permits measurements of transmission cross sections with relatively small samples of rare elements or separated isotopes. Since the spot source of neutrons can be held to a region about $\frac{1}{16}$-inch diameter and the angular spread of the detector is only 2°, many materials might be measured with fair accuracy in the region of a resonance with a sample (about $\frac{5}{16}$-inch diameter) weighing about one gram.

The closing of the gap in knowledge of neutron cross sections between 2 and 15 kev will now permit more accurate calculations of interesting thermal neutron properties of many elements, including such properties as the phase of scattering and the coherent and incoherent scattering amplitudes.

We wish to thank J. R. Wallace for extensive help in operating the electrostatic generator and aid in measurements and trouble-shooting during the course of the experiments.

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# Fission Neutron Spectrum of $U^{235}$*

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The energy distribution of neutrons (prompt and delayed) arising from thermal fission of $U^{235}$ has been studied with nuclear plates. The spectrum obtained shows a maximum in the region of 0.7–0.8 Mev and an exponential slope of 3.9±0.2 Mev per decade of intensity in the energy region above 2 Mev. The data extend over the energy range from 0.4 to 7 Mev.

## I. INTRODUCTION

A NUCLEAR plate measurement of the spectrum of neutrons from the fission of $U^{235}$ was first made by Richards[1] using Ilford half-tone plates. These measurements agreed with the original ionization chamber measurements of Staub and Nicodemus[2] above an energy of 1.8 Mev but at lower energies uncertainty existed as to the shape of the spectrum curve. The ionization chamber data showed a maximum height around 1 Mev and indicated more low energy neutrons than the plate data which gave a broad maximum centering in the vicinity of 1.8 Mev. In the light of present knowledge, this discrepancy was most likely caused by the unreliability of the half-tone emulsions for tracks corresponding to energies less than 1.5 Mev. A measurement by Hill[3] using a coincidence-anticoincidence counter arrangement (the Argonne "ranger") gave an energy distribution with a maximum around 0.8 Mev and an exponential shape in the 2–7 Mev region.

The present experiment (performed in 1948–1949) was planned to furnish additional data concerning the uncertain region of the spectrum in the vicinity of the maximum as well as to contribute more information over the energy region from 0.5 to 6 Mev. The experiment was improved over Richards' preliminary work by (1) using the newer Ilford C2 plates which are reliable down to an energy of 0.5 Mev and (2) measuring more tracks so as to improve the statistics. Other contemporary experiments on the fission spectrum have provided detailed data over certain limited energy regions of the spectrum, *viz.*, Bonner's[4] cloud-chamber work covering the low energy region from 0.075 to 0.6 Mev and Watt's[5] coincidence counter arrangement emphasizing the high energy region from 3 to 16 Mev.

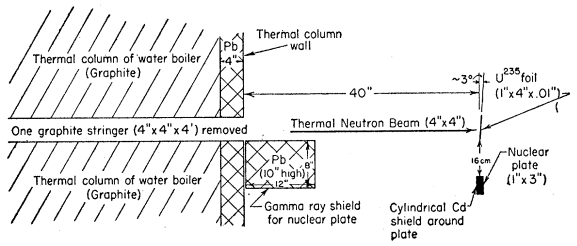


FIG. 1. Experimental arrangement.

## II. EXPERIMENTAL ARRANGEMENT

The experimental arrangement is illustrated in Fig. 1. A 10-mil thickness of $U^{235}O_3$ was coated on a $\frac{1}{32}$-inch thick aluminum strip measuring 1×4 inches. The thermal column of the Los Alamos water boiler was arranged to provide a thermal neutron beam of cross section 4×4 inches. The uranium foil was placed approximately perpendicular to this beam with the aluminum side facing the beam. Ilford type C2 nuclear plates, 100 microns thick, were placed outside of the

* This work was performed under the auspices of the AEC.
[1] H. Richards, private communications (1944–1945).
[2] H. Staub and D. Nicodemus, private communication (1944). (To be published.)
[3] D. Hill, private communication (1947).
[4] T. Bonner, private communication (1948).
[5] B. Watt, private communication (1948). Using the results of the previous two references and those of his own experiment, Watt has obtained a semi-empirical formula which fits the $U^{235}$ fission spectrum over the energy region from 0.075 to 16 Mev. (In process of publication.)

