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Mesonic Proper-Field*

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A method is presented of treating the meson cloud around the nucleon. The meson cloud is described in terms of the free field operators which coincide, on the world point on the space-like surface, with the usual field operators in the Heisenberg representation. In the second and the third sections, the relation between the field and its source on an arbitrary space-like surface is studied. The fourth and fifth sections are concerned with the method of the phenomenological investigation of the meson cloud in our formalism and with the treatment of the problem of the multiple production of mesons and the magnetic moment of the nucleon-meson system. This treatment of the multiple production of mesons is shown to be the covariant generalization of Bloch-Nordsiecks' method for the multiple production of low energy photons.

1. INTRODUCTION

IT is well known that the meson theory, based on the quantum field theory, has been a foundation for the study of models of elementary particles, namely the kinds of actual particles and their mutual interactions. The experimental evidence of the creation of π-mesons by nucleon-nucleon or γ-nucleon collisions seems to afford testimony of the existence of the meson field interacting with the nucleon. This meson field gives the effect of the field reaction on the nucleon. Some of the difficulties in meson theory have been resolved by modifying the model into a two-meson theory, but the problem of the field reaction still remains an important problem to be investigated.

In quantum electrodynamics the problem of the field reaction has been studied in detail in Tomonaga-Schwinger's covariant formalism, and it has become clear that the effect of the field reaction plays an indispensable role in the explanation of experimental results (Lamb-Retherford-shift, etc.).

In the field of the meson theory not only are there many problems which cannot be solved by renormalization, but also various results depend on the method of treating divergences.[1] This is probably due to the meson having a heavier mass than the electron, the coupling between nucleon and meson not being invariably weak, the meson cloud having a greater singularity in the vicinity of a nucleon, and so on. Therefore, in the meson theory, it is probable that the "structure" of the elementary particle will present a problem in the investigation of its model, and that one of the most important problems in the theory of the "structure" of elementary particles may consist in the clarification of the relation between the nucleon and the meson field (meson cloud) attached to it.

In this respect, Heisenberg[2] has attempted, using the results of Bloch and Nordsieck,[3] to consider the spectrum of multiple production of mesons as the difference between the proper fields of the nucleon as it is and as it should be after the scattering process; and further he has assumed that:

(I) the "spectrum" of meson cloud can be obtained on the classical basis, and

(II) the probability of the emission of mesons is subject to the Poisson distribution.

In this paper we shall attempt a general formulation of the quantum theoretical treatment of the meson cloud. To this end we introduce an operator which denotes the "spectrum" of the meson field around a nucleon, i.e., the meson cloud. Without making any restrictions on the magnitude of the coupling strength,

* The contents of this paper were briefly reported in Prog. Theoret. Phys. **6**, 426, 628 (1951), but the publication of the paper has been delayed.

[1] For example, the C-meson introduced by Sakata in order to construct a stable electron model does not show any observable effects in the region of quantum electrodynamics, but it does have an observable effect in the case of heavy particles. S. Sakata and H. Umezawa, Prog. Theoret. Phys. **5**, 682 (1950).

[2] W. Heisenberg, Z. Physik **101**, 533 (1936); **113**, 61 (1939).

[3] F. Bloch and A. Nordsieck, Phys. Rev. **52**, 54 (1937).

the discussions are developed exactly by a method based on the quantum theory.

To express this meson spectrum in terms of time-dependent operators of the meson field, we shall use the Heisenberg representation. The various characteristics of the meson cloud will be discussed in Secs. 2, 3, and 4. The discussion of the relation between the meson cloud and certain phenomena (the multiple production of mesons and the magnetic moment of nucleons) is given in Sec. 5.

This treatment of the multiple production of mesons is the covariant generalization of Bloch-Nordsiecks' method for the multiple production of low energy photons (and so of Heisenberg's semiclassical method).

A detailed consideration of the spectrum, i.e., comparing the various theoretical spectra with experimental results and looking for their actual forms suitable to the explanation of the latter, etc., are problems for further investigation.

2. SPECTRUM OF THE MESON FIELD[4]

In this paper we shall discuss, for simplicity, a zero-spin meson field, i.e., a scalar or pseudoscalar field.

The Lagrange function (in Heisenberg representation) is given by

$$\bar{L}=\int\{L^m+L^n+L^{mn}-\bar{\psi}V\psi\}dv, \tag{2.1}$$

where L^m, L^n are the free parts of the Lagrange densities of the meson and the nucleon fields, respectively, and L^{mn} is the interaction part between them, which takes the following forms:

$$L^{mn}=\begin{cases}-g^\nu\bar{\psi}O^\nu\psi U^\nu & \text{(scalar type)}, \quad (2.2)\\ -g^\nu\bar{\psi}O_\mu{}^\nu\psi\partial U^\nu/\partial x_\mu & \text{(vector type)}. \quad (2.2')\end{cases}$$

V is an external potential and ψ, U^ν are field operators of the nucleon and the meson, respectively.

From (2.1) we get the following equations of motion:

$$\left.\begin{aligned}&(\square-\kappa^2)U^\nu=g^\nu\bar{\psi}O^\nu\psi\equiv g^\nu O^\nu(x),\\&(\gamma_\mu\partial/\partial x_\mu+\mu)\psi=-(g^\nu O^\nu U^\nu+V)\psi.\end{aligned}\right\} \tag{2.3}$$

Firstly we shall consider, for brevity, the case of scalar coupling. If there is no interaction in the remote past, the solutions of (2.3) will be as follows:

$$U^\nu(x)=U_{\text{in}}{}^\nu(x)+g^\nu\int\Delta_{\text{ret}}(x-x')O^\nu(x')d^4x', \tag{2.4}$$

$$\psi(x)=\psi_{\text{in}}(x)-\int S_{\text{ret}}(x-x')\{g^\nu O^\nu U^\nu(x')+V(x')\}\psi(x')d^4x', \tag{2.5}$$

where $U_{\text{in}}{}^\nu(x)$ and $\psi_{\text{in}}(x)$ are incoming fields which coincide in the infinite past with $U^\nu(x)$ and $\psi(x)$, respectively, and satisfy respective homogeneous free field equations. Therefore, they satisfy the following commutation relations:

$$\begin{aligned}&[\psi_{\text{in}}{}^\alpha(x),\bar{\psi}_{\text{in}}{}^\beta(x')]_+=-iS_{\alpha\beta}(x-x'),\\&[U_{\text{in}}{}^\nu(x),U_{\text{in}}{}^\mu(x')]=i\delta_{\mu\nu}\Delta(x-x').\end{aligned} \tag{2.6}$$

Furthermore:

$$\begin{aligned}&\Delta_{\text{ret}}(x-x')=\tfrac{1}{2}\{1+\epsilon(x,x')\}\Delta(x-x'),\\&S_{\text{ret}}(x-x')=\tfrac{1}{2}\{1+\epsilon(x,x')\}S(x-x'),\\&\epsilon(x,x')=\begin{cases}+1 & \text{for}\quad \sigma(x)>\sigma'(x'),\\-1 & \text{for}\quad \sigma(x)<\sigma'(x').\end{cases}\end{aligned} \tag{2.7}$$

A given free meson field $U_f(x)$ can be decomposed into the positive frequency part ${}^+U_f(x)$ and the negative frequency part ${}^-U_f(x)$ in the same way as in Schwinger's paper.[5] The Fourier components of $U_f(x)$, are introduced in

$$U_f(x)=\int U_f(k)\delta(k^2+\kappa^2)e^{ik_\mu x_\mu}d^4k, \tag{2.8}$$

and $U(k)(-k_\mu\epsilon_\mu>0)$ and $U(k)(k_\mu\epsilon_\mu>0)$ are written with ${}^+U(k)$ and ${}^-U(k)$, respectively, where ϵ is a unit time-like vector. As the magnitude of the vector ${}^+U_f(k)$, i.e., ${}^+U_f{}^*(k){}^+U_f(k)$, represents the number of mesons with momentum k_μ, it is convenient to use the positive frequency parts of the free meson fields $U^\nu(x,\sigma)$, which coincide with $U^\nu(x)$ of (2.4) on a space-like surface σ, in order to express the number of mesons on the surface σ. From (2.4), we obtain such free meson fields $U^\nu(x,\sigma)$ as follows:

$$U^\nu(x,\sigma)=U_{\text{in}}{}^\nu(x)+g^\nu\int_{-\infty}^{\sigma}\Delta(x-x')O^\nu(x')d^4x'. \tag{2.9}$$

$U^\nu(x,\sigma)$ satisfies the homogeneous free equation for fixed σ and

$$\left.\begin{aligned}&[\psi^\alpha(x,\sigma),\bar{\psi}^\beta(x',\sigma)]_+=-iS_{\alpha\beta}(x-x'),\\&[U^\nu(x,\sigma),U^\mu(x',\sigma)]=i\delta_{\mu\nu}\Delta(x-x').\end{aligned}\right\} \tag{2.10}$$

$$U^\nu(x/\sigma)\equiv[U^\nu(x,\sigma)]_{x\text{ on }\sigma}=U^\nu(x), \tag{2.11}$$

using the notation x/σ for x on σ. Yang and Feldman[6] have shown that $U^\nu(x,\sigma)$ can be obtained by the unitary transformation S from $U^\nu(x,\sigma')$:

$$U^\nu(x,\sigma)=S^{-1}(\sigma,\sigma')U^\nu(x,\sigma')S(\sigma,\sigma'). \tag{2.12}$$

It follows from (2.12) that

$$U^\nu(x/\sigma)=[S^{-1}(\sigma)U_{\text{in}}{}^\nu(x)S(\sigma)]_{x/\sigma}, \tag{2.13}$$

[4] Throughout this paper natural units are used, i.e., $\hbar=C=1$. and κ and μ represent the rest mass of the meson and the nucleon, respectively. A^* is the hermite conjugate operator of A. A_μ is A_1, A_2, A_3 and $A_4=iA_0$. In particular we may write $x_0=t$. The formulation in this section is closely related to Heisenberg's theory appearing in Ann. Physik 5, 339 (1931).

