clear from the way in which the new perturbed zone boundaries intersect with the energy contours of the unperturbed zone that the gaps at $\frac{1}{4}$ and $\frac{3}{4}$ will be mended. Only the gap at $\frac{1}{2}$, which follows the square-like energy contour closely, has a good chance of materializing.

A more systematic study of qualitative relations of this type is under way. Preliminary results indicate that gaps for $\frac{1}{2}$-filled bands are favored more frequently than for other ratios.

[1] J. C. Slater, Phys. Rev. 84, 179 (1951).

Elastic Photoproduction of π^0-Mesons in Deuterium*

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THE standard weak coupling which is used to calculate the photoproduction of π^+-mesons in hydrogen predicts much too small a cross section for π^0-meson production.[1] However, when the anomalous magnetic moment of the proton is assumed to interact directly with the electromagnetic field and the π^0-meson is taken as a pseudoscalar particle, the cross section for π^0-production becomes comparable to the π^+-cross section.[2] If the anomalous magnetic moment of the proton is responsible for π^0-production in hydrogen, then the anomalous magnetic moment of the neutron should contribute an almost equal amount in a nucleus like deuterium. Indeed, the elastic production process

$$\gamma+d\rightarrow d+\pi^0 \tag{1}$$

is especially interesting because one cannot only explore the role of the neutron in π^0-production but by taking advantage of interference effects, determine the relative sign of the proton and neutron π^0-coupling constants.

If the anomalous magnetic moments of both proton and neutron are treated phenomenologically as in the case of hydrogen, the elastic production cross section in deuterium, to first order in μ/M (μ and M are the masses of meson and nucleon, respectively), can be written,

$$(d\sigma/d\Omega)_{\mathrm{D}}=\{\tfrac{2}{3}F[(g_P\mu_P+g_N\mu_N),\,g_P]+G[(g_P\mu_P+g_N\mu_N),\,g_P]\}I^2, \tag{2}$$

where F is the "spin" contribution to the cross section, G is the "no spin" contribution, g_P and g_N are the π^0 coupling constants to proton and neutron, respectively, μ_P and μ_N are the magnetic moments of proton and neutron respectively, and I is the usual overlap integral,

$$I=\int \psi_{\mathrm{D}}^2(\mathbf{r})e^{\frac{1}{2}i(\mathbf{k}-\mathbf{q})\cdot\mathbf{r}}d\mathbf{r}, \tag{2a}$$

where ψ_{D} is the deuteron wave function, $\mathbf{k}$ is the photon momentum, $\mathbf{q}$ is the meson momentum, and the integration is over all space. A term in F or G depending on $(g_P\mu_P+g_N\mu_N)^2$ is pure magnetic, on g_P^2 is pure electric, and on $g_P(g_P\mu_P+g_N\mu_N)$ is an interference term.

We have calculated F and G according to weak coupling $PS(PV)$ theory with the result,

$$\begin{aligned}F[(g_P\mu_P+g_N\mu_N),\,g_P]=&\tfrac{1}{8}(e^2/M^2)(q^3/k\mu^2)\{(1+\cos^2\theta)\\&\times[2(g_P\mu_P+g_N\mu_N)-g_P]^2+[2(k^2+\mu^2)^2g_P^2/q^2k^2]\\&-[4(k^2+\mu^2)/qk]g_P\cos\theta[2(g_P\mu_P+g_N\mu_N)-g_P]\}\\G[(g_P\mu_P+g_N\mu_N),\,g_P]=&\tfrac{1}{8}(e^2/M^2)(q^3/\mu^2k)g_P^2\sin^2\theta,\end{aligned} \tag{3}$$

where θ is the angle between the photon and meson directions and e is the electric charge. The proton cross section is $(F+G)$ with $g_N\mu_N$ set equal to zero; the neutron cross section is $(F+G)$ with $g_P\mu_P$ and g_P both set equal to zero. The proton, neutron, and deuteron elastic differential cross sections are plotted in Fig. 1 for an incident photon energy of 300 Mev, assuming $g_N=-g_P$

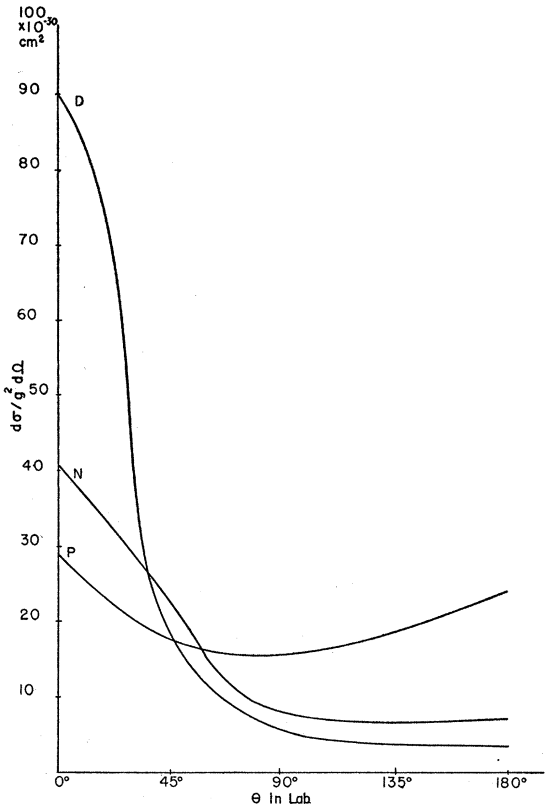

FIG. 1. Weak coupling photo π^0-production cross sections at 300 Mev, assuming $g_P=-g_N=g$.

