rev. **83**, 608, 1207 (1951).

[4] Phys. Rev. **83**, 608 (1951), Section XII.

[5] See reference 4. The error in the argument lay in using the concept "the number of times that an interaction operates" in an intuitive and imprecise way.

[6] R. P. Feynman, Phys. Rev. **76**, 769 (1949), Eq. (4); Phys. Rev. **80**, 440 (1950), Appendix B.