The Pairing Effect in Nuclei and the Beta-Labile Elements*

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It has been shown previously that in certain regions of nuclides the pairing energy of protons exceeds that of neutrons. Statistical examination of the beta-stable odd-mass nuclides shows that this situation is probably fairly general except for light elements. Theoretical explanations of the effect are suggested. Its bearing on the beta-lability of technetium and promethium and the possible beta-lability of astatine and indium is discussed.

I. EMPIRICAL EVIDENCE

FROM an analysis of alpha- and beta-disintegration energies of the heavy nuclides, Glueckauf[1] has shown that the pairing energy of protons exceeds that of neutrons in heavy nuclei. Thus the masses of odd-A nuclides in this region do not lie on a single smooth surface, but on two parallel surfaces separated by about 0.2 Mev, with even-Z nuclides on the lower surface and odd-Z nuclides on the upper. The effect is similar to but much less pronounced than the well-known even-odd effect for even-A nuclides. By a somewhat different method of analysis for which more experimental data were available, this finding has been substantiated.[2]

Kowarski[3] has postulated that such a situation exists in regions of open-shell neutrons, in order to explain the occurrence of the beta-labile elements $_{43}$Tc and $_{61}$Pm beyond closed neutron shells. Suess[4] has adduced experimental evidence for this from beta-disintegration energies.

A classification of the beta-stable odd-A nuclides according to parity of Z, Table I, shows a greater frequency of even values than of odd, everywhere except in the region of light nuclei. The inequality seems too great to be a matter of chance and strongly suggests that the effect in question is fairly general for medium-weight as well as heavy nuclei.

TABLE I. Classification of beta-stable odd-A nuclides.

Range of A	Number of beta-stable odd-A nuclides with Even Z	Odd Z
1–39	8	11
41–79	9	11
81–119	13	7
121–159	11	8
161–199	12	8
201–243	9	7
Total	62	52

The apparent generality of this effect suggests that the pairing energy term of atomic mass equations based on statistical concepts, such as those of Bethe and Bacher[5] and of Bohr and Wheeler,[6] should be modified as follows:

$$-\tfrac{1}{2}\delta_A \text{ for even } A, \text{ even } Z,$$
$$-\tfrac{1}{2}\epsilon_A \text{ for odd } A, \text{ even } Z,$$
$$+\tfrac{1}{2}\epsilon_A \text{ for odd } A, \text{ odd } Z,$$
$$+\tfrac{1}{2}\delta_A \text{ for even } A, \text{ odd } Z.$$

If π_A is the excess energy associated with an unpaired proton and ν_A that for an unpaired neutron in nuclei of mass number $\sim A$, then $\delta_A=\pi_A+\nu_A$ and $\epsilon_A=\pi_A-\nu_A$.

II. THEORETICAL CONSIDERATIONS

An explanation of this inequality in pairing energy of protons and neutrons has been sought in terms of several possible contributions to the pairing energy:

1. Exclusion Principle

Consider an independent-particle nuclear model in which protons and neutrons move freely in the same potential well, with nondegenerate energy levels fairly evenly spaced. Since the Pauli exclusion principle allows two nucleons of each kind to a level, in the ground state of a nucleus with $N>Z$ the levels are occupied by neutrons to a higher energy than by protons (the difference being approximately the electrostatic energy of a proton in the field of the other protons). In any such potential, the average spacing between levels decreases with increasing energy, so that the spacing at the top of the proton distribution, S_p, is greater than that at the top of the neutron distribution, S_n. It can be shown from simple considerations that in this model $\pi_A=\frac{1}{4}S_p$ and $\nu_A=\frac{1}{4}S_n$, so that $\delta_A=\frac{1}{4}(S_p+S_n)$ and $\epsilon_A=\frac{1}{4}(S_p-S_n)$. Using the statistical mechanical formula[7] for average level spacings of noninteracting particles of mass m and twofold spin degeneracy confined to a sphere of radius r:

$$S_p=\tfrac{1}{3}(3/4\pi)^{4/3}(h^2/mr^2)(\tfrac{1}{2}Z)^{-\frac{1}{3}},$$
$$S_n=\tfrac{1}{3}(3/4\pi)^{4/3}(h^2/mr^2)(\tfrac{1}{2}N)^{-\frac{1}{3}}$$

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[1] E. Glueckauf, Proc. Phys. Soc. (London) **61**, 25 (1948).
[2] Unpublished work with G. T. Seaborg.
[3] L. Kowarski, Phys. Rev. **78**, 477 (1950).
[4] H. E. Suess, Phys. Rev. **81**, 1071 (1951).
[5] H. A. Bethe and R. F. Bacher, Revs. Modern Phys. **8**, 82 (1936).
[6] N. Bohr and J. A. Wheeler, Phys. Rev. **56**, 426 (1939).
[7] E. Feenberg, Revs. Modern Phys. **19**, 239 (1947).

there results for a typical heavy nucleus:

$$S_p=0.28\text{ Mev},\quad \delta_A=0.13\text{ Mev},$$
$$S_n=0.24\text{ Mev},\quad \epsilon_A=0.01\text{ Mev}.$$

A more realistic (tapering) nuclear potential would give a more rapid decrease of level spacing with increasing energy, but this could hardly more than double ϵ_A. Thus the exclusion principle accounts for only about a tenth of the experimentally observed values of both δ_A and ϵ_A.