("symmetrical theory") for the deuteron. The cross section for the deuteron, assuming $g_P=g_N$ ("neutral theory"), is much smaller; e.g., at 300 Mev and $\theta=0$, the "neutral" cross section is 140 times smaller than the "symmetrical" cross section.

One objection[3] which can be raised against our calculation is that while the proton cross section agrees well with experiment[4] both as regards absolute magnitude and variation with energy, it predicts an approximate isotropic angular distribution, contrary to experiment. The origin of the approximately isotropic distribution can be traced to interference effects between the relativistic charge interaction and the odd part of the $PS(PV)$ meson-nucleon interaction operator (destructive in the forward direction and constructive in the backward direction[5]). If for an as yet unknown reason, the odd part of the $PS(PV)$ operator is suppressed[6] in the energy region under consideration,[7] and only the $\boldsymbol{\sigma}\cdot\mathbf{q}$ part ($\boldsymbol{\sigma}$ is the nucleon spin) is retained, the differential proton cross section becomes

$$(d\sigma/d\Omega)_P=F[(g_P\mu_P),\,g_P], \tag{4}$$

where

$$F[(g_P\mu_P),\,g_P]=\tfrac{1}{2}(e^2/M^2)(q^3/k\mu^2)\times\{(g_P\mu_P)^2(1+\cos^2\theta)+g_P^2\sin^2\theta\}. \tag{4a}$$

The neutron cross section is

$$(d\sigma/d\Omega)_N=F[(g_N\mu_N),\,0], \tag{5}$$

and the deuteron elastic cross section is

$$(d\sigma/d\Omega)_{\mathrm{D}}=\{\tfrac{2}{3}F[(g_P\mu_P+g_N\mu_N),\,g_P]\}I^2. \tag{6}$$

These cross sections are plotted in Fig. 2 at a photon energy of 300 Mev and assuming $g_N=-g_P$ for the deuteron as before; it is seen that the proton cross section is now peaked in the forward direction, in much better agreement with the hydrogen

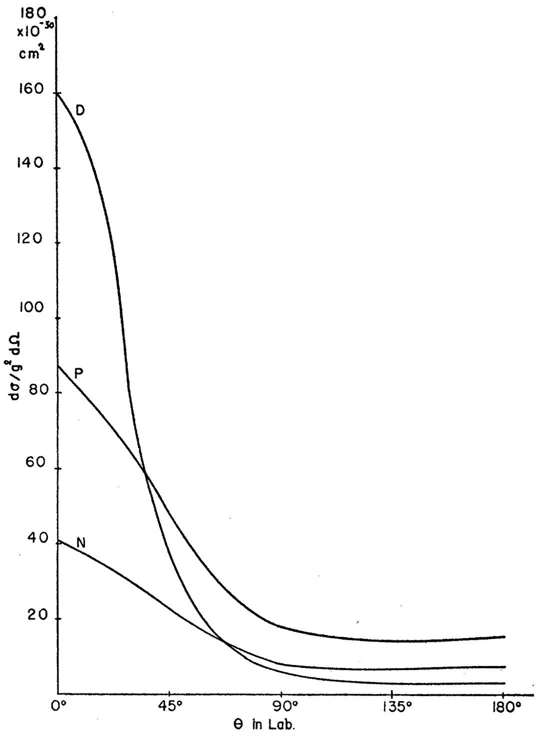

FIG. 2. Phenomenological photo π^0-production cross sections at 300 Mev, assuming $g_P=-g_N=g$.

experiment. However, the important point is that the striking difference between the "neutral" and "symmetrical" deuteron cross sections persists (the "neutral" value is now 1/30 of the "symmetrical" value at 300 Mev and $\theta=0°$). It should, therefore, be possible to determine the relative sign of g_P and g_N in a straightforward fashion from a measurement of the cross section for the elastic photoproduction of π^0-mesons in deuterium.

* This work was assisted by the AEC.

[1] See K. A. Brueckner, Phys. Rev. **79**, 641 (1950).

[2] M. F. Kaplon, Phys. Rev. **83**, 712 (1951); and Aidzu, Fujimoto, and Fukuda, Prog. Theor. Phys. **6**, 193 (1951).

