for the branching ratio determination, an upper limit of 3 minutes can be placed on the half-life of this postulated short-lived Ag^{115} isomer.

The fact that the 14-Mev neutron fission yield of each of the cadmium isomers is one hundred times the yield from thermal neutrons indicates that both isomers have a common ancestor whose yield increases the hundred-fold. This common ancestor could be the short-lived Ag^{115} isomer (postulated in the previous paragraph), which branches, 75 percent undergoing isomeric transition to the 21-minute silver and 25 percent decaying by beta-emission directly to the 53-hour cadmium isomer. The common ancestor could also be a short-lived palladium isotope which branch decays to the Ag^{115} isomers.

V. ACKNOWLEDGMENTS

It is a pleasure to thank Dr. R. W. Spence and the members of his group for their hospitality and assistance during the summer of 1950 when this work was carried out. We also wish to express our appreciation to the groups responsible for the operation of the water boiler, fast reactor, Cockroft-Walton, and betatron for making their facilities available to us.

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Neutron Yield from the Nuclear Photoeffect

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The yields of neutrons produced in photonuclear reactions by a bremsstrahlung-photon-spectrum are analyzed in terms of the multiplicity of neutron production. Approximating the neutron multiplicity as proportional to the photon energy, we derive the integrated photonuclear cross section from the experimental neutron yields. The integrated cross section from copper to bismuth is $\int_0^\infty \sigma dW = 0.14NZ/A$. (Uranium has a neutron yield 35 percent higher than given by this relation, probably due to photofission.) This relation has the correct form, but a somewhat higher absolute value than the theoretical relation $\int_0^\infty \sigma dW = 0.060(NZ/A)(1+0.8x)$, where x is the fraction of exchange force.

I. INTRODUCTION

IN a previous paper,[1] we calculated the integrated photonuclear cross section, and found satisfactory agreement between our theoretical result and preliminary experiments.[2–5] Since that time very many new experimental results have appeared, some of which are in apparent contradiction with theory.

First, let us summarize the theoretical results of our previous paper,[1] and the assumptions behind them. The summed oscillator strength $\sum_n f_{0n}$ for electric dipole transitions by a nucleus containing N neutrons and Z protons is

$$\sum_n f_{0n} = (NZ/A)(1+0.8x). \tag{1}$$

Here x is the fraction of the neutron-proton force that has an exchange character. High energy neutron-proton scattering experiments[6] indicate the value $x=\frac{1}{2}$. The cross section σ for photon absorption is proportional to the oscillator strength f. The cross section for photon absorption integrated over all photon energies W is given by

$$\int_0^\infty \sigma dW = (2\pi^2 e^2\hbar/Mc)\sum_n f_{0n}$$
$$= 0.060(NZ/A)(1+0.8x)$$
$$= 0.015A(1+0.8x) \text{ Mev-barns.} \tag{2}$$

The last numerical result is for the case $N=Z=A/2$.

The sum rule $\sum_n f_{0n} = NZ/A$ is completely independent of any nuclear model. The modification shown by the $0.8x$ term in Eq. (1) occurs for any potential that does not commute with the position, such as an exchange potential or a velocity dependent potential. This modification was first found by Feenberg.[7] Recently Austern and Sachs[8] have considered the general problem of modifications for all multipole transitions due to potentials that do not commute with position. The coefficient 0.8 in Eq. (1) is based on the

[1] J. S. Levinger and H. A. Bethe, Phys. Rev. **78**, 115 (1950).
[2] J. L. Lawson and M. L. Perlman, Phys. Rev. **74**, 1190 (1948).
[3] G. C. Baldwin and G. S. Klaiber, Phys. Rev. **73**, 1156 (1948).
[4] M. L. Perlman and G. Friedlander, Phys. Rev. **74**, 442 (1948); and Phys. Rev. **75**, 988 (1949).
[5] E. R. Gaerttner and M. L. Yeater, Phys. Rev. **77**, 714 (1950).
[6] Hadley, Kelly, Leith, Segrè, Wiegand, and York, Phys. Rev. **75**, 351 (1949); R. S. Christian and E. W. Hart, Phys. Rev. **77**, 441 (1950); Kelly, Leith, Segrè, and Wiegand, Phys. Rev. **79**, 96 (1950).
[7] E. Feenberg, Phys. Rev. **49**, 328 (1936).
[8] N. Austern and R. G. Sachs, Phys. Rev. **81**, 710 (1951).

nuclear model of a degenerate Fermi gas with parameters specified in our previous paper.