[5] J. Schwinger, Phys. Rev. 75, 651 (1949).

[6] C. N. Yang and D. Feldman, Phys. Rev. 79, 972 (1950).

with

$$S(\sigma)=S(\sigma,-\infty). \quad (2.14)$$

Furthermore, they have shown that

$$i\delta S(\sigma)/\delta\sigma(x)=H^i(x)S(\sigma) \quad (2.15)$$

and that, for various types of meson fields, $H^i(x)$ coincides with the interaction Hamiltonian in the interaction representation in which $U^\nu(x)$ and $\psi(x)$ are replaced by $U_{\text{in}}{}^\nu(x)$ and $\psi_{\text{in}}(x)$, respectively.[6]

Since $U^\nu(x,\sigma)$ satisfies the homogeneous free field equation, we can split it into two parts, and from (2.12), for the positive frequency part, we get

$$^+U^\nu(x,\sigma)=S^{-1}(\sigma)^+U_{\text{in}}{}^\nu(x)S(\sigma), \quad (2.16)$$

which can be written, by (2.9), as follows:

$$^+U^\nu(x,\sigma)={}^+U_{\text{in}}{}^\nu(x)+g^\nu\int_{-\infty}^{\sigma}{}^+\Delta(x-x')O^\nu(x')d^4x' \quad \text{(scalar type)}, \quad (2.17)$$

$$O^\nu(x')=[S^{-1}(\sigma)\bar{\psi}_{\text{in}}(x')O^\nu\psi_{\text{in}}(x')S(\sigma)]_{x'/\sigma}, \quad (2.17')$$

where $^+\Delta(x)$ is the positive frequency part of $\Delta(x)$. We call this $^+U(x,\sigma)$ "the cloud spectrum in the Heisenberg representation" (c-spectrum), by which one can express the number of mesons at x/σ.[7]

As is evident from the definition, $O(x)$ denotes the change of the nucleon spin density due to the interaction H^i between meson and nucleon. This will be called, in the following pages, the "effective spin" (e-spin) of a nucleon in the meson cloud. It describes the nucleon states in the meson cloud and, therefore, plays an important role in the treatment of the meson cloud. Now, we introduce ^+u as follows:

$$^+U^\nu(x,\sigma)\equiv{}^+U_{\text{in}}{}^\nu(x)+(-i)\int_{-\infty}^{\infty}{}^+u^\nu(x,\sigma;x')d^4x'. \quad (2.18)$$

Here ^+u is an operator in the Heisenberg representation and is called the "spectrum density" (d-spectrum) with respect to x' contributing to the meson field at σ.

Equation (2.18) means that the c-spectrum at x/σ is the sum of the contributions of the d-spectrum at all the points denoted by x' and $^+U_{\text{in}}$.

In the absence of an external potential we shall denote $S(\sigma)$, the e-spin and the spectra by $S_0(\sigma)$, $\bar{O}(x)$, and $(^+U_0, {}^+u_0)$, respectively. These spectra are related to the mesonic proper-field.

$(^+U_0, {}^+u_0)$ are obtained by replacing $S(\sigma)$ by $S_0(\sigma)$ respectively in (2.16) and (2.17). When we restrict ourselves to the case in which one nucleon and no mesons exist in the remote past, the c-spectrum at a definite time is called the "c-spectrum of the single nucleon." In this case, there is the following relation:

$$^+U_{\text{in}}{}^\nu(x)\rangle_{m=0}=0$$

and so from (2.18) we have:

$$^+U_0{}^\nu(x,\sigma)\rangle_{m=0}=(-i)\int_{-\infty}^{\infty}{}^+u_0{}^\nu(x,\sigma;x')\rangle_{m=0}d^4x'. \quad (2.19)$$

Operators $^+U^\nu$ and $^+u^\nu$ have forms of nonlinear interactions containing many operators $\bar{\psi}_{\text{in}}$, ψ_{in}, and $U_{\text{in}}{}^\nu$. From (2.18) and (2.17) we obtain

$$^+u^\nu(x,\sigma;x')=ig^\nu\left(\frac{1+\epsilon(\sigma,x')}{2}\right){}^+\Delta(x-x')O^\nu(x'). \quad (2.20)$$

Similarly, in the case of the vector coupling (2.2′), we can obtain the corresponding d-spectrum:

$$^+u^\nu(x,\sigma;x')=ig^\nu\left(\frac{1+\epsilon(\sigma,x')}{2}\right)\frac{\partial^+\Delta(x-x')}{\partial x_\mu}O_\mu{}^\nu(x') \quad (2.20')$$

with

$$O_\mu{}^\nu(x)\equiv[S^{-1}(\sigma)\bar{\psi}_{\text{in}}(x)O_\mu{}^\nu\psi_{\text{in}}(x)S(\sigma)]_{x/\sigma}. \quad (2.20'')$$

$\bar{O}(x)$ is uniquely determined from $^+U_0(x,\sigma)$ as will be pointed out in Sec. 4.

Now, we shall proceed to define the "e-spin with the forced vibration $\bar{O}(x)F[\sigma(x)]$" of the nucleon, the oscillation of which is forced by the effect of its past processes that is described by $F[\sigma(x)]$. Such a forced vibration of the nucleon may be possible when the external disturbance is present. (See Sec. 5.) We shall define "the c-spectrum of a nucleon with the forced vibration" as follows:

$$[^+U_0{}^\nu F\}(x_0,\sigma)\equiv\begin{cases}{}^+U_{\text{in}}{}^\nu(x_0)+g^\nu\int^{\sigma}{}^+\Delta(x_0-x)\bar{O}^\nu(x)F[\sigma(x)]d^4x & \text{(scalar coupling)}, \quad (2.21)\\ {}^+U_{\text{in}}{}^\nu(x_0)+g^\nu\int^{\sigma}\frac{\partial^+\Delta(x_0-x)}{\partial x_\mu}\bar{O}_\mu{}^\nu(x)F[\sigma(x)]d^4x & \text{(vector coupling)}. \quad (2.21')\end{cases}$$

(2.21), (2.21′) also satisfy the homogeneous free equation of the meson field. From (2.19), we obtain

$$[^+U_0F\}(x_0,\sigma)\rangle_{m=0}=(-i)\int_{-\infty}^{\infty}{}^+u_0{}^\nu(x_0,\sigma;x)F[\sigma(x)]d^4x\rangle_{m=0}. \quad (2.22)$$

More generally we use the notation $[AF\}$ for the operator in which the e-spin $\bar{O}(x)$ of the nucleon with the meson cloud contained in the operator A is replaced by $\bar{O}(x)F[\sigma(x)]$. Making the surface σ flat, and using

[7] We can expect that when the interaction between the meson and the nucleon is switched off on a surface σ, mesons are emitted according to the spectrum amplitude $^+U(x/\sigma)$. This was pointed out by Professor Tomonaga.

the relations

$$\left.\begin{aligned} \bar{O}^{\nu}(x) &\equiv \int \bar{O}^{\nu}(k) e^{ikx} d^4k, \\ F[\sigma(x)] &= \int F(b) e^{ibt} db, \\ K &\equiv (\mathbf{k}^2+\kappa^2)^{\frac{1}{2}}, \\ k_\mu &\equiv (k_1, k_2, k_3, k_4 = ik_0), \\ K_\mu &\equiv (k_1, k_2, k_3, iK), \end{aligned}\right\} \quad (2.23)$$

we find after some calculations the following relations:

$$[{}^+U_0{}^{\nu}F\}(x_0,\sigma) = {}^+U_{\text{in}}{}^{\nu}(x_0) + \frac{g}{2}\int \frac{\bar{O}^{\nu}(k)F(b)}{K[b(k)+b]} e^{iKx_0} e^{i[b(k)+b]t} d^4k db \quad \text{(scalar coupling)}, \quad (2.24)$$

$$[{}^+U_0{}^{\nu}F\}(x_0,\sigma) = {}^+U_{\text{in}}{}^{\nu}(x_0) + \frac{g}{2}\int \frac{(iK_\mu)\bar{O}_\mu{}^{\nu}(k)F(b)}{K[b(k)+b]} \times e^{iKx_0} e^{i[b(k)+b]t} d^4k db \quad \text{(vector coupling)}, \quad (2.24')$$

where t is the time of the flat surface σ and

$$b(k) \equiv k_0 - K. \quad (2.25)$$

As seen in (2.24) and (2.24′), k_0 is the energy transfer on σ of the nucleon with the meson cloud, and so $\bar{O}(\mathbf{k}, k_0)$ and $b(k)$ depend on the state of the nucleon on the surface σ through the variable k_0. Now, ${}^+U_0{}^{\nu}(x_0,\sigma)$ can be obtained as the particular case of (2.24), (2.24′) in which $F(x)=1$ and so $F(b)=\delta(b)$. It will then be

$${}^+U_0{}^{\nu}(x_0,\sigma) = {}^+U_{\text{in}}{}^{\nu}(x_0) - \frac{g}{2}\int \frac{\bar{O}^{\nu}(k)}{K(K-k_0)} \times e^{iKx_0} e^{-i(K-k_0)t} d^4k \quad \text{(scalar coupling)}, \quad (2.26)$$

or

$${}^+U_0{}^{\nu}(x_0,\sigma) = {}^+U_{\text{in}}{}^{\nu}(x_0) - \frac{g}{2}\int \frac{iK_\mu \bar{O}_\mu{}^{\nu}(k)}{K(K-k_0)} \times e^{iKx_0} e^{-i(K-k_0)t} d^4k \quad \text{(vector coupling)}. \quad (2.26')$$