[3] There is a much deeper objection, of course, that the anomalous magnetic moment of the nucleon should itself be a consequence of the meson-nucleon interaction. In the absence of a correct theory of the anomalous magnetic moment, we have merely taken cognizance of the fact that the nucleon magnetic moment is coupled to the electromagnetic field. Moreover, if the magnetic moment of the nucleon extends over its Compton wavelength (as it well may, considering that the π-meson is pseudoscalar), the static value which is assigned to the magnetic moment in our calculations would be justified.

[4] A. Silverman and M. Stearns, Phys. Rev. **83**, 206 (1951).

[5] It should be mentioned that the rise of the proton cross section in the backward direction (see Fig. 1) is the result of our infinite mass approximation; for forward angles ($<90°$), the infinite mass approximation introduces at most an error of 10 percent.

[6] A similar ad hoc assumption is needed to explain the forward maximum in the differential cross section for the reaction $p+p\rightarrow d+\pi^+$ [see Chew, Goldberger, Steinberger, and Yang, Phys. Rev. **84**, 581 (1951)].

[7] The odd part of the $PS(PV)$ meson-nucleon interaction operator is needed to explain the absorption of slow π^--mesons in hydrogen and in deuterium [see R. E. Marshak, Revs. Modern Phys. **23**, 137 (1951)].

Inelastic Cross Sections for 240-Mev Protons*

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THE mean-free path for inelastic collisions of 240-Mev protons in nuclear emulsions has been measured. These collisions include stars, stoppings (which are presumed to be charge exchange scatters), and all ordinary scatters not ascribed to the elastic processes, Coulomb scattering, and diffraction scattering. The observations were made by scanning along individual proton tracks at 600× magnification. Ilford G-5 plates were used.

In 99.07 meters of proton track, 198 stars, 10 stoppings, 124 scatters of 4° or more, and 5 $p-p$ scatters were found. While the efficiency for observing stars and stoppings is believed to be 100 percent, it is possible to miss very small scatters, or scatters with small horizontal projections. Corrections were applied on the assumption that the scattering is axially symmetric. Shrinkage of the emulsion during processing was, of course, taken into account. Corrections varied from 43 percent for scatters in the interval 4–6° to 10 percent for the interval 15–60°.

In deducing the cross sections of the constituent elements of the emulsions from the mean-free path, λ, it has sometimes been assumed that the ratio of the cross section, σ, to the geometrical area, σ_0, is the same for all of the elements in the emulsion. It seems more realistic to assume for the protons a mean-free path in nuclear matter, λ_0, which depends on the energy of the proton, but is the same for all nuclei. One can then show that the ratio σ/σ_0 is given by

$$\sigma/\sigma_0\equiv P=1-[1-(1+2KR)e^{-2KR}]/2K^2R^2,$$

where $K=\lambda_0^{-1}$, and the nuclear radius $R=r_0A^{1/3}$; r_0 is a constant. P is calculated as a function of λ_0 for an average of the heavy elements, silver and bromine, and for an average of the light elements, carbon, nitrogen, and oxygen. From these values of P and from the known composition of the emulsion, a curve λ *vs* λ_0 is obtained, as shown in Fig. 1. Determination of λ by experiment thus fixes λ_0, the values of P for the light and heavy elements, and the cross sections of the elements. For a given λ, the values of λ_0, P, and σ_0 depend on the choice of r_0, but $\sigma=P\times\sigma_0$ is practically independent of r_0. The value $r_0=1.37\times10^{-13}$ cm, given by 90-Mev neutron cross section measurements,[1,2] is used here.

The number of diffraction scatters to be subtracted from the observed scatters is estimated by use of the optical nuclear model of Fernbach, Serber, and Taylor.[2] The calculation involves the parameters $K=\lambda_0^{-1}$ and $k_1=k[(1+V/E)^{1/2}-1]$, where k is the wave propagation vector of the proton outside the nucleus, and E

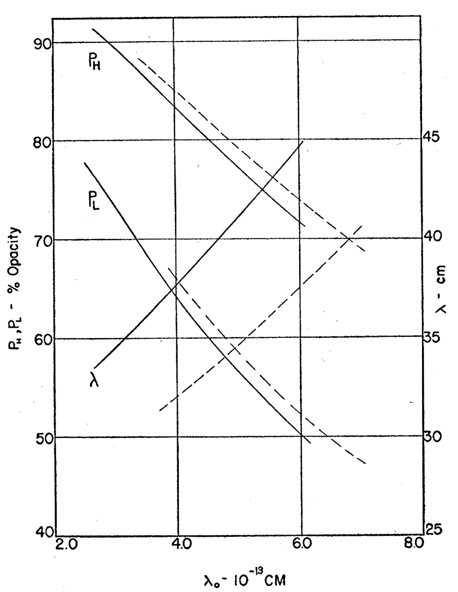

FIG. 1. Nuclear opacities, P, and emulsion mean-free path, λ, as functions of λ_0, the mean-free path in nuclear material. Solid curves for $r_0=1.37\times10^{-13}$ cm. Dashed curves for $r_0=1.47\times10^{-13}$ cm.