neutron beam at a distance of 16 cm from the nearest edge of the foil. In order to prevent fission neutrons from traveling through the aluminum backing of the foil, the foil was slightly tilted so that it made an angle of about 3° with the center line of the nuclear plates. A cylindrical cadmium enclosure, $\frac{1}{32}$-inch thick, was placed around the plates in order to shield them from thermal neutrons present in the room. This precaution was taken to guard against the possibility of thermal neutrons entering the emulsion and producing 0.6-Mev protons by means of the $N^{14}(n,p)C^{14}$ reaction.

Both the plates and the $U^{235}$ foil were 40 inches from the wall of the thermal column and 52 inches above the floor. The above dimensions were made large compared to the distance of the plates from the foil; this insured a minimum degradation of the neutron spectrum by back-scattering of fission neutrons from the wall and floor into the plates. The thermal neutrons emerging from the water boiler had a cadmium ratio of about 500 and, at a power of 5.5 kilowatts, the flux at the position of the foil was $6\times10^5$ neutrons per $cm^2$ per sec. This flux produced a source of approximately $4\times10^7$ neutrons per sec coming from the $U^{235}$ foil. A 15-hour run at a power of 5.5 kilowatts gave an appropriate proton recoil track density on the nuclear plates.

In order to increase the shielding of the plates against gamma-rays, a lead shield $8\times12\times10$ inches was erected at one side of the thermal port so it was positioned between the plates and the thermal column wall. This shield primarily acted to screen the plate from gamma-rays coming from the vicinity of the open port of the thermal column. Subsequent experiments showed that this shield aided in reducing plate fogging but was not essential in obtaining readable plates.

A background experiment was performed by removing the $U^{235}$ foil and substituting a piece of aluminum which was identical to that on which the uranium was coated. A similar irradiation was given with a nuclear plate in place.

### III. NUCLEAR PLATE ANALYSIS

The nuclear plates were analyzed by the method of measuring proton recoils about a small angle of the forward neutron direction. This technique has been discussed previously[6,7] and only the necessary facts relating to the method for this particular experiment will be presented here.

The area analyzed on the $1\times3$-inch plates was restricted to an area $\frac{3}{4}$ inch wide$\times1$ inch long located at the end of the plate nearest to the foil. The proton recoils measured in the processed emulsion were those which made less than a 10° angle with the center line of the nuclear plate. The above criteria were simplified in practice to the following procedure: from any point in the processed emulsion, all recoils contained within the emulsion surfaces were measured which lay within a rectangular pyramid having half-angles of 10° in the horizontal and vertical planes. Since these angular criteria were applied to the processed emulsion and since the emulsion thickness shrinks vertically by a factor 2.5 upon processing, acceptable proton recoils in the unprocessed emulsion lay within a rectangular pyramid of half-angle 10° in the horizontal plane and half-angle 25° in the vertical plane. In an analysis, the above angular limits represent average values since the actual limits fluctuate about the chosen values on account of (1) microscopist error in the determination of the dip angle,[7] and (2) the fact that the forward neutron direction is not always parallel to the center line of the plate. In the present geometry, the maximum angles which a fission neutron entering the analysis area could make with the plate center line were 3.4° in the horizontal plane and 4.5° in the vertical plane. However, the average entrance angles were 1.2° and 1.6° in the horizontal and vertical planes, respectively; these angles are quite small compared to the average track angular limits of 10° and 25° in the unprocessed emulsion.

The energy resolution resulting from these angular critieria is 12 percent at 0.5 Mev and 6 percent at 2 Mev.[7] Energy resolution is defined as the energy half-width of the proton recoil distribution obtained from neutrons of energy $E_n$ divided by $E_n$. Thus a neutron peak possessing a half-width (at half-maximum) of 60 kev could be resolved at 0.5 Mev and 120 kev at 2 Mev. This is ample resolution for the spectral curve obtained in the present experiment.