Here, let us introduce the number operator of mesons. Since $U(x_0,\sigma)$ satisfies the homogeneous free field equation, the "number operator" $N(\sigma)$ of this meson field on the surface σ is defined as follows:

$$N^{\nu}(\sigma) \equiv \frac{2}{i}\left[\int_\sigma ({}^+U(x,\sigma))^* \frac{\partial {}^+U^{\nu}(x,\sigma)}{\partial x_\mu} d\sigma_\mu\right]_{x/\sigma} \equiv \int N_\sigma{}^{\nu}(k) d^4k. \quad (2.27)$$

Using (2.26) and (2.26′), the expectation value of (2.27) for the state of a nucleon can be written in the following forms (making the surface σ flat):

$$\langle N(\sigma)\rangle_{\substack{n=1\\m=0}} = \frac{g^2}{2}\int \frac{\langle O^{\nu}(k)^* O^{\nu}(k)\rangle_{\substack{n=1\\m=0}}}{K(K-k_0)^2} d^4k \quad \text{(scalar coupling)}, \quad (2.28)$$

$$\langle N(\sigma)\rangle_{\substack{n=1\\m=0}} = \frac{g^2}{2}\int \frac{K_\mu K_\rho}{K(K-k_0)^2} \langle O_\mu{}^{\nu}(k)^* O_\rho{}^{\nu}(k)\rangle_{\substack{n=1\\m=0}} d^4k \quad \text{(vector coupling)}, \quad (2.28')$$

which implicitly depend on σ through the variable k_0.

Finally we will examine the relation between $S(\sigma)$ and $S_0(\sigma)$, when an external field $\bar{\psi}V\psi$ is present. When we denote $S(\sigma)$ as follows as the power series of $V(x)$:

$$S(\sigma) = S_0(\sigma) + S_1(\sigma) + \cdots \quad (2.29)$$

then it is easily seen that

$$S_m(\sigma) = S_0(\sigma)\int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{\infty} dx_2 \cdots \int_{-\infty}^{\infty} dx_m (-i)^m \times V(\sigma, x_1/\sigma_1) V(\sigma_1, x_2/\sigma_2) \cdots V(\sigma_{m-1}, x_m/\sigma_m), \quad (2.30)$$

where

$$V(\sigma; x/\sigma') \equiv \tfrac{1}{2}[1+\epsilon(\sigma,x)] S_0{}^{-1}(\sigma') \bar{\psi}_{\text{in}}(x) V(x) \psi_{\text{in}}(x) S_0(\sigma') \quad (2.31)$$

is the external disturbance acting on the nucleon with the meson cloud. Using these relations, we can obtain the connection between ${}^+U^{\nu}(x,\sigma)$ and ${}^+U_0{}^{\nu}(x,\sigma)$.

Here we remark that for the state Ψ_0 in which there exist one nucleon and no mesons, we have the relation

$$S_0(\infty)\Psi_0 \doteqdot \Psi_0, \quad (2.32)$$

(where the notation $\doteqdot$ means the equality of both sides except for a constant phase factor) under the assumption that the interaction between the meson and the nucleon is adiabatically switched on and switched off at $\sigma=-\infty$ and $\sigma=+\infty$, respectively.[8]†

In our formalism which constantly makes use of the Heisenberg operator, the transition probabilities are

[8] κ and μ are the masses of the bare meson and the bare nucleon respectively, and so they differ from the observable masses κ_{ob} and μ_{ob}:

$$\kappa = \kappa_{\text{ob}} - \delta_m, \qquad \mu = \mu_{\text{ob}} - \delta_n.$$

δ_n and δ_m are the self-energies of a nucleon and a meson, respectively, which are obtained in Sec. 3. Their values are infinite in the usual quantum field theory. Masses appearing in the Δ-functions and the spectrum ${}^+u$ are not $(\kappa_{\text{ob}}, \mu_{\text{ob}})$ but (κ, μ) in the present formalism, which differs from the usual renormalization formalism in which masses appearing in the Δ-functions are $(\kappa_{\text{ob}}, \mu_{\text{ob}})$, and in which (δ_m, δ_n) appears in the interaction terms. Considering the success of the usual mass renormalization in perturbation procedure, it is expected that the (δ_m, δ_n) of the (κ, μ) appearing in the Δ-functions cancels the infinities which originate from the self-energies, although a general proof independent of the perturbation procedure has not yet been given. If so, the infinities of the constant phase factor in (2.32) will be calcelled. Such a renormalization procedure with the use of (δ_m, δ_n) in the Δ-functions is now being investigated.

† *Note added in proof.*—This problem will be discussed in detail by H. Umezawa and S. Kamefuchi, Prog. Theoret. Phys. (to be published).

obtained in the following way. In general, the state of the system can be represented by the numbers $n_1, n_2, \cdots$ of the mesons with the momenta $\mathbf{k}_1, \mathbf{k}_2, \cdots$. Then, the average number of mesons is given by

$$\bar{N}(\sigma)=\langle i|N(\sigma)|i\rangle$$
$$=\sum\langle i|S^{-1}(\sigma)|v\rangle\langle v|S(\sigma)|i\rangle N_{\text{in}}(\sigma, v) \quad \langle 2.33\rangle$$

with

$$N_{\text{in}}(a)=-2i\int {}^{+}U_{\text{in}}{}^{*}(x)[\partial^{+}U_{\text{in}}(x)/\partial x_{\mu}]d\sigma_{\mu}$$

and

$$\langle v'|N_{\text{in}}(\sigma)|v\rangle=N_{\text{in}}(\sigma, v')\delta_{vv'},$$

because $N_{\text{in}}(\sigma)$ is diagonal in this representation. The coefficient $|\langle v|S(\sigma)|i\rangle|^2$ of $N_{\text{in}}(\sigma, v)$ in $\langle 2.33\rangle$ is to be interpreted as the probability that the state on the surface σ is found in v. Thus, we find that the transition matrix between the states i and v is given by

$$\langle v|S\langle\sigma\rangle|i\rangle. \quad \langle 2.34\rangle$$

3. RELATION BETWEEN THE MESONIC PROPER-FIELD AND THE SPECTRUM

We shall discuss in more detail the relation between the mesonic proper-field and the c-spectrum as defined above.

First, we consider intuitively the energy of the interacting meson and nucleon to be composed of the following three parts:

(1) The kinetic energy E^m of the free meson field (meson cloud) which is given by

$$E^m(\sigma)=\left\langle\int\left[\frac{\partial^{+}U^*(x,\sigma)}{\partial x_i}\frac{\partial^{+}U(x,\sigma)}{\partial x_i}+\frac{\partial^{+}U^*(x,\sigma)}{\partial t}\frac{\partial^{+}U(x,\sigma)}{\partial t}+\kappa^{2+}U^*(x,\sigma)^{+}U(x,\sigma)\right]dv\right\rangle_{x/\sigma} \quad (3.1)$$

$$=\left\langle\int KN_{\sigma}(k)d^4k\right\rangle. \quad (3.2)$$

(2) The interaction energy E^{mn} between a nucleon, the meson cloud ${}^{+}U(x,\sigma)$, and the fluctuation field ${}^{-}U(x,\sigma)$, which, taking account of the Hermitian character of $O(x)$, is given by

$$E^{mn}(\sigma)=\left\langle g\int O(x)\{{}^{+}U(x,\sigma)+{}^{+}U^*(x,\sigma)\}dv\right\rangle_{x/\sigma}$$

(scalar coupling), (3.3)

$$E^{mn}(\sigma)=\left\langle g\int O_{\mu}(x)\frac{\partial}{\partial x_{\mu}}\{{}^{+}U(x,\sigma)+{}^{+}U^*(x,\sigma)\}dv-\tfrac{1}{2}g^2\int O_4{}^*(x)O_4(x)dv\right\rangle_{x/\sigma}$$

(vector coupling). (3.3′)

Taking account of the properties ${}^{+}U_{\text{in}}\rangle_{m=0}=0$, we see that the term $U_{\text{in}}(x)$ in $U(x,\sigma)$ does not contribute to $E^{mn}(\sigma)$ for the state $\Psi(m=0)$. From (3.3), we obtain, by means of commutability of $O(x)$ and ${}^{+}U(x,\sigma)$ for x/σ;

$$\langle E^{mn}(\sigma)\rangle_{m=0}=-\left\langle\int\frac{g^2O^*(k)O(k)}{K(K-k_0)}d^4k\right\rangle_{m=0},$$

(scalar coupling). (3.3″)

(3) The kinetic-energy $E^n(\sigma)$ of a free nucleon $\psi(x,\sigma)$, which is

$$E^n(\sigma)=\left\langle\int\left[\bar{\psi}(x,\sigma)\gamma_i\frac{\partial}{\partial x_i}\psi(x,\sigma)+\mu\bar{\psi}(x,\sigma)\psi(x,\sigma)\right]dv\right\rangle_{x/\sigma}, \quad (3.4)$$

where $\psi(x,\sigma)=S^{-1}(\sigma)\psi_{\text{in}}(x)S(\sigma)$ and $\bar{\psi}(x,\sigma)$ satisfies the individual free field equations.