Equation (2) predicts that the integrated photonuclear cross section should be a smooth function of atomic number. In our previous paper we explained the lack of smoothness in the experimental results[4] as due to Perlman and Friedlander's use of the method of measuring induced radioactivity. This method measures only partial cross sections which are always less than the cross section for photon absorption by an unknown and fluctuating factor. Further the less endothermic nuclear reactions—those leading to stable isotopes—are not measured by this technique. Also the disintegration schemes of the induced activities may not be accurately known.

Measurements of particle yields—neutrons or protons—avoid the above difficulties but have the problem of multiplicity of particle production, which is discussed below. Measurements on neutron yields from photonuclear reaction induced by bremsstrahlung radiation have recently been made by Baldwin and Elder[9] at G.E., by Price and Kerst[10,11] at the University of Illinois and by Terwilliger, Jones, and Jarmie[12] at Berkeley, while Halpern and Mann[13] at the University of Pennsylvania have studied proton yields from photonuclear reactions. Gaerttner and Yeater[5,14] have observed photonuclear reactions induced in gas in a cloud chamber. This method has the advantage that photon absorption leads to one and only one observable event. It has the disadvantages of limited applicability and of poor statistics. Their data for He, C, N, and O agree with our Eq. (2) within the rather large experimental uncertainties. The measurements[9–12] of neutron yield *vs* atomic number disagree with our Eq. (2) in two respects: First, the yield is not a smooth function of atomic number for elements up to copper; second, while for heavier nuclei the yield is a fairly smooth function of Z, it increases more rapidly with Z than predicted by Eq. (2), i.e., about as Z^2 rather than as NZ/A or $Z^{1.2}$.

The first discrepancy has been removed by the proton yield measurements of Halpern and Mann.[13] They found that the proton yield was also not a smooth function of Z, but that the sum of their proton yield and the neutron yield of Price and Kerst[10] was a smooth function of Z within experimental error. (The measurements of Halpern and Mann, and Price and Kerst are made for similar conditions: the former for protons from bremsstrahlung from 25-Mev electrons, the latter for neutrons from bremsstrahlung from 22-Mev electrons.) For 8 elements from $_{12}$Mg to $_{30}$Zn the combined neutron plus proton yield is proportional to NZ/A, with the sole exception of $_{22}$Ti, which has a low particle yield.

The competition between proton emission and neutron emission following photon absorption is significant up to $_{30}$Zn. For higher Z the proton yield is greatly decreased due to the increased Coulomb barrier. The experimental values of the proton/neutron yield ratio are in reasonable agreement with the calculation of Heidmann and Bethe.[15] They calculated the proton/neutron yield ratio for 17.5-Mev excitation using the compound nucleus model. The fluctuations in this ratio are due to fluctuations from element to element of the relative values of the proton and neutron binding energies.

While competition between proton and neutron emission was the source of the discrepancy for low Z mentioned previously, it cannot be significant for high Z as according to both experiment and theory, the proton yield is very small for this case. The explanation for the high Z discrepancy is suggested by Heidmann and Bethe who calculate that the average number of neutrons emitted at 17.5-Mev excitation is 0.83 for $_{29}$Cu, 1.22 for $_{53}$I, and 1.98 for $_{73}$Ta, the increase with Z being principally due to the decrease of neutron binding energy. The possible importance of multiple neutron production is also suggested by the measurements of Sugarman and Peters,[16] who measured induced activities produced in bismuth by 86-Mev bremsstrahlung. They found activities due to the emission of from 3 up to perhaps 10 neutrons. (Tl^{199} could be produced by $\gamma-10n$ or $\gamma-9n$, p followed by electron capture or by $\gamma-8n$, $2p$.) As the counting efficiencies for many of the induced activities are not known well, and as the $\gamma-n$ and $\gamma-2n$ reactions could not be measured by this method due to the very long half-lives of the produced activities, we cannot at present interpret these measurements quantitatively.

