

FIG. 1. Comparison of the observed hyperfine structure of the $J=2 \rightarrow 3$ transition of OCSe^{79} with theoretical patterns.

Nuclear shell structure considerations³ lead to an expected $g_{9/2}$ state for the ${}_{34}\text{Se}_{45}^{79}$ nucleus, although a $p_{1/2}$ state would not be improbable. The observed spin of $7/2$ is a clear exception to the single particle strong spin-orbit coupling model. This spin has been predicted, however, by Goldhaber and Sunyar⁴ for Se^{79} and several other neighboring nuclei from a study of isomeric states. The other known exceptions of this type are $\text{Na}^{23}(I=3/2)$ and $\text{Mn}^{55}(I=5/2)$, which have been assumed to arise from $(d_{5/2})^3$ and $(f_{7/2})^3$ configurations, respectively. The Se^{79} state probably involves a $(g_{9/2})^3$ configuration, in which two particles fail to pair off and give zero angular momentum, but give unit momentum instead. The positive quadrupole moment of Se^{79} is consistent with such a state.

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² Geschwind, Minden, and Townes, *Phys. Rev.* **78**, 174 (1950).

³ M. Mayer, *Phys. Rev.* **78**, 16 (1950).

⁴ M. Goldhaber and A. W. Sunyar, *Phys. Rev.* **83**, 906 (1951).

Splitting of Bands in Solids

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THE natural explanation of the existence of metals with a partially filled band and insulators or intrinsic semiconductors with full or empty bands is one of the greatest early successes of the band theory of solids. However, metals of even valence required special consideration. Such materials would be insulators in a one-dimensional world, but become metals on account of a kind of overlapping or merging of bands which is only possible in more dimensions.

Recently Slater¹ devised a scheme by which the reverse can be achieved. Materials which would naively be expected to be metallic conductors may now be insulators or semiconductors (NiO or Au at low temperatures) by introducing suitable energy gaps into bands. This band splitting is a consequence of the assumption that the actual basic cell of the lattice may be an integral multiple of the naive basic cell, if certain refinements are considered, such as may arise from spin and possibly from a number of other causes. According to Slater's paper gaps may thus appear in principle almost anywhere in a band.

The purpose of the present note is to draw attention to the fact that the Slater gaps can only develop under certain conditions

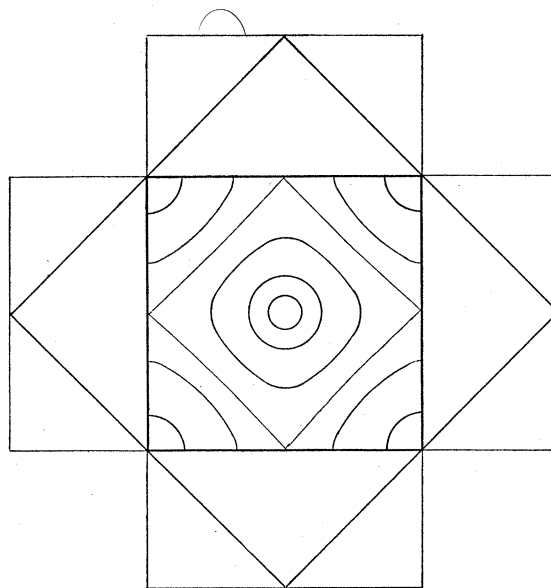


FIG. 1. The first three zones of the two-dimensional square lattice, with energy contours drawn in the first zone.

which greatly reduce their incidence. The potential in cases under consideration is assumed to consist of a major part, with the naive or unperturbed periods, and superimposed on this, a ripple or perturbation with larger periods. The resulting small gaps will almost certainly be mended by the same more dimensional mechanism, responsible for the merging of bands in metals of even valence. The condition that the gap persists is evidently that the new zone boundary, introduced by the perturbation, coincides very nearly with an equi-energy contour (in two dimensions) or surface (in three dimensions) of the unperturbed lattice in reciprocal space.

An example with the two-dimensional square lattice may illustrate this point. The reciprocal lattice is also square, and the zone boundaries are indicated in Fig. 1. The energy contours are like circles around the center of the first zone and around its corners. A square-like contour separates the center region from the corner regions of the first zone. The corner contours must meet the zone boundary at right angles, except the square-like contour, which has a double point where it meets the zone boundary and consequently may make any angle with the latter.