We shall first discuss, under the approximation of Bloch-Nordsieck, the total energy obtained by the above intuitional consideration. The approximation that σ-spin and τ-spin matrices are regarded as c-number unit vectors and $O(k)$ is treated as the change of these classical vectors corresponds to the treatment of Bloch-Nordsieck[3] and Lewis-Oppenheimer-Wouthuysen.[9]

In this case it is easily seen from (2.23) and (2.24) that k_0 is equal to the energy transfer E_p-E_q of the nucleon. Introducing the nucleon mean velocity $\mathbf{v}=(\mathbf{v}_p+\mathbf{v}_q)/2$, as in the Bloch-Nordsieck paper, we may write

$$k_0=E_p-E_q\approx(\mathbf{v},\,\mathbf{p}-\mathbf{q})=(\mathbf{v}\cdot\mathbf{k}),$$

and, therefore,

$$N(\sigma)\simeq\frac{1}{2}\int\frac{|g(\mathbf{q}|O(\mathbf{k})|\mathbf{p})|^2}{K(K-(\mathbf{v}\cdot\mathbf{k}))^2}d\mathbf{k}\simeq\int N(\mathbf{k})d^3\mathbf{k}$$

(scalar coupling). (3.5)

Thus, we obtain

$$E^m(\sigma)\simeq\frac{1}{2}\int\frac{|g(\mathbf{q}|O(\mathbf{k})|\mathbf{p})|^2}{(K-(\mathbf{v}\cdot\mathbf{k}))^2}d^3\mathbf{k}\ \text{(scalar coupling)}. \quad (3.6)$$

Now, in the nonrelativistic and classical approximation, as the nucleon suffers the recoil $-\mathbf{k}$ from the meson of momentum $\mathbf{k}$, we obtain the following kinetic energy:

$$E^n(\sigma)\simeq-\int(\mathbf{v}\cdot\mathbf{k})N(\mathbf{k})d\mathbf{k}$$
$$=-\frac{1}{2}\int\frac{|g(\mathbf{q}|O(\mathbf{k})|\mathbf{p})|^2}{K(K-(\mathbf{v}/\mathbf{k}))^2}(\mathbf{vk})d\mathbf{k}. \quad (3.7)$$

[9] Lewis, Oppenheimer, and Wouthuysen, Phys. Rev. **73**, 127 (1948).

Adding to this the interaction energy $E^{mn}(\sigma)$ and free meson energy $E^m(\sigma)$, the total energy $E(\sigma)$ will be as follows:

$$E=E^n+E^m+E^{mn}=-\frac{1}{2}\int\frac{|g(\mathbf{p}+\mathbf{k}|O(\mathbf{k})|\mathbf{p})|^2}{K(K-(\mathbf{v}\cdot\mathbf{k}))}d\mathbf{k}\quad\text{(scalar coupling).}\quad(3.8)$$

Equation (3.8) agrees exactly with the self-energy obtained by Bloch-Nordsieck[3] and Pauli-Fierz[10] by a canonical transformation.

It can easily be seen that the energy of the meson-nucleon system is given exactly by $E=E^n+E^m+E^{mn}$.

The total energy of the meson-nucleon system is, in the Heisenberg representation,

$$-\int T_{44}(x)dv=-\int T_{44}{}^m(x)dv-\int T_{44}{}^n(x)dv,\quad(3.9)$$

where

$$T_{44}{}^m(x)=\begin{cases}\dfrac{\partial U^\nu(x)}{\partial x_4}\dfrac{\partial U^\nu(x)}{\partial x_4}+L^m(x) & \text{(scalar coupling),}\quad(3.10)\\[2ex] \left(\dfrac{\partial U^\nu(x)}{\partial x_4}+g^\nu\bar{\psi}O_4{}^\nu\psi\right)\dfrac{\partial U^\nu(x)}{\partial x_4}+L^m(x) & \text{(vector coupling),}\quad(3.11)\end{cases}$$

$$T_{44}{}^n(x)=\frac{1}{2}\left\{\bar{\psi}(x)\gamma_4\frac{\partial}{\partial x_4}\psi(x)+\bar{\psi}'(x)\gamma_4\frac{\partial}{\partial x_4}\psi'(x)\right\},\quad(3.12)$$

$$L^m=\frac{1}{2}\left\{\frac{\partial U^\nu(x)}{\partial x_\mu}\frac{\partial U^\nu(x)}{\partial x_\mu}+\kappa^2U^\nu(x)U^\nu(x)\right\}.\quad(3.13)$$

On the other hand, the canonical transformation $S(\sigma)$ requires the following replacement:

$$\frac{\partial U(x/\sigma)}{\partial x_\mu}=\begin{cases}\left[\dfrac{\partial U(x,\sigma)}{\partial x_\mu}\right]_{x/\sigma} & \text{(scalar coupling),}\\[2ex] \left[\dfrac{\partial U(x,\sigma)}{\partial x_\mu}+n_\mu n_\nu g\bar{\psi}(x,\sigma)O_\nu\psi(x,\sigma)\right]_{x/\sigma} & \text{(vector coupling),}\end{cases}\quad(3.14)$$

$$\frac{\partial\psi(x/\sigma)}{\partial x_\mu}=\left[\frac{\partial\psi(x,\sigma)}{\partial x_\mu}+\begin{matrix}gn_\mu(n\gamma)U(x,\sigma)O\psi(x,\sigma) & \text{(scalar coupling),}\\ gn_\mu(n\gamma)\dfrac{\partial U(x,\sigma)}{\partial x_\nu}O_\nu\psi(x,\sigma) & \text{(vector coupling).}\end{matrix}\right]_{x/\sigma}\quad(3.15)$$

From (3.14) and (3.15), it is clear that $-\{\int T_{44}{}^n(x)dv\}$ is equal to $E^{mn}+E^n$. Using the zero-point energy E_0, it is easily shown that

$$-\{T_{44}{}^m(x)dv\}=E^m+E_0,\quad(3.16)$$

where

$$E_0=-i\int_\sigma\left[\frac{\partial^2}{\partial x_4{}^2}+\Delta(x-x')\right]_{x=x'}dv=\frac{1}{2}\int Kd\mathbf{k}.\quad(3.17)$$

If the zero-point energy is subtracted by the vacuum-subtraction method, $-\int T_{44}(x)dv$, (3.9), coincides with $E^m+E^n+E^{mn}$ obtained by intuitional consideration.[11] The fact that we must not neglect E^{mn} means that the cloud and the nucleon cannot be considered separately. (The usual intuitive picture of the meson cloud is not valid in this sense.)

It is noteworthy that E^m and E^{mn} are determined only by the c-spectrum, since $O(x)$ is calculated from a given spectrum ${}^+U(x,\sigma)$ as shown in the next section.

So far we have considered only the real field operator ${}^+U^\nu(x,\sigma)$, but the charged meson field with complex field operators U and U^* can also be treated in a similar manner.

${}^+U^\pm$ (positive frequency parts of $\pm$ charged meson fields $U^\pm(x_0,\sigma)$) are introduced as follows:

$${}^+U^\pm(x_0,\sigma)={}^+U_{\text{in}}{}^\pm(x_0)+(-i)\int_\infty^\infty{}^+u^\pm(x_0,\sigma;x)d^4x.\quad(3.18)$$

When the interaction is of the form $g(\bar{\psi}O^+\psi U+\bar{\psi}O^-\psi U^*)$, we get

$$\left.\begin{aligned}{}^+u^+(x_0/\sigma;x)&=ig{}^+\Delta_{\text{ret}}(x_0-x)O^-(x),\\ {}^+u^-(x_0/\sigma;x)&=ig{}^+\Delta_{\text{ret}}(x_0-x)O^+(x).\end{aligned}\right\}\quad(3.19)$$

4. AN EXAMPLE OF THE SPECTRUM

Since it is very difficult actually to calculate the exact c-spectrum from the present quantum field theory, we must solve the problem by using suitable methods of approximation in various cases.

Here we shall give an example in which the d-spectrum and the e-spin of a nucleon in the meson cloud can be obtained from the given c-spectrum.

Let us consider the pseudoscalar meson with pseudovector coupling under the strong coupling approxima-

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[10] W. Pauli and M. Fierz, Nuovo cimento **15**, No. 3 (1938).