The experimental results indicate that emission of one neutron is the predominant photonuclear process only for elements in the neighborhood of copper. For lower atomic number, proton emission is generally favored, while for higher atomic number, multiple particle emission appears to be the predominant process. Even for the case of copper, the cross sections for processes other than the $\gamma-n$ are by no means negligible, amounting to about one third of the absorption cross section.[17] Ge^{76} appears to be the isotope for which emission of a single neutron is most strongly favored.[1] For elements either much lighter or much heavier than copper, study of the reactions by the method of induced activity[18,19] may well not provide reliable information as to the shape of the curve of absorption cross section

[9] G. C. Baldwin and F. R. Elder, Phys. Rev. **78**, 76 (1950).
[10] G. A. Price and D. W. Kerst, Phys. Rev. **77**, 806 (1950).
[11] D. W. Kerst and G. A. Price, Phys. Rev. **79**, 725 (1950).
[12] Terwilliger, Jones, and Jarmie, Phys. Rev. **82**, 820 (1951).
[13] J. Halpern and A. K. Mann, Phys. Rev. **82**, 733 (1951).
[14] E. R. Gaerttner and M. L. Yeater, Phys. Rev. **82**, 461 (1951); and Phys. Rev. **83**, 145 (1951).
[15] J. Heidmann and H. A. Bethe, Phys. Rev. **84**, 274 (1951).
[16] N. Sugarman and R. Peters, Phys. Rev. **81**, 951 (1951).
[17] P. R. Byerly, Jr., and W. E. Stephens, Phys. Rev. **83**, 54 (1951).
[18] Katz *et al.*, Phys. Rev. **80**, 1062 (1950); Phys. Rev. **81**, 815 (1951); Phys. Rev. **82**, 270 (1951); Phys. Rev. **82**, 271 (1951).
[19] R. Sagane, Phys. Rev. **83**, 174 (1951).

against photon energy, or as to the area under this curve.

The purpose of this paper is to interpret the measurements of neutron yield *vs* Z for large Z ($Z \geq 29$) in terms of multiple neutron production. We shall show that Eq. (2) is in accord with experimental results on the neutron yield. We wish to make clear that establishing the validity of Eq. (2) does not verify any particular nuclear model or any specific mechanism for photonuclear reactions. Equation (2), or a rather similar equation, must hold for any theory of electric dipole transitions. The comparison with experiment does indicate that electric dipole transitions are indeed the predominant effect, as we would expect from theory.[1]

II. NEUTRON YIELD

Let $\sigma(W)$ be the cross section for absorption of a photon of energy W; let $\gamma(W)$ be the differential energy spectrum of the photon beam; and let $\nu(W)$ be the neutron multiplicity for the nucleus subsequent to photon absorption. (Note that $\nu(W)$ can be less than unity, due to proton competition.) The measured neutron yield Y is then

$$Y = \int \sigma(W)\gamma(W)\nu(W). \tag{3}$$

To relate Eq. (3) for the measured yield to the theoretical result given in Eq. (2) we make the following two approximations. For the photon spectrum $\gamma(W)$ we take

$$\gamma(W) \cong \begin{cases} ndW/W, & W \leq W_m \\ 0, & W > W_m. \end{cases} \tag{4}$$

The neutron yields are given[11,12] as neutrons/mole per erg/cm^2 of bremsstrahlung striking the sample. For the low energy end of the bremsstrahlung spectrum the number of photons in an erg is[20] $(1.3/W_m)dW/W$, where the maximum energy W_m is expressed in ergs. Then $n = 1.3/W_m$. Second, the neutron multiplicity $\nu(W)$ is approximated as

$$\nu(W) \cong W/E_n. \tag{5}$$

Here E_n represents the average energy spent in producing a neutron, and varies from isotope to isotope but generally decreases with increasing atomic number.

Substituting in Eq. (3) we have the neutron yield

$$Y = (n/E_n)\int_0^{W_m} \sigma dW. \tag{6}$$

If the maximum bremsstrahlung energy is very high, such as 325-Mev, we make a third approximation by replacing W_m by infinity, and find the integrated cross section in terms of the neutron yield

$$\int_0^{\infty} \sigma dW = E_n Y/0.602n = E_n Y/1500 \text{ Mev-barns}. \tag{7}$$

We have taken $W_m = 325$-Mev. The factor 0.602 is to convert from cm^2/mole to barns per nucleus.

TABLE I. Neutron multiplicity.