In Fig. 2 an added perturbation potential is assumed, with periods of twice the original lattice periods in both directions. This gives rise to a new zone pattern, similar to the one in Fig. 1 but at half scale. The original first zone thus contains roughly four perturbed zones, and naively new gaps may be expected, corresponding to $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$ filling of the original band. However, it is

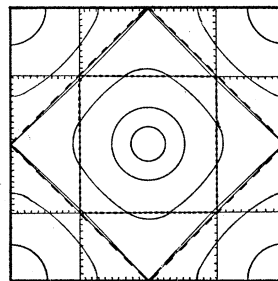


FIG. 2. New zone boundaries of the perturbed lattice within the first zone of the unperturbed lattice. The energy contours are the same as in Fig. 1.

clear from the way in which the new perturbed zone boundaries intersect with the energy contours of the unperturbed zone that the gaps at $\frac{1}{2}$ and $\frac{3}{4}$ will be mended. Only the gap at $\frac{1}{2}$, which follows the square-like energy contour closely, has a good chance of materializing.

A more systematic study of qualitative relations of this type is under way. Preliminary results indicate that gaps for $\frac{1}{2}$ -filled bands are favored more frequently than for other ratios.

¹J. C. Slater, Phys. Rev. **84**, 179 (1951).

Elastic Photoproduction of π^0 -Mesons in Deuterium*

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THE standard weak coupling which is used to calculate the photoproduction of π^+ -mesons in hydrogen predicts much too small a cross section for π^0 -meson production.¹ However, when the anomalous magnetic moment of the proton is assumed to interact directly with the electromagnetic field and the π^0 -meson is taken as a pseudoscalar particle, the cross section for π^0 -production becomes comparable to the π^+ -cross section.² If the anomalous magnetic moment of the proton is responsible for π^0 -production in hydrogen, then the anomalous magnetic moment of the neutron should contribute an almost equal amount in a nucleus like deuterium. Indeed, the elastic production process



is especially interesting because one cannot only explore the role of the neutron in π^0 -production but by taking advantage of interference effects, determine the relative sign of the proton and neutron π^0 -coupling constants.

If the anomalous magnetic moments of both proton and neutron are treated phenomenologically as in the case of hydrogen, the elastic production cross section in deuterium, to first order in μ/M (μ and M are the masses of meson and nucleon, respectively), can be written,

$$\left(\frac{d\sigma}{d\Omega}\right)_D = \left\{ \frac{2}{3} F[(g_P \mu_P + g_N \mu_N), g_P] + G[(g_P \mu_P + g_N \mu_N), g_P] \right\} I^2, \quad (2)$$

where F is the "spin" contribution to the cross section, G is the "no spin" contribution, g_P and g_N are the π^0 coupling constants to proton and neutron, respectively, μ_P and μ_N are the magnetic moments of proton and neutron respectively, and I is the usual overlap integral,

$$I = \int \psi_D^2(\mathbf{r}) e^{i(\mathbf{k}-\mathbf{q}) \cdot \mathbf{r}} d\mathbf{r}, \quad (2a)$$

where ψ_D is the deuteron wave function, \mathbf{k} is the photon momentum, \mathbf{q} is the meson momentum, and the integration is over all space. A term in F or G depending on $(g_P \mu_P + g_N \mu_N)^2$ is pure magnetic, on g_P^2 is pure electric, and on $g_P(g_P \mu_P + g_N \mu_N)$ is an interference term.

We have calculated F and G according to weak coupling $PS(PV)$ theory with the result,

$$\begin{aligned} F[(g_P \mu_P + g_N \mu_N), g_P] &= \frac{1}{8} (e^2/M^2) (q^3/k\mu^2) \{ (1 + \cos^2\theta) \\ &\times [2(g_P \mu_P + g_N \mu_N) - g_P]^2 + [2(k^2 + \mu^2)g_P^2/q^2k^2] \\ &- [4(k^2 + \mu^2)/qk] g_P \cos\theta [2(g_P \mu_P + g_N \mu_N) - g_P] \} \\ G[(g_P \mu_P + g_N \mu_N), g_P] &= \frac{1}{8} (e^2/M^2) (q^3/\mu^2k) g_P^2 \sin^2\theta, \end{aligned} \quad (3)$$

where θ is the angle between the photon and meson directions and e is the electric charge. The proton cross section is $(F+G)$ with $g_N \mu_N$ set equal to zero; the neutron cross section is $(F+G)$ with $g_P \mu_P$ and g_P both set equal to zero. The proton, neutron, and deuteron elastic differential cross sections are plotted in Fig. 1 for an incident photon energy of 300 Mev, assuming $g_N = -g_P$