The track data were accumulated by taking successive swaths in the analysis area along the 3-inch dimension of the plate; these swaths were 1 inch long and about 0.2 mm apart. Consecutive fields were examined along each swath; using this method, every acceptable track originating in a swath would be measured. This horizontal projection of each track was measured to an accuracy of $\pm1$ microscope scale division$\cong\pm1$ micron. The track data was tabulated in microscope scale division intervals corresponding to proton recoil energy intervals of approximately 0.1 Mev. After observing the curve shape, the 0.1-Mev intervals were kept if the curve necessitated this resolution, but if not, the data were grouped into larger intervals (about 0.5 Mev) and averaged. The procedure of first grouping the tracks into scale division intervals and then converting to proton energy seemed to be the most accurate method of treating the data. In the method of first sorting the tracks into even energy intervals, the scale division corresponding to the energy interval end points usually come out in fractions of a division; since the track lengths are only measured to $\pm1$ division, the question often arises as to the proper energy interval for a measured track length. This uncertainty does not occur with the present method.

In order to obtain the true number of proton recoil tracks, the number of measured tracks must be cor-

[6] H. Richards, Phys. Rev. **59**, 796 (1941).
[7] N. Nereson and F. Reines, Rev. Sci. Instr. **21**, 534 (1950).

rected for those tracks leaving the emulsion surfaces. Since 100-micron thick emulsions[8] were used in the experiment, the usual method of analyzing the entire emulsion thickness was slightly modified so as to reduce the percentage of long tracks leaving the emulsion surfaces. The method consisted of analyzing a restricted central portion of emulsion thickness $t$ and ignoring a small thickness $t_s$ adjacent to each emulsion surface. In the present case, the processed emulsion thickness measured 35 microns and the values of $t$ and $t_s$ chosen were 25 and 5 microns, respectively. The probability of a track leaving the emulsion surface from a particular point in the thickness $t$ is $A_0/A_T$ where $A_T$ is the total area traced out by the end of the track as it moves within its prescribed angular limits and $A_0$ is the area formed by the end of the track moving outside of the emulsion surfaces. The average probability $P$ of a track leaving the emulsion surfaces from any point in the thickness $t$ is

$$P=\int_0^{t/2} A_0/A_T dt.$$

This derivation assumes that the proton recoil tracks were uniformly distributed within the angular limits about the forward neutron direction. This assumption results in only a 3 percent error in the correction if the average angle of the proton recoils is 15° in the unprocessed emulsion.

The above integral was evaluated graphically by summing $A_0/A_T$ over six points in the thickness $t$. A column in Table I shows the correction factor $C=1/(1-P)$ by which the number of measured tracks are to be multiplied to obtain the actual number of tracks.

The number of corrected proton recoil tracks is proportional to the collision cross section of hydrogen $\sigma_p$ and to the neutron flux $N(E)$. The neutron spectrum or $N(E)$ can then be obtained after correcting the measured tracks for recoils leaving the emulsion surface and for $\sigma_p(E)$.[9] Since it is desirable to plot the spectral curve in terms of neutron energy $E$ rather than proton recoil energy $E_p$, it is necessary to convert the track data from $E_p$ to $E$. This conversion can be obtained by either calculating the energy correction of $\theta^2/2$[10] or by using experimental values.[7] In either case the correction is approximately 6 percent or $E=1.06E_p$. Table I shows the values of $E_p$ and $E$.

## IV. RESULTS

The data obtained from the experiment are tabulated in Table I. The column listing microscope scale divisions per 0.1 Mev was obtained from a smoothed curve of the above quantity plotted *vs* $E_p$; the points for this curve were calculated from range-energy values.[11] The data represent a measurement of about 4700 proton recoil tracks and were obtained by two microscopists working independently. The separate results of the two microscopists showed good agreement. The lowest energy point at 0.39 Mev is uncertain by more than

TABLE I. Data and calculations on fission neutron spectrum of $U^{235}$.