Element	Neutron binding energies (Mev) L_1	L_2	L_3	L_4	Level density parameter a (Mev^{-1})	Average energy E_n (Mev)
$_{29}$Cu	10.5	18.7	29.7	38.4	1.9	21
$_{53}$I	9.3	16.8	26.6	34.6	7.3	13
$_{73}$Ta	7.7	14.0	22.2	28.6	9.4	11
$_{83}$Bi	7.4	13.5	21.4	28.0	10.2	10
$_{92}$U	5.6	10.0	16.1	21.0	11.5	8

The average energy expended per neutron emitted, E_n, has been evaluated by means of the statistical compound nucleus model[21,22] for the 5 elements, $_{29}$Cu, $_{53}$I, $_{73}$Ta, $_{83}$Bi, and $_{92}$U. In each case we calculate and plot the multiplicity as a function of excitation energy of the compound nucleus—i.e., the function $\nu(W)$—and approximate it by a straight line through the origin.

The nuclear parameters used, and the results obtained, are summarized in Table I. The neutron binding energies L_1, L_2 (L_i for removal of i neutrons) are taken from Heidmann and Bethe.[15] L_1 for removal of the first neutron is based on thresholds for $\gamma - n$ reactions; the higher L's are based on the semi-empirical Weizsäcker mass equation. The level density parameter a occurs in determining the nuclear temperature, $T = (E/a)^{\frac{1}{2}}$, where E is the nuclear excitation energy. The values for a are found by interpolation from Blatt and Weisskopf.[21] The values for E_n are found by approximating the functions $\nu(W)$ by a straight line. Figure 1 shows the curve $\nu(W)$ up to $W = 30$-Mev, together with the straight line approximation for Ta. The maximum deviation from the approximation is 30 percent. For the case of copper we calculated $\nu(W)$ and $E_n = 15$-Mev for neutron emission, neglecting the proton competition; and subsequently corrected for proton competition using the experimental result[17] neutron yield/particle yield$=0.7$. Proton competition is negligible for the heavier nuclei.

We note that to a good approximation $E_n = \frac{1}{2}(L_1 + L_2)$, which is evident from Fig. 1.

We can now compare the relation between experimental values of the integrated cross section derived from the yield by Eq. (7) with the quantity NZ/A, to find if Eq. (2) holds. We shall use the experimental data of Kerst and Price[11] and Terwilliger, Jones, and Jarmie[12] for neutrons/mole erg/cm^2 of bremsstrahlung. (The lower energy measurements of Baldwin and Elder[9] and Price and Kerst[10] agree with the general features of

[20] B. Rossi and K. I. Greisen, Revs. Modern Phys. **13**, 240 (1941).

[21] J. M. Blatt and V. F. Weisskopf, Massachusetts Institute of Technology Laboratory Nuclear Science and Engineering Technical Report 42 (1950), Chapter 6.

[22] Orear, Rosenfeld, and Schluter, *Nuclear Physics* (University Press, Chicago, 1950), p. 162.

the measurements referred to above. We have selected the high energy measurements since we are making the approximation W_m = infinity.) The former measured neutrons produced by 320-Mev bremsstrahlung, the latter by 330-Mev bremsstrahlung. The energy in the photon beam was measured calorimetrically by the former group, and using the method of Panofsky *et al.*[23] by the latter group. The neutrons were detected at 90° in each case, and additional measurements showed that they were emitted isotopically. The former group used rhodium foils in paraffin as neutron detectors, while the latter group used a large BF_3 counter in paraffin. The efficiency of detection of the second method should be less sensitive to the neutron energy. The two sets of data are in fairly good agreement: the ratio of the Berkeley to Illinois neutron yields vary between 1.44 and 1.17 for the 5 elements considered here. We shall average the two sets of data, since there is no clear basis for preference of one against the other.

The product

$$YE_n/1500=\int_0^\infty \sigma dW$$

is plotted against NZ/A in Fig. 2. We find that the linear relationship between them predicted by theory holds quite well for the four elements $_{29}$Cu, $_{53}$I, $_{73}$Ta, and $_{83}$Bi. The agreement is almost within the experimental errors of measuring Y, and well within the approximations we have made. However, $_{92}$U falls 35 percent above the straight line predicted from theory.