[11] Thus, strictly speaking, in order to determine the binding energy of the stationary state of the A nucleons system, we must take into account not only the kinetic energy $E^n(A)$ of the A nucleons and the interaction energy $E^{mn}(A)$, but also the kinetic energy $E^m(A)$ of the meson cloud.

tion. In this case the interaction is given by:

$$L^{mn}=-g(\tau^+U+\tau^-U^*+2^{-\frac{1}{2}}\tau^3)D(x), \qquad (4.1)$$

where $D(x)$ is a spherical symmetric function representing the extended source. The Fourier component of $D(x)$ is

$$F(k)=\int D(x)e^{-i\mathbf{kr}}dv, \qquad (4.2)$$

and if we use r_0, as the radius of the source, then we may write as follows:

$$\left.\begin{aligned} F(k)&=1 \quad \text{for} \quad k\lesssim 1/(4\pi)^2 r_0,\\ F(k)&=0 \quad \text{for} \quad k\gtrsim 1/(4\pi)^2 r_0.\end{aligned}\right\} \qquad (4.3)$$

Under the condition of the strong coupling ($\kappa g \gg r_0\kappa$, $r_0\kappa \ll 1$) we get in the zeroth-order approximation the following meson field:[12]

$$\left.\begin{aligned} U^\pm(x/\sigma)\rangle_{m=0}&=(g/8\pi)(\mathbf{e}^1\pm i\mathbf{e}^2)\mathrm{grad}K(x)e^{-ik_0t}\\ U^3(x/\sigma)\rangle_{m=0}&=(g/4\pi 2^{\frac{1}{2}})\mathbf{e}^3\,\mathrm{grad}K(x)e^{-ik_0t},\end{aligned}\right\} \qquad (4.4)$$

where

$$K(x)=\int D(x')\exp\{-\kappa|\mathbf{r}-\mathbf{r}'|\}/|\mathbf{r}-\mathbf{r}'|\cdot dv.$$

We can obtain the source $\bar{O}(x)$ by means of (4.7):

$$\left.\begin{aligned} O^\pm(x)\rangle_{m=0}&=-\frac{\mathbf{e}^1\pm i\mathbf{e}^2}{2}\int\frac{(K^2-k_0^2)}{2K^2}F(k)e^{ik_\mu x_\mu}\frac{d\mathbf{k}}{(2\pi)^3}\\ O^3(x)\rangle_{m=0}&=-\frac{\mathbf{e}^3}{2^{\frac{1}{2}}}\int\frac{(K^2-k_0^2)}{2K^2}F(k)e^{ik_\mu x_\mu}\frac{d\mathbf{k}}{(2\pi)^3}\end{aligned}\right\} \qquad (4.5)$$

where $\mathbf{e}^i(i=1,2,3)$ is such a q-number as to make the interaction Hamiltonian diagonal.

${}^+U_0(x,\sigma)$ can be obtained from (2.26′) as follows:

$$\left.\begin{aligned} &{}^+U_0^\pm(x,\sigma)\rangle_{m=0}\\ &\quad=\frac{i}{2}g\int\frac{(\mathbf{e}^1\mp i\mathbf{e}^2,\mathbf{k})}{2}\frac{(K^2-k_0^2)}{K^2}\frac{F(k)}{2K(K-k_0)}\\ &\qquad\qquad\times e^{iK_\mu x_\mu+i(K-k_0)t_0}\frac{d\mathbf{k}}{(2\pi)^3}\\ &{}^+U_0^3(x,\sigma)\rangle_{m=0}\\ &\quad=\frac{i}{2}g\int\frac{(\mathbf{e}^3,\mathbf{k})}{2^{\frac{1}{2}}}\frac{(K^2-k_0^2)}{K^2}\frac{F(k)}{2K(K-k_0)}\\ &\qquad\qquad\times e^{iK_\mu x_\mu+i(K-k_0)t_0}\frac{d\mathbf{k}}{(2\pi)^3}.\end{aligned}\right\} \qquad (4.6)$$

Thus, if the meson field $U_0(x/\sigma)$ is obtained, by whatever means it may be, the e-spins of the nucleon in the meson cloud are uniquely determined by the following relations:

$$\left.\begin{aligned} g\bar{O}(x)&=(\Box-\kappa^2)U_0(x/\sigma)\\ &\qquad\text{(scalar coupling)}\\ -g\partial\bar{O}_\mu(x)/\partial x_\mu&=(\Box-\kappa^2)U_0(x/\sigma)\\ &\qquad\text{(vector coupling).}\end{aligned}\right\} \qquad (4.7)$$

${}^+U_0(x,\sigma)$ can be determined by means of (2.26), (2.26′), and the Fourier representation of $\bar{O}(x)$ and $\bar{O}_\mu(x)$. Such ${}^+U_0(x,\sigma)$ is obtained by replacing $\exp(ik_0t)$ and $k_\mu\exp(ik_0t)$ in the Fourier representation of $U(x/\sigma)$ by $(K^2-k_0^2)/2K(K-k_0)\cdot\exp\{iKx_0-i(K-k_0)t\}$ (for the scalar coupling case) and $K_\mu(K^2-k_0^2)/2K(K-k_0)\cdot\exp\{iKx_0-i(K-k_0)t\}$ (for the vector coupling case), respectively, where t is the time on the flat surface σ.

5. TREATMENT OF SOME PHENOMENA BY THE SPECTRUM

It is, of course, not possible to observe directly the meson cloud, and so its physical properties must be found in the correlated phenomena. In this connection we have considered, in Sec. 3, the self-energy of the nucleon, which is rather an inadequate phenomenon for our consideration in spite of its immediate relation to the cloud, since its contribution is already contained in the experimental mass. To find out the nature of the proper field experimentally, therefore, we must put it under the disturbance of external fields. We may take up here the multiple production of mesons and the anomalous magnetic moment of the nucleon, discuss the relations between the spectrum of the proper field and these two phenomena, and re-examine along these lines the various theoretical results so far presented.

(1) Multiple Production of Mesons (Covariant Generalization of the Bloch-Nordsieck Method)[13]

The process of the production of mesons is described by the transition from the mesonic vacuum state Ψ_0 to the state $\Psi(n)$ with n mesons. Introducing an operator

$$W^n(x_0)\equiv[{}^+U_{\mathrm{in}}(x_0)]^n/n!,^{14} \qquad (5.1)$$

the state $\Psi(m=n)$ is given by $W_n^*(x_0)\Psi_0$. (Taking into account the change of the normalization factor[15] $\Pi_i(2K_i)$ in (5.8)).

[12] W. Pauli and S. M. Dancoff, Phys. Rev. 62, 85 (1942).

[13] After completion of the work, we have received Phil. Mag. 42, 244 (1951) in which W. Thirring and B. Touschek describe a method of treatment of this problem, similar to the method of this paper.

[14] $\psi(\mathbf{k}_1,\cdots,\mathbf{k}_n)$, in which there are n mesons with momenta $\mathbf{k}_i(i=1,\cdots,n)$ is equivalent, apart from the constant phase factor $\exp(i\Sigma\mathbf{k}_ix_0)$ (x_0 is a given space-time point), to

$$(\mathbf{k}_1,\cdots,\mathbf{k}_n|\Pi_i(2K_i)^{\frac{1}{2}}W(x_0)^*\Psi_0.$$

This fact is taken into consideration when we use (5.1) and (5.8) below.

[15] Of course, the Fourier transformation of $W^n(x_0)$ with respect to x_0 corresponds to the creation operators of final mesons with the given momenta $\mathbf{k}_1,\cdots,\mathbf{k}_n$.

Then, the transition matrix element for the production of n mesons on the surface σ is equivalent to the following matrix element between Ψ_0^* and Ψ_0. [See (2.34)]:

$$S^n(x_0/\sigma)=\langle W^n(x_0)S(\sigma)\rangle_{m=0}. \tag{5.2}$$

We have the following relation:

$$i\frac{\delta}{\delta\sigma(x)}S_0^{-1}(\sigma)W^n(x_0)S(\sigma) ={}^+u_0(x_0,\sigma';x)S_0^{-1}(\sigma)W^{n-1}(x_0)S(\sigma) +V(\sigma';x)S_0^{-1}(\sigma)W^n(x_0)S(\sigma), \tag{5.3}$$

where σ' is a certain surface later than σ, $V(\sigma';x)$ is

$$\tfrac{1}{2}\{1+\epsilon(\sigma,x)\}[S_0^{-1}(\sigma)\bar{\psi}_{\rm in}(x)V(x)\psi_{\rm in}(x)S_0(\sigma)]_{x/\sigma}.$$

$V(\sigma';x)$ means the external interaction acting on the nucleon accompanied by the meson cloud.

