high energy rays observed with the cloud chamber appear to be inconsistent with a high multiplicity of cores.

The apparent contradiction stated above might possibly be eliminated if better approximations were made, (*a*) not only in the arguments that led to the energy estimates, but (*b*) also in the arguments that led to the core identifications.

(*a*) The transition effect in $\frac{1}{2}$ inch of Dural was estimated to be small, but if it were actually a factor of two near the core, the multiplicity would be reduced to about five. The fact that the absolute intensity agrees with ion-chamber and counter measurements is, however, an argument that the transition effect is indeed small. The normalization involved in finding the total number (N) of electrons depends on the Molière distribution, which has been verified for distances greater than a few meters. The present work indicates a less steep distribution near the axis, but this means that we have underestimated N by assuming the steeper distribution. The shower theory for longitudinal development should be at its best in this case of high energy air showers, but any change of cross sections that resulted in a more rapid degradation of energy or added processes that diverted energy to mesons would aid in the explanation of the contradiction.

(*b*) The observed result could be explained by the addition of one or two cores of high energy and many cores of low energy; the latter would add greatly to the total number of electrons but would add little either to the high energy end of the local spectrum or to the number of distinguishable cores.

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Soft Radiation at Balloon Altitudes*

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Results are given on cloud-chamber observations of electron showers and "spray" events at high altitude (18 millibars). The fluxes derived for initiating radiation are consistent with a common origin for showers and "sprays," and thus support the conclusion by Rau and Wightman that "sprays" are showers produced in a single lead plate. However, less than one-fourth as many such events occur as would be expected from observations on mu-mesons if mesons are produced with a spectrum that does not vary with altitude. If all soft radiation at 18 millibars were of primary origin, it would constitute less than 0.6 percent of the total cosmic-ray flux.

INTRODUCTION

DAYTIME operation of cloud chambers which are carried to high altitudes by balloons is a well-established procedure. The chambers, together with controlling and recording mechanisms, are enclosed in a pressurized gondola. Solar radiation maintains a temperature in the gondola that is approximately 70° Fahrenheit. By means of lead plates, placed in the useful part of the chamber. the various components of the cosmic radiation at high altitude can be identified on the basis of their ionizing and penetrating properties in the usual way.

Among the cosmic-ray events, electron showers are readily identified through their characteristic multiplication in passing through the lead plates (which are 1.2-radiation units thick in the experiments reported here). These events have been analyzed previously[1] to establish an upper limit on the flux of soft component in the primary cosmic radiation. The present paper includes greater detail of such an analysis. A typical electron shower is shown in Fig. 1.

The second type of event considered in this study is the "spray" phenomenon reported by Oppenheimer and Ney,[2] which is the multiple production in lead of ionizing tracks that are near minimum ionization but that, for the most part, do not penetrate neighboring lead plates (see Fig. 2).

The cloud-chamber pictures obtained in seven flights at altitude, at 55°N magnetic latitude, were examined for both electron showers and sprays. In six of these flights the plates in the cloud chambers were horizontal. In the remaining flight the plates were vertical so as to change the response of the system relative to the vertical and horizontal components of the initiating rays. The

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[1] Critchfield, Ney, and Oleksa, Phys. Rev. **79**, 402 (1950).
[2] F. Oppenheimer and E. P. Ney, Phys. Rev. **76**, 1418 (1949).

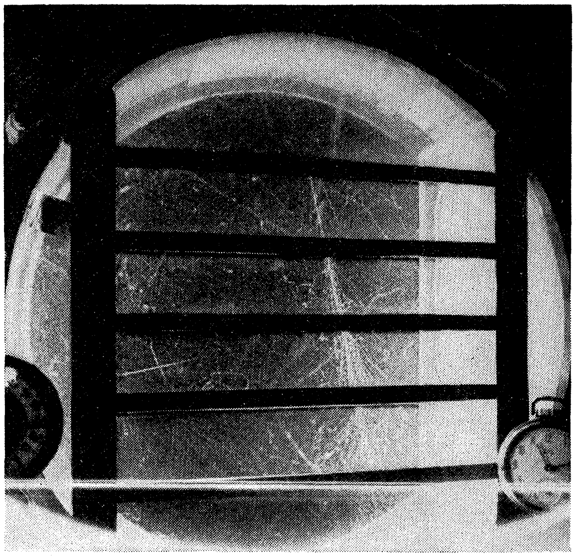

FIG. 1. An electron shower obtained in the flight of July 16, 1948. It has the fifth highest energy of those obtained with horizontal plates. The estimated total energy is 1.6 Bev.