| Microscope scale division interval[a] | Number of measured tracks | Scale divisions per 0.1 Mev | Proton recoil energy $=E_p$[b] | Correction factor $=C$ | Neutron energy $=E$ | $\sigma_p(E)$ (barns) | Neutron flux $=N(E) \propto N_C(E)$[c]$/\sigma_p \times 10^{24}$ |
|---|---|---|---|---|---|---|---|
| 3– 4 | 263 | 1.33 | 0.37 Mev | 1.00 | 0.39 Mev | 7.15 | 58.6 ±3.6[d] |
| 4– 5 | 221 | 1.40 | 0.45 | 1.00 | 0.48 | 6.25 | 49.5 ±3.3 |
| 5– 6 | 228 | 1.46 | 0.53 | 1.00 | 0.56 | 5.85 | 57.0 ±3.8 |
| 6– 8 | 432 | 1.53 | 0.64 | 1.00 | 0.68 | 5.35 | 61.8 ±3.0 |
| 8– 10 | 377 | 1.65 | 0.76 | 1.00 | 0.80 | 4.90 | 63.5 ±3.3 |
| 10– 14 | 523 | 1.85 | 0.94 | 1.00 | 0.98 | 4.45 | 54.4 ±2.4 |
| 14– 18 | 446 | 2.00 | 1.13 | 1.00 | 1.20 | 4.00 | 55.7 ±2.6 |
| 18– 24 | 471 | 2.22 | 1.36 | 1.00 | 1.44 | 3.60 | 48.5 ±2.2 |
| 24– 32 | 437 | 2.47 | 1.65 | 1.00 | 1.75 | 3.20 | 42.2 ±2.0 |
| 32– 46 | 521 | 2.80 | 2.06 | 1.02 | 2.18 | 2.80 | 37.9 ±1.7 |
| 46– 62 | 348 | 3.18 | 2.56 | 1.06 | 2.71 | 2.45 | 30.0 ±1.6 |
| 62– 82 | 173 | 3.68 | 3.06 | 1.12 | 3.24 | 2.18 | 16.3 ±1.3 |
| 82–105 | 106 | 4.10 | 3.57 | 1.20 | 3.78 | 1.98 | 11.5 ±1.1 |
| 105–130 | 79 | 4.45 | 4.14 | 1.34 | 4.39 | 1.78 | 10.6 ±1.2 |
| 130–155 | 33 | 4.83 | 4.68 | 1.54 | 4.95 | 1.63 | 6.0 ±1.1 |
| 155–180 | 29 | 5.20 | 5.17 | 1.79 | 5.48 | 1.52 | 7.1 ±1.3 |
| 180–210 | 18 | 5.60 | 5.69 | 2.09 | 6.03 | 1.42 | 4.9 ±1.2 |
| 210–240 | 7 | 6.00 | 6.17 | 2.40 | 6.55 | 1.33 | 2.5 ±0.9 |
| 240–270 | 1 | 6.30 | 6.63 | 2.72 | 7.02 | 1.28 | 0.45±0.5 |

[a] Microscope scale calibration: 1 scale division = 1.06 micron.
[b] Since the horizontal projections of the tracks were measured, a 3-percent correction was added to the midpoint of the microscope scale division interval to obtain the average true track length; this length was used to determine $E_p$.
[c] $N_C(E)$ = Number of corrected tracks per 0.1 Mev = number of measured tracks per scale division × number of scale divisions per 0.1 Mev × $C$.
[d] This result at $E=0.39$ Mev has been increased by 20 percent to adjust for emulsion inefficiency at low energies (see reference 7).

[8] The actual measured thickness of the unprocessed emulsions ranged from 85 to 90 microns. Emulsions of 200-micron thickness would have been more appropriate for this experiment but, at the time, none were available.
[9] The values of $\sigma_p(E)$ were obtained from Goldsmith, Ibser, and Feld, Revs. Modern Phys. **19**, 259 (1947).
[10] M. Livingston and H. Bethe, Revs. Modern Phys. **9**, 290 (1937). The angle $\theta$ in this expression is the half-angle (in radians) for a cone in which proton recoils are measured. From proton recoil energy considerations, a rectangular pyramid having half-angles of 10° and 25° is approximately equivalent to a cone of half-angle 19°.
[11] Lattes, Fowler, and Cuer, Proc. Phys. Soc. (London) **A59**, 883 (1947).

the statistical error shows but the remaining points above this energy should be reliable to the extent that the statistical error indicates. Beyond the 5- to 6-Mev region the statistical errors are large since few tracks were measured in this energy region due to these long tracks leaving the emulsion and also on account of the shape of the spectral curve.