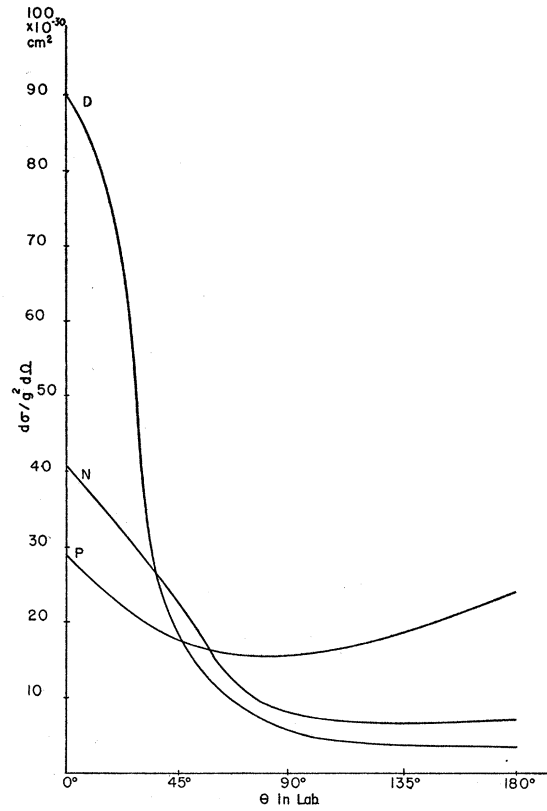


FIG. 1. Weak coupling photo π^0 -production cross sections at 300 Mev, assuming $g_P = -g_N = g$.

("symmetrical theory") for the deuteron. The cross section for the deuteron, assuming $g_P = g_N$ ("neutral theory"), is much smaller; e.g., at 300 Mev and $\theta = 0$, the "neutral" cross section is 140 times smaller than the "symmetrical" cross section.

One objection³ which can be raised against our calculation is that while the proton cross section agrees well with experiment⁴ both as regards absolute magnitude and variation with energy, it predicts an approximate isotropic angular distribution, contrary to experiment. The origin of the approximately isotropic distribution can be traced to interference effects between the relativistic charge interaction and the odd part of the $PS(PV)$ meson-nucleon interaction operator (destructive in the forward direction and constructive in the backward direction⁵). If for an as yet unknown reason, the odd part of the $PS(PV)$ operator is suppressed⁶ in the energy region under consideration,⁷ and only the $\boldsymbol{\sigma} \cdot \mathbf{q}$ part ($\boldsymbol{\sigma}$ is the nucleon spin) is retained, the differential proton cross section becomes

$$\left(\frac{d\sigma}{d\Omega}\right)_P = F[(g_P \mu_P), g_P], \quad (4)$$

where

$$F[(g_P \mu_P), g_P] = \frac{1}{8} (e^2/M^2) (q^3/k\mu^2) \times \{ (g_P \mu_P)^2 (1 + \cos^2\theta) + g_P^2 \sin^2\theta \}. \quad (4a)$$

The neutron cross section is

$$\left(\frac{d\sigma}{d\Omega}\right)_N = F[(g_N \mu_N), 0], \quad (5)$$

and the deuteron elastic cross section is

$$\left(\frac{d\sigma}{d\Omega}\right)_D = \left\{ \frac{2}{3} F[(g_P \mu_P + g_N \mu_N), g_P] \right\} I^2. \quad (6)$$

These cross sections are plotted in Fig. 2 at a photon energy of 300 Mev and assuming $g_N = -g_P$ for the deuteron as before; it is seen that the proton cross section is now peaked in the forward direction, in much better agreement with the hydrogen