The solution of (5.3), which has the zero initial value for $\sigma=-\infty$ $(n>0)$, is given as follows:

$$\begin{aligned}S_0^{-1}(\sigma)W^n(x_0)S(\sigma)\rangle_{m=0} &=\sum_{j=0}^{\infty}\sum_{m_1=0}^{m_2}\cdots\sum_{m_j=0}^{n}\int_{-\infty}^{\sigma}\cdots\int_{-\infty}^{\sigma}dx_1\cdots \\ &\times dx_{m_1}dy_1dx_{m_1+1}\cdots dx_{m_j}dy_jdx_{m_j+1}\cdots dx_n \\ &\times{}^+u_0(x_0,\sigma;x_1){}^+u_0(x_0,\sigma_1;x_2)\cdots{}^+u_0(x_0,\sigma_{m_1-1};x_{m_1}) \\ &\times V(\sigma_{m_1};y_1){}^+u_0(x_0,\sigma_{y_1};x_{m_1+1})\cdots \\ &\times{}^+u_0(x_0,\sigma_{m_j-1};x_{m_j})V(\sigma_{m_j};y_j){}^+u_0(x_0,\sigma_{y_j};x_{m_j+1})\cdots \\ &\times{}^+u_0(x_0,\sigma_{n-1};x_n)\rangle_{m=0}.\end{aligned} \tag{5.3'}$$

This solution can be easily obtained by the following consideration: From (5.3), we can get[16]

$$\left.\begin{aligned} i\frac{\delta}{\delta\sigma(x)}T&=\{a^+u(x_0,\sigma';x)+V(\sigma';x)\}T,\\ T&=\sum_{n=0}^{\infty}a^nS_0^{-1}(\sigma)W^n(x_0)S(\sigma)\rangle,\\ T(\sigma&\to-\infty)=1.\end{aligned}\right\} \tag{5.4}$$

Thus, we get $S_0^{-1}(\sigma)W^nS(\sigma)$ as the nth-order term with respect to $a^+u_0(x_0,\sigma;x)$ in T. We may also see, from (5.4), that T has the form of the S-matrix with the interaction a^+u_0+V. Of course, we can see directly, without using (5.4), that (5.3′) satisfies Eq. (5.3). Thus, when we restrict ourselves to the approximation of the first order with respect to $V(x)$ (though we can also treat similarly the case of the higher order), we obtain $S^n(x_0/\sigma)$ from T:

$$\begin{aligned}S^n(x_0/\sigma)=(-i)^{n+1}\Big\langle\int_{-\infty}^{\sigma}&\cdots\int_{-\infty}^{\sigma}dx_1\cdots dx_mdydx_{m+1}\cdots \\ &\times dx_m{}^+u_0(x_0/\sigma;x_1){}^+u_0(x_0,\sigma_1;x_2)\cdots \\ &\times{}^+u_0(x_0,\sigma_{m-1};x_m)V(\sigma_m;y){}^+u_0(x_0,\sigma_y;x_{m+1})\cdots \\ &\times{}^+u_0(x_0,\sigma_{n-1};x_n)\Big\rangle_{m=0},\end{aligned} \tag{5.5}$$

where we have eliminated $S_0(\infty)$ by the use of the relation (2.32). By means of (2.22), we obtain, therefore, from (5.5),

$$S^n(x_0/\infty) =\Big\langle\sum_{m=0}^{n}(-i)^{n+1}[({}^+U_0)^mV({}^+U_0)^{n-m}\}(x_0/\infty)\Big\rangle_{m=0}, \tag{5.6}$$

where we have used the notation $[A_1A_2\cdots A_n\}\equiv[A_1[A_2[\cdots[A_{n-1},A_n\}\}\cdots\}$. Equations (5.5) and (5.6) show that the production of mesons is described by the iteration of the spectra, the vibration of which is influenced by the effect of the meson-production in the preceding processes. Equation (5.5) shows that ${}^+u_0(x_0/\sigma,x')$ has a physical meaning which may correspond to that of Heisenberg's η-matrix.[17] Moreover, we may say that (5.6) is the quantum theoretical relation which corresponds to the Heisenberg's classical treatment of the multiple production of mesons[4] based on the classical meson spectrum which has been briefly explained in Sec. 1.

The probability that n mesons are produced is given by

$$dw_n=\{\prod_{i=1}^{n}(2K_id\mathbf{k}_i)\}|S(\mathbf{k}_1,\cdots,\mathbf{k}_n)|^2, \tag{5.7}$$

$$S^n(x_0/\infty)\equiv\int S(\mathbf{k}_1,\cdots,\mathbf{k}_n)e^{i\Sigma\mathbf{k}_ix}d\mathbf{k}_1\cdots d\mathbf{k}_n, \tag{5.8}$$

where $S(\mathbf{k}_1,\cdots,\mathbf{k}_n)$ is the matrix element of $S^n(x_0/\infty)$ responsible for the process under consideration.

Equations (5.5), (5.7), and (5.8) will afford a starting-point for our discussion of the multiple production of mesons. This is an exact relation obtained by the quantum field theory and is valid for any magnitude of the coupling strength between meson and nucleon fields.

Now, we shall investigate the relation (5.5) in detail. We introduce $v(l)$ as follows:

$$\bar{\psi}(y)V(y)\psi(y)\equiv\int v(l)e^{ily}d^4l. \tag{5.9}$$

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[16] a is a constant introduced from the dimensional consideration.

[17] W. Heisenberg, Z. Physik **120**, 673 (1943).

Then, using (2.24) and (2.24′), we obtain, from (5.6):

$$S(\mathbf{k}_1,\cdots,\mathbf{k}_n)=\left\{\prod_{i=1}^{n}\frac{1}{(2K_i)}\right\}\sum_{a=0}^{n}\times\sum_{(\mathrm{perm})}^{(1,\cdots,n)}\langle\bar{O}(k_1)\bar{O}(k_2)\cdots\bar{O}(k_a)v(l)\bar{O}(k_{a+1})\cdots\bar{O}(k_n)\rangle_{m=0}$$
$$\times\frac{(-1)^{n-a}\delta(b_1+\cdots+b_n+l_0)d^4l}{b_n\cdots(b_{a+1}+\cdots+b_n)(b_1+b_2+\cdots+b_a)\cdots(b_1+b_2)b_1},\quad(5.10)$$

where $\sum_{(\mathrm{perm})}^{(1,\cdots,n)}$ means the summation over all permutations of the group $(\mathbf{k}_1,\cdots,\mathbf{k}_n)$ and $b_i\equiv b(k_i)$. From (5.9) it is seen that $l_\mu=(\mathbf{l}, il_0)$ is the energy-momentum transfer of the nucleon accompanying the meson cloud during the scattering process by the external potential $V(y)$.

In (5.10), $\bar{O}(k)$ is defined by

$$\bar{O}(k)\equiv\begin{cases}\bar{O}(k) & \text{(scalar coupling)}, \quad (5.11)\\ iK_\mu\bar{O}_\mu(k) & \text{(vector coupling)}. \quad (5.12)\end{cases}$$

Equation (5.10) gives the intuitive picture of the multiple production of mesons: the meson-production is caused by the oscillation of the e-spin of the nucleon with the meson cloud in the external potential. Now, in order to compare the exact formulas (5.5) and (5.10) with the approximate result of Bloch and Nordsieck, we examine (5.10) under the following conditions, which are satisfied in the Bloch-Nordsieck case:

(1) $O(k_i)(i=1, 2, \cdots, n)$ and $v(l)$ are commutable with one another.

(2) The nucleon with the meson cloud before and after the scattering due to the external potential $v(l)$ can be assumed to be in the states ( )″ and ( )′, respectively. We denote $O(k)$, $b(k)$ in the states ( )′ and ( )″ as $O(k)'$, $b(k)'$, $O(k)''$, and $b(k)''$, respectively.

Under these approximations it is shown below that mesons are produced according to the Poisson distribution.

In rewriting (5.10) it is convenient to use the following relation:

$$\sum_{(\mathrm{perm})}^{(1,2,\cdots,n)}1/b_1(b_1+b_2)\cdots(b_1+\cdots+b_n)=1/b_1b_2\cdots b_n,\quad(5.13)$$

and so we obtain

$$\left\langle\sum_{a=0}^{n}\sum_{(\mathrm{perm})}^{(1,2,\cdots,n)}\frac{\bar{O}(k_1)'\cdots\bar{O}(k_a)'v(l)\bar{O}(k_{a+1})''\cdots\bar{O}(k_{n-1})''\bar{O}(k_n)''(-1)^{n-a}}{b_1'(b_1'+b_2')\cdots(b_1'+\cdots+b_a')(b_{a+1}''+\cdots+b_n'')\cdots(b_{n-1}''+b_n'')b_n''}\right\rangle_{m=0}$$
$$=\left[\frac{\bar{O}(k_1)'\cdots\bar{O}(k_n)'}{b_1'b_2'\cdots b_n'}-\left(\frac{\bar{O}(k_1)''\bar{O}(k_2)'\cdots\bar{O}(k_n')}{b_1''b_2'\cdots b_n'}+\cdots\right)+\left(\frac{\bar{O}(k_1)''\bar{O}(k_2)''\bar{O}(k_3)'\cdots\bar{O}(k_n)'}{b_1''b_2''b_3'\cdots b_n'}+\cdots\right)\right.$$
$$\left.+\cdots+(-1)^n\frac{\bar{O}(k_1)''\bar{O}(k_2)''\cdots\bar{O}(k_n)''}{b_1''b_2''\cdots b_n''}\right]v(l)\quad(5.14)$$
$$=\left\{\prod_{i=1}^{n}\Delta\frac{\bar{O}(k_i)}{b_i}\right\}v(l),$$

where

$$\Delta\frac{\bar{O}(k_i)}{b_i}\equiv\frac{\bar{O}(k_i)'}{b_i'}-\frac{\bar{O}(k_i)''}{b_i''},\quad(5.16)$$

which denotes the momentum distribution of the difference of two spectra in states ( )′ and ( )″, and so is equivalent to $(\sigma_{s\lambda}-\tau_{s\lambda})$ appearing in Bloch and Nordsieck's paper.