As Kerst and Price[10] have pointed out, it is most likely that this excess neutron emission by U is due to the possibility of photofission. Koch, McElhinney, and Gasteiger[24] and Anderson and Duffield[25] have measured the threshold for photofission. Both groups found it to be 5.1 Mev, close to the neutron binding energy of 5.6 Mev, in conformity with the prediction of the theory of Bohr and Wheeler.[26] Charbonnier, Scherrer, and Wäffler[27] have measured the photofission cross section at 17-Mev photon energy and found it to be 9 percent of the cross section for neutron emission at the same energy. Goward, Jones, Watson, and Lees[28] find a value of 14 percent for this ratio, for 23-Mev bremsstrahlung. Let us assume that generally fission may be substituted for neutron emission with a probability y. Then, e.g., at a γ-ray energy at which normally 3 neutrons are emitted, fission may occur instead of the emission of the first neutron with a probability y. With a probability $1-y$, a neutron is emitted first; but in this case, there is again a probability that fission occurs as the second step, and similarly for the third step. The total probability of fission is then

$$y+y(1-y)+y(1-y)^2=1-(1-y)^3=3y(1-y+\tfrac{1}{3}y^2).$$

If y is small, as in fact it is, this is close to $3y$, or ny if normally n neutrons are emitted. If fission occurs, the remaining $n-1$ neutrons can still be emitted because the necessary energy is still available; they may be emitted either before or after the fission. In the fission of U^{235} by slow neutrons, $\nu_F=2.5$ neutrons are emitted, so that if fission occurs after γ-ray absorption, the total number of neutrons emitted is $n+\nu_F-1=n+1.5$, or 1.5 more than without fission. The probability of obtaining these 1.5 additional neutrons was found above to be about ny, so that on the average the number of neutrons is increased by $1.5ny$, or by a factor

$$F=1+(\nu_F-1)y. \qquad (8)$$

To get agreement with the 35 percent increase which we deduced above from the observations, we would have to assume that $y=0.23$. Although this is considerably higher than the values of y observed,[27,28] it is in good agreement with the branching ratio for the fission of U^{238} by fast neutrons.

The straight line drawn in Fig. 2 gives the experimental result

$$\int_0^\infty \sigma dW=0.14NZ/A. \qquad (9)$$

In Eq. (2) the value of the right-hand side is $0.060\times(1+0.8x)(NZ/A)$. Thus the experimental value of the coefficient is 30 percent larger than the theoretical value 0.108 for the case of complete exchange force ($x=1$), or 65 percent larger if we use the value $x=\frac{1}{2}$. This disagreement seems within the errors of the absolute measurement of neutron yield, combined with the approximations of our analysis. Gaerttner and Yeater[5,14] also found some integrated cross sections higher than

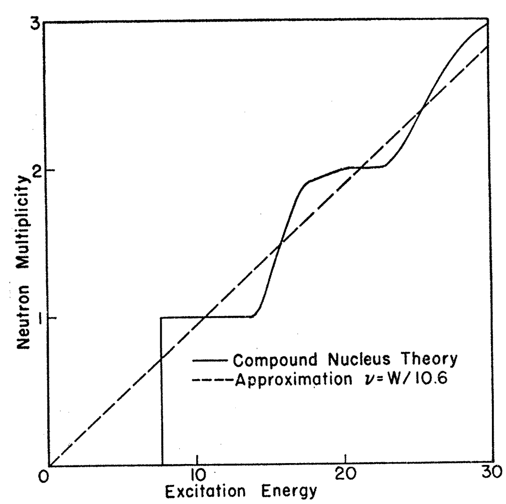

FIG. 1. Neutron multiplicity from Ta^{181} *vs* excitation energy in Mev.

[23] Blocker, Kenney, and Panofsky, Phys. Rev. **79**, 419 (1950).
[24] Koch, McElhinney, and Gasteiger, Phys. Rev. **77**, 329 (1950).
[25] R. E. Anderson and R. B. Duffield, Phys. Rev. **85**, 728(A) (1952).
[26] N. Bohr and J. A. Wheeler, Phys. Rev. **56**, 426 (1939).
[27] Charbonnier, Scherrer, and Wäffler, Helv. Phys. Acta **22**, 385 (1949).
[28] Goward, Jones, Watson, and Lees, Proc. Phys. Soc. (London) **A64**, 95 (1951).