2. Electromagnetic Interaction

Besides the electrostatic interaction between two protons, there should be an electromagnetic interaction between any two nucleons due to their magnetic dipole moments, which approximates the coupling between two permanent magnets having the same moments and orientation and separated by a distance of the order of nuclear dimensions. If the measured moment of an odd-nucleon nucleus can be attributed entirely to the odd particle, the observation that odd-proton nuclei usually have larger moments than odd-neutron nuclei can be considered as evidence for larger moments for protons than for neutrons in general. This results from the fact that the orbital charge motion of a proton can make an important contribution to its net moment. The paired nucleons are assumed to have magnetic moments equal in magnitude and opposite in direction. The coupling energy is given roughly by $\xi\sim\mu^2/d^3$, where d is an appropriate distance. Taking as representative values $\mu_p=3$ and $\mu_n=1$ in nuclear magnetons and $d\sim3\times10^{-13}$ cm, there results $\xi_p\sim0.005$ Mev and $\xi_n\sim0.0006$ Mev, so that the contribution to $\epsilon_A=\xi_p-\xi_n$ ~0.004 Mev. Again the contribution is in the right direction but is only a small part of the observed magnitude.

3. Overlap Effect

The main part of the pairing energy is presumably due to the similarity of the spatial parts of the wave functions of the pairing nucleons and the consequent extensive overlap.[8] Thus the mutual attraction of the pairing nucleons is greater than the average for two random nucleons in the same nucleus. In nuclei having considerably more particles of one kind than the other, the general character of the orbitals at the tops of the two distributions will be different, so it is quite plausible that there might be a difference in the magnitude of the overlap energy for the two types of particles. Unfortunately, there is no simple way of estimating the magnitude of this effect, nor is any reason apparent why the effect should be consistently greater for protons than for neutrons. In fact Suess[4] suggests that in certain regions of nuclides the neutron pairing energy exceeds that of protons; in such regions ϵ_A would take on negative values.

[8] E. Feenberg, Phys. Rev. **76**, 1275 (1949).

III. THE BETA-LABILE ELEMENTS

A favoring of even values of Z for odd-A nuclides can help to explain the complete beta-lability of the odd-Z elements mentioned, as noted by Kowarski[3] and Suess.[4] However, from the evidence cited above it appears that this effect is not limited to the regions in which these elements occur, but is fairly general. Therefore it alone could hardly be the whole explanation, or there would presumably be a considerable number of such beta-labile elements.

On the other hand, it has been noted on several occasions[3, 9–12] that both of these beta-labile elements occur near closed neutron shells, with $N=50$ and $N=82$. Aten[10] has explained how the closing of a shell perturbs the normal mass valley so that the line of maximum beta-stability[6,13] tends to run more nearly parallel to the line joining the closed-shell isotopes or isotones at the point of crossing. Its slope is thus decreased near closed proton shells and increased near closed neutron shells, and in the latter case the possibility of beta-stable odd-A isotones is enhanced. However, only right at the point of crossing is the increase in the slope ever sufficiently great to stabilize a pair of odd-A isotones unassisted. This does occur at $N=20$ and $N=82$, but here the labilized nuclides have even Z, so beta-labile elements do not result.

In the combination of these two effects, individually insufficient, probably lies the explanation of the beta-lability of technetium and promethium. It is not necessary, however, that such an occurrence should be associated with every pronounced neutron shell closure, since other, seemingly accidental, factors are also involved. There is a distinct possibility on experimental grounds that $_{85}$At is completely beta-labile,[12] but this is not necessarily to be expected as has been postulated.[12]

It seems likely that $_{49}$In is also beta-labile. In^{115} is known[14–16] to be unstable with respect to Sn^{115}. Evidence has been obtained[15] for cadmium L x-rays associated with electron-capture decay of In^{113} to Cd^{113}, and the anomalously high absorption half-thickness found[16] for the In^{115} β-rays might be accounted for by an admixture of cadmium K x-rays. The beta-lability of this element may likewise be explained by a combination of two effects. The stronger pairing of protons, as compared to neutrons, helps to stabilize Cd^{113} relative to In^{113}, but this is certainly insufficient to make both indium isotopes unstable in a region where the beta-stability curve has such a low slope. The lability of In^{115} relative to Sn^{115} is attributable to the especial stability of 50-proton configurations.

[9] A. Broniewski, Compt. rend. **228**, 916 (1949).
[10] A. H. W. Aten, Science **110**, 260 (1949).
[11] W. D. Harkins, Phys. Rev. **76**, 1538 (1949).
[12] Perlman, Ghiorso, and Seaborg, Phys. Rev. **77**, 26 (1950).
[13] T. P. Kohman, Phys. Rev. **73**, 16 (1948).
[14] Bell, Ketelle, and Cassidy, Phys. Rev. **76**, 574 (1949).
[15] S. G. Cohen, Nature **167**, 779 (1951).
[16] E. A. Martell and W. F. Libby, Phys. Rev. **80**, 977 (1950).