A graph of the results on the fission neutron spectrum is shown in Fig. 2. The spectral curve shows a maximum in the 0.7–0.8 Mev energy region and at energies greater than about 2 Mev the curve diminishes exponentially with a "relaxation energy" of 1.7±0.1 Mev. This result is very similar to that of Hill[3] who obtained a maximum in the same energy region and a relaxation energy of 1.55 Mev. The semi-empirical relation, $e^{-E}\sinh\sqrt{(2E)}$, where $E$ is in Mev, obtained by Watt[5] fits the data of this experiment quite well; the above equation was matched to the experimental data in the vicinity of 1 to 2 Mev which is considered the most accurate region of the data. An experimental curve drawn through the data points would be only slightly higher in the region of the maximum and at energies above 5 Mev.

Tracks on the background plate were less than 5 percent of the number of measured tracks on the regular data plate. Therefore, the background was neglected in the analysis of the data.

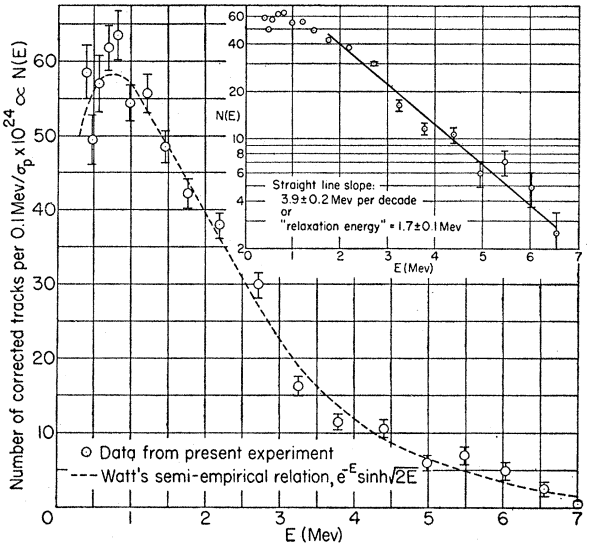


FIG. 2. Fission neutron spectrum of $U^{235}$. Inset: Spectrum plotted on semi-log paper.

The author wishes to thank Esther Allison and Dorothy Squires of the nuclear plate laboratory for their services in reading the plates used in this experiment.

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# Spin-Spin Relaxation in a Simple System*

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A system of two particles of spin ½ coupled by their dipolar interaction and in an arbitrary magnetic field is considered. It is shown that exact expressions for the energy levels of the system can be obtained. The density matrix is calculated exactly for the cases where a magnetic field is suddenly applied, parallel or perpendicular, to the line joining the dipoles. It is used in the evaluation of the magnetic moment of the system. Instead of there being a gradual approach to an equilibrium situation the magnetic moment varies harmonically in time. A partially successful attempt is made to calculate the density matrix when the functional dependence of the magnetic field on the time is more complicated than a step function. The exact calculations for this simple system are compared with the approximate calculations of Waller for a system of $N$ spins. It is pointed out that there may be no gradual approach to equilibrium in the $N$-spin system, either.

THE problem of calculating the time required for the establishment of statistical equilibrium in a system of interacting spins, when the external magnetic field is varied, was first considered by Waller[1] in 1932. Although it has been known for some years that Waller's results are of limited validity when applied to an actual paramagnetic crystal, his basic conclusion that there is a spin-spin relaxation process seems to have gone unchallenged. Qualitative discussions of spin-spin relaxation are frequently based on his calculations. According to Waller, for temperatures not too low and for external magnetic fields small in comparison with the internal field, statistical equilibrium is established in the spin system about $10^{-11}$ sec after the external field is varied.

Broer[2] applied a different, but none the less approximate, method to the problem. He calculated a relaxa-

* This work was supported in part by the AEC.
[1] I. Waller, Z. Physik **79**, 370 (1932).

[2] L. J. F. Broer, Physica **10**, 801 (1943).