the maximum theoretical value, while Halpern and Mann[13] found

$$\int_0^{25} \sigma dW = 0.090NZ/A.$$

We can suggest two corrections to make in comparing the absolute cross sections given by theory and experiment. First, the measured neutron yield includes neutrons produced in high energy nuclear stars, which appear to be in large part caused by photomeson production, and subsequent reabsorption of the meson in the same nucleus, producing a star. The calculations which led to Eqs. (1) and (2) did not include interaction of mesons with photons, so the experimental neutron yield could be larger than that calculated in Eq. (2). The integrated cross sections for photomesonic effects can be estimated from the measurements of Miller,[29] and Kikuchi.[30] Miller found that the cross section of the silver nucleus for production of stars of 3 or more prongs was about 7 mb for photons from 160 to 240-Mev, and 8 mb for photons from 240 to 320-Mev. A large part of this cross section—say 5 mb—is probably due to the process of meson production and reabsorption. Using the mesonic cross section $\sigma_m = 5$ mb from 160 to 320-Mev, and using Eq. (2) with $x=1$ for the denominator we have as an order of magnitude estimate for this correction C_1

$$C_1 = \int_0^{325} \sigma_m dW \Big/ \int_0^{\infty} \sigma dW = 0.3. \qquad (10)$$

Second, the nonmesonic part of the photonuclear cross section is not negligible above $W_m = 325$-Mev, so that the experimental measurement is expected to be somewhat smaller than the theoretical result. A theoretical estimate of the high energy nuclear photoeffect[31] gives us a correction for this effect

$$C_2 = \int_{325}^{\infty} \sigma dW \Big/ \int_0^{\infty} \sigma dW = 0.1. \qquad (11)$$

The ratio between experimental and theoretical yields should be $1 + C_1 - C_2$ or roughly 1.2, but the corrections can only be made very approximately at the present.

Note added in proof.—Eyges has recently discussed the contribution of high energy mesonic processes to the neutron yield; his estimate is somewhat higher than ours. The significance of mesonic processes in the high energy deuteron photoeffect has been shown by the high cross sections found recently at Berkeley (Kikuchi, and also Gilbert and Rose) and at Cornell (Benedict and Woodward, and also Keck and Littauer).

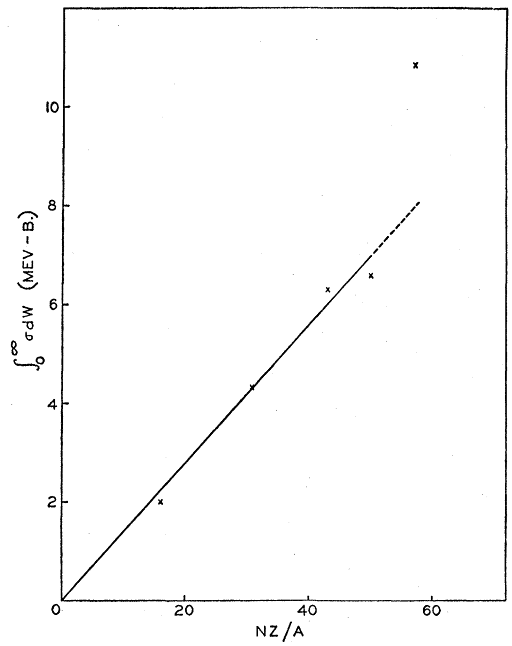

FIG. 2. Integrated photonuclear cross sections. The five points shown are for the elements. $_{29}$Cu, $_{53}$I, $_{73}$Ta, $_{83}$Bi, and $_{92}$U. $N = A - Z$ is the number of neutrons in the nucleus.

We conclude that there is no clear discrepancy between theoretical and experimental results for the absolute value of the neutron yield from photonuclear reactions. There is quite good agreement between theory and experiment on the relative values of the particle yield (neutrons plus protons) for the various nuclei, uranium being high probably due to photofission. This confirms that the photonuclear process is predominantly electric dipole in character, and that the integrated cross section is increased due to exchange forces between neutron and proton. Much more experimental and theoretical work must be done to confirm, or refute, any given nuclear model for the dipole transitions.

One of us (J.S.L.) did much of this work while employed as a research associate at Cornell University under an ONR contract. We are grateful to G. C. Baldwin, D. W. Kerst, and N. Sugarman for discussions of their experiments.

[29] R. D. Miller, Phys. Rev. **82**, 260 (1951).
[30] S. Kikuchi, Phys. Rev. **81**, 1060 (1951).
[31] J. S. Levinger, Phys. Rev. **84**, 523 (1951).