Then we obtain from (5.15) and (5.8),

$$dw_n=\left\{\prod_{i=1}^{n}\frac{d\mathbf{k}_i}{2K_i}\left(\Delta\frac{\bar{O}(k_i)}{b_i}\right)^2\right\}|v(l)|^2,\quad(5.17)$$

which is integrated over $\mathbf{k}_i(i=1, 2, \cdots)$ to give

$$w_n=(1/n!)\left\{\int\Delta N(\mathbf{k})d\mathbf{k}\right\}^n|v(l)|^2,\quad(5.18)$$

with

$$\Delta N(\mathbf{k})\equiv\frac{g^2}{2}\frac{1}{K}\left(\Delta\frac{O(k)}{b(k)}\right)^2=N(k)'-N(k)''.\quad(5.19)$$

In (5.18) the three-fold integration in momentum space is to be performed in such a way as to satisfy the energy momentum conservation law. (5.17) and (5.18) show that the multiple production of mesons is caused by the change of the state of the nucleon accompanying the meson cloud (spin, τ-spin states, etc.). (5.18), which has been obtained under the conditions (1) and (2), agrees with the Bloch-Nordsieck and Lewis-Oppenheimer-Wouthuysen formulas for multiple production of bosons. In fact, by substituting the approximate spectra[18] into (5.19) we can show that in the case of scalar or pseudoscalar meson fields (5.18) gives the same results as obtained by Lewis, Oppenheimer, and Wouthuysen[10] (see Appendix). Therefore, (5.5) and (5.11), from which (5.18) has been obtained, can be regarded as the generalized formulas of the Bloch-Nordsieck result.

According to (5.18), the probability for the production of n mesons due to a nucleon (a)-nucleon (b) collision is found to be, under the conditions (1) and (2) (Poisson distribution),

$$w_n = \sum_{n_a=0}^{n} \binom{n}{n_a} \frac{1}{n_a!} \left\{ \int \Delta N_a(\mathbf{k}) d\mathbf{k} \right\}^{n_a} \times \frac{1}{n_b!} \left\{ \int \Delta N_b(\mathbf{k}) d\mathbf{k} \right\}^{n_b}. \tag{5.18'}$$

A detailed account of this case, in which the conditions (1) and (2) (Poisson distribution) are fulfilled, (the relation between the energy spectrum of produced mesons and the c-spectrum of the meson field around a nucleon) will be given in the Appendix.

(2) Magnetic Moment of the Nucleon

The magnetic moment of the nucleon may also be interpreted as a phenomenon which expresses the effect of the meson cloud around it.[19] It is obtained by computing the corrections due to the mesonic interaction of the operator

$$\mathbf{M} = \frac{1}{2} \int [\mathbf{r} \times \mathbf{J}] dv,$$

and for the case of a charged spinless meson field, $\mathbf{J}$ is given by

$$\begin{aligned}
J_\mu &= J_\mu{}^n + J_\mu{}^m + J_\mu{}^{mn}, \\
J_\mu{}^n &= -\tfrac{1}{2} ie\bar{\psi}\gamma_\mu(1-\tau_3)\psi, \\
J_\mu{}^m &= ie\{(\partial U^*/\partial x_\mu)U - (\partial U/\partial x_\mu)U^*\}, \qquad (5.20) \\
J_\mu{}^{mn} &= -eg\bar{\psi}O_\mu{}^+\psi U - eg\bar{\psi}O_\mu{}^-\psi U^* \\
\mu &= 1, 2, 3. \qquad \text{(vector coupling)}
\end{aligned}$$

The mesonic correction of the moment $\mathbf{M}$ arising from the mesonic current $\mathbf{J}^m$ is given by

$$\langle \mathbf{M}^m \rangle = \frac{1}{2} \int [\mathbf{r} \times \langle \mathbf{J}^m(x) \rangle] dv. \tag{5.21}$$

The external magnetic field exerts on the meson cloud the following effects: (1) the scattering of mesons, and (2) the pair creation and annihilation of mesons, by which the current due to the meson cloud is disturbed and then returns to its original stationary state. In a manner similar to the self-energy case, $\mathbf{J}^m$ can be written in terms of the free meson field, i.e.,

$$\langle J_\mu{}^m(x) \rangle = ie\langle \partial U^*(x,\sigma)/\partial x_\mu \cdot U(x,\sigma) - \partial U(x,\sigma)/\partial x_\mu \cdot U^*(x,\sigma) \rangle_{x/\sigma}, \qquad (5.21)$$

where the terms ${}^-U^{++}U^+$, ${}^-U^{-+}U^-$ denote the scattering effects, and ${}^-U^{+-}U^-$, ${}^+U^{++}U^-$ denote the pair creation and annihilation, respectively. Actually, $\mathbf{M}^m$ has a nonvanishing value even for the spinless meson, due to the fact that the positively and negatively charged parts of the meson cloud have their respective angular momenta. In the weak coupling theory, for instance, the perturbation calculations show that the contribution from $\mathbf{M}^m$ and, $\mathbf{M}^{mn}$ is of the same order of magnitude as the nucleon current.[20]

By the same argument as in the derivation of (3.3), the magnetic moment due to the current $\mathbf{J}^{mn}$ is found to be

$$\langle \mathbf{M}^{mn} \rangle = \tfrac{1}{2} eg^2 \left\langle \int [\mathbf{r} \times (\mathbf{O}^+(x) U(x,\sigma) - \mathbf{O}^-(x) U^*(x,\sigma))] dv \right\rangle_{x/\sigma}. \tag{5.22}$$

It is worth noticing that $\mathbf{M}^m$ and $\mathbf{M}^{mn}$ can be uniquely determined in this way by the given c-spectrum ${}^+U(x,\sigma)$.

$(\mathbf{M}^m + \mathbf{M}^{mn})$ has the same magnitude but opposite signs for proton and neutron, due to the different signs of electric charge of the meson cloud.[21] Remembering that the existence of a small difference in the anomalous magnetic moments of the proton and the neutron has been experimentally confirmed, we may say that the nucleon current contributes slightly to its magnetic moment.

The magnetic moment $\mathbf{M}^n$ due to the current $\mathbf{J}^n$ is determined by the behavior of τ_3- and σ-spins in the meson cloud; that is, it is given by the expectation

[18] Such an approximation leads to the erroneous nuclear forces, i.e., the ratio of exchange to ordinary forces becomes very small. R. Serber and S. M. Dancoff, Phys. Rev. 63, 143 (1943).

[19] While, by expressing the charge distribution ρ around the nucleon in terms of the spectrum, the problem of neutron-electron interaction is more easily treated than that of the magnetic moment, the information obtained from the former is restricted; i.e., it gives only the depth of the equivalent square well (symmetric static part of the cloud).

[20] From the Case's results for the anomalous magnetic moment of the nucleon, the meson cloud is supposed to spread over a considerably wide region around the nucleon. According to Brueckner, on the contrary, to explain the isotropic angular distribution of the mesons produced by γ-rays it is desirable that the cloud should be closely bound to the nucleon. In the problem of the meson cloud, therefore, it is very important to look for a consistent model which satisfies the above two requirements. Recently, an example of such a model has been suggested by Y. Fujimoto and H. Miyazawa, who attempted to explain the isotropic angular distribution of produced mesons by assuming the existence of isobar energy levels of the cloud, an extension of which satisfies the requirement of the magnetic moment. Y. Fujimoto and H. Miyazawa, Prog. Theoret. Phys. 5, 1052 (1950).

[21] This point has been proved, including the cases of higher order approximation, in the perturbation calculation. Y. Takahashi, Prog. Theoret. Phys. 6, 624 (1951).

values of[22]

$$\int[\mathbf{r}\times\Delta\{\tfrac{1}{2}(1-\tau_3)\boldsymbol{\gamma}\}]dv.$$

But, in general, it is not always possible to calculate $\Delta\{\frac{1}{2}(1-\tau_3)\boldsymbol{\gamma}\}$ from a given spectrum. Therefore, we have to be content with the following intuitive and rough estimation: From an intuitive model, we shall estimate $\mathbf{M}^n$ as follows.

$$\mathbf{M}^n\sim(e/2\mu)P,$$

where eP is the average value of the charge of a nucleon in the meson cloud. Making a transformation (3.14), (3.15) and calculating in the same way as in the self-energy case, we get

$$\begin{aligned} e_m=\left\langle\int\rho dv\right\rangle_{\substack{m=0\\n=1}}=\Big\langle e\int[\{&\partial^-U^+(x,\sigma)/\partial x_4\cdot{}^+U^+(x,\sigma)\\ &-\partial^+U^+(x,\sigma)/\partial x_4\cdot{}^-U^+(x,\sigma)\}\\ &-\{\partial^-U^-(x,\sigma)/\partial x_4\cdot{}^+U^-(x,\sigma)\\ &-\partial^+U^-(x,\sigma)/\partial x_4\cdot{}^-U^-(x,\sigma)\}]_{x/\sigma}dv\Big\rangle_{\substack{m=0\\n=1}}. \end{aligned}\quad(5.23)$$

Further rewriting gives the expression

$$e_m=e\left\langle\int(N^+(\mathbf{k})-N^-(\mathbf{k}))d\mathbf{k}\right\rangle.\quad(5.24)$$

The average value of the charge of a nucleon in the meson cloud is obtained by the relation

$$eP=e_t-e_m\quad(5.25)$$

where e_t is the total charge of the nucleon-meson system. ($e_t=e$ for an apparent proton and $e_t=0$ for an apparent neutron).

For the spectrum (4.5) given in Sec. IV, we can calculate $\mathbf{M}^m$ from (5.21) and obtain the result, for the system of a nucleon and no meson,

$$\begin{gathered}\langle\mathbf{M}^m\rangle_{\substack{m=0\\n=1}}=-2eg^2\mathbf{e}_3/12(4\pi r_0),\\ [\mathbf{e}_1\times\mathbf{e}_2]=\mathbf{e}_3.\end{gathered}\quad(5.26)$$

For the spectrum (4.5), we get, from (4.6) and (5.22),

$$\langle\mathbf{M}^{mn}\rangle_{\substack{m=0\\n=1}}=eg^2\mathbf{e}_3/12(4\pi r_0).\quad(5.27)$$

Adding (5.26) to the above expression (5.27) we obtain

$$\langle\mathbf{M}^{mn}+\mathbf{M}^m\rangle_{\substack{m=0\\n=1}}=-eg^2\mathbf{e}_3/12(4\pi r_0)\quad(\text{for (4.5)}),\quad(5.28)$$

which is exactly the result obtained by Pauli and Dancoff. Equation (5.28) has opposite signs for proton and neutron since the vector $\mathbf{e}_3$ has opposite signs for them. For the spectrum (4.5), we find $e_m=0$ and $P=1$ for an apparent proton and $P=0$ for an apparent neutron and so $\mathbf{M}^n\approx0$. Here it is to be remembered that, since (4.5) is merely a leading term in the strong coupling theory, e_m does not vanish in the higher approximations. Therefore, we expect that the next approximation may give a nonvanishing and small $\mathbf{M}^n$.[23]

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APPENDIX

Under the conditions (1) and (2), the relation between the most probable number of mesons and the c-spectrum of the meson field around a nucleon can be obtained from (5.18′).

If we make the assumptions

$$\begin{aligned}K\Delta N_a(\mathbf{k})&=f(a)g(\mathbf{k}),\\ K\Delta N_b(\mathbf{k})&=f(b)g(\mathbf{k}),\end{aligned}\quad(\text{A}.1)$$

where the function f depends on the nucleon variables only and g on the meson variables only, w_n will be

$$w_n=\sum_{n_a=0}^{n}n!/(n_a!)^2(n_b!)^2\cdot(f(a))^{n_a}(f(b))^{n_b}\left\{\int\frac{d\mathbf{k}}{K}g(\mathbf{k})\right\}^n.\quad(\text{A}.2)$$

For the large value of n, the above expression with the summation over n_a can be replaced by

$$w_n\sim\frac{(2\pi)^n}{n!}\left\{\frac{f(a)+f(b)}{d_1^{d_1}d_2^{d_2}}\right\}^n\left\{\int\frac{d\mathbf{k}}{K}g(\mathbf{k})\right\}^n,\quad(\text{A}.3)$$

with

$$d_1=f(a)/[f(a)+f(b)],\quad d_2=f(b)/[f(a)+f(b)].\quad(\text{A}.4)$$

The most probable number $\bar{n}$ of emitted mesons is determined by the following relation

$$\frac{w_{n+1}}{w_n}=\frac{2\pi}{\bar{n}}\frac{f(a)+f(b)}{d_1^{d_1}d_2^{d_2}}\left\{\int\frac{d\mathbf{k}}{K}g(\mathbf{k})\right\}^{\bar{n}+1}\Big/\left\{\int\frac{d\mathbf{k}}{K}g(\mathbf{k})\right\}^{\bar{n}}\sim1.\quad(\text{A}.5)$$

Now, let us consider the case in which the spectrum ${}^+U$ has the asymptotic form

$${}^+U(\mathbf{k})\sim K^q/K^{\frac{1}{2}}\quad(\text{A}.6)$$

[22] $\Delta\{\frac{1}{2}(1-\tau_3)\boldsymbol{\gamma}\}$ means $\mathbf{O}(x)$, where $\mathbf{O}=\frac{1}{2}(1-\tau_3)\boldsymbol{\gamma}$.

[23] In fact, Pauli and Dancoff obtained $-e\mathbf{e}_3/2\mu$ for $\mathbf{M}^n$ (see reference 12).

in the high energy region. Then

$$N(\mathbf{k}) \propto K^{2q}, \quad g(\mathbf{k}) \propto K^{2q+1}. \tag{A.7}$$

Therefore, it follows from Dirichlet's formula that

$$\left\{\int \frac{d\mathbf{k}}{K} g(\mathbf{k})\right\}^n \propto \left\{4\pi \int K^{2q+2} dK\right\}^n_{\Sigma K_i \leqq \epsilon_{\max}}$$

$$= (4\pi)^n \frac{\{\Gamma(2q+3)\}^n}{\Gamma(2nq+3n+1)} \epsilon_{\max}{}^{2nq+3n}, \tag{A.8}$$

where $\epsilon_{\max}$ is the maximum energy transfer from the nucleons to the emitted mesons and $\bar{n}$ will be

$$\bar{n} \sim 2\pi \frac{f(a)+f(b)}{d_1{}^{d_1} d_2{}^{d_2}} \left\{\int \frac{d\mathbf{k}}{K} g(\mathbf{k})\right\}^{\bar{n}} \Big/ \left\{\int \frac{d\mathbf{k}}{K} g(\mathbf{k})\right\}^{\bar{n}-1}. \tag{A.9}$$

When $f(a)$ behaves as

$$f(a) \propto (\epsilon_{\max})^p, \tag{A.10}$$

from (A.8), (A.9), (A.4), and (A.1), we get the result

$$\bar{n} \propto \left.\begin{matrix} \epsilon_{\max}{}^{(2q+3+p)/(2q+4)} & \text{for} \quad q > -\frac{3}{2}, \\ \epsilon_{\max}{}^{p} \log \epsilon_{\max} & \text{for} \quad q = -\frac{3}{2}. \end{matrix}\right\} \tag{A.11}$$

Under these approximations, the spectrum ($p=0$, $q=-\frac{3}{2}$) gives

$$\bar{n} \propto \log \epsilon_{\max},$$

which agrees with the result obtained by Lewis *et al.* in the case of the scalar meson with scalar coupling. The spectrum ($p=0$, $q=-\frac{1}{2}$) gives

$$\bar{n} \propto \epsilon_{\max}{}^{\frac{2}{3}},$$

which is the result obtained by them in the case of pseudoscalar meson with pseudovector coupling. Further, from the spectrum

$$^{+}U(\mathbf{k}) = F(\mathbf{k})(\mathbf{e}\cdot\mathbf{k})/K^2 (p=0,\, q=-\tfrac{1}{2}) \tag{A.12}$$

obtained in Sec. 4 from the strong coupling theory of pseudoscalar meson with pseudovector coupling, it follows

$$\bar{n} \propto \epsilon_{\max}{}^{\frac{2}{3}}. \tag{A.13}$$

Recently, Fermi[24] has presented a new theory of the multiple production of mesons based on statistical considerations. His method consists in assuming that the meson cloud is so rigidly attached to the nucleon that an external disturbance is rapidly transferred to the whole cloud. As a result of this assumption, the probabilities for multiple production of mesons are determined only by the statistical weight, i.e., the magnitude of volume in phase space. Fermi's idea is, therefore, essentially different from our theory in which probabilities are determined essentially by the cloud spectrum itself. However, the result of his theory is reinterpreted in terms of the c-spectrum theory in the following way. Translating his theory into ours, it is easily found that Fermi's assumption

$$w_n \propto \text{volume in phase space}$$

is equivalent, in our theory, to assuming that

$$N(\mathbf{k}) \text{ is independent of } K,$$

i.e.,

$$q=0, \tag{A.14}$$

and

$$\Omega \propto (\epsilon_{\max})^{-1}, \tag{A.15}$$

i.e.,

$$f(a) \propto (\epsilon_{\max})^{-1}. \tag{A.16}$$

From (5.34) and (5.36) we obtain, by means of (5.29),

$$\bar{n} \propto \epsilon_{\max}{}^{\frac{1}{4}}, \tag{A.17}$$

which is Fermi's result.

Apart from the above assumption, it may also be interesting to discuss the statistical probability factor in connection with its Lorentz invariancy. Assuming that the number of mesons in the Lorentz invariant volume element $d\mathbf{k}/K$ of phase space is independent of momentum $\mathbf{k}$, i.e.,

$$N(\mathbf{k})d\mathbf{k} \propto d\mathbf{k}/K, \quad (q=-\tfrac{1}{2}), \tag{A.18}$$

we obtain, in high energy region, the following result:

$$\bar{n} \propto \epsilon_{\max}{}^{\frac{2}{3}}. \tag{A.19}$$

[24] E. Fermi, Prog. Theoret. Phys. 5, 570 (1950).