number of plates, each $\frac{1}{4}$ in. thick, in the cloud chamber varied from time to time. In all, four different arrangements were used and, in the following article, they have been denoted as types 1 to 4. Type 1 consisted of 5 lead plates; type 2 had a top plate of carbon and 3 lead plates beneath it; type 3 had 2 lead plates in a four-plate sequence: lead, carbon, lead, aluminum (bottom); type 4, which was flown with the plates vertical, had 4 lead plates. Plates in a given chamber are equally spaced; the distances between plates, lengths and widths of the illuminated regions, etc., will be given later. Although some of the flights were counter-controlled, only random expansion events were considered. A careful determination of the sensitive time was made. An effort was made to considere events of approximately the same age. Both stereos were examined several times for the electron showers and "sprays."

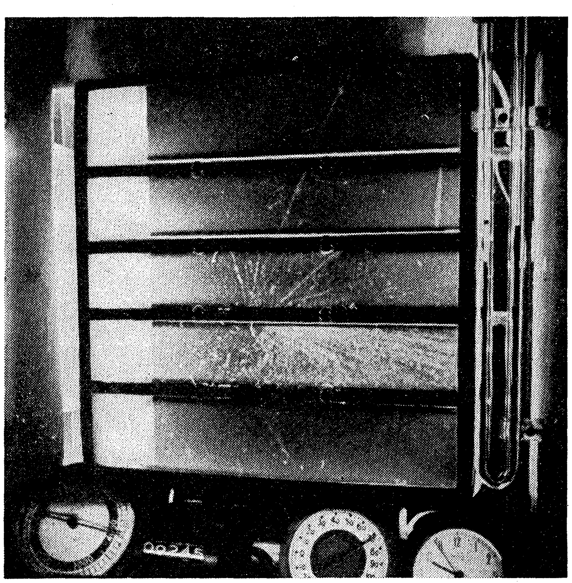

FIG. 2. A large spray event recorded on a flight over Cuba.

It is the purpose of this paper to present the results of the observations and to determine the absolute flux of the soft radiation at high altitude. The relationship of the flux obtained to the mu-meson flux is also investigated.

RESULTS ON ELECTRON SHOWERS

Among the six flights carrying cloud chambers with horizontal plates, one thousand sets of pictures were selected on the basis that they were clear enough to permit counting the prongs of an electron shower if it occurred. From the one flight carrying vertical plates, 148 frames at altitude were usable. These frames were searched for electron shower events. Such events were considered acceptable if there were at least ten charged particles, i.e., tracks at minimum ionization, in the region of maximum development of the shower and if the apparent axis of the shower passed through illuminated areas of at least four of the plates in the chamber (whether lead or not). The first criterion sets a lower limit on the energy of the incident shower producing particle; the second criterion defines the geometrical factor in each experiment. The latter varies a little among the four types of plate arrangement. Moreover, showers definitely associated with nuclear events in the plates were not included. The results are given in Table I.

The most probable energy of each shower was estimated on the basis of the calculations of Bhabha and Chakrabarty.[3] Only the leading term was used in their series for the average number of particles beneath various thicknesses. The reason for using the main term only is as follows. Our observations were made in regions between lead plates, each 1.2-radiation units thick. Hence, very low energy electrons will be mostly stopped in the plates and not observed. This loss is approximated by omitting the higher order terms in the formula obtained in reference 3 because the formula is essentially a power series in the argument $\beta/(E+\beta)$, where β is the energy lost by collision in one radiation unit (assumed to be constant in the calculation) and E is the energy of the electron. Therefore, the higher order terms in the expansion become of importance at those electron energies for which absorption is serious. Furthermore, derivation of the series formula was based on the asymptotic forms of the Bethe-Heitler cross sections and, as pointed out by Messel,[4] this assumption leads to too many low energy electrons because the cross sections at lower energy are smaller than the asymptotic

[3] H. J. Bhabha and S. K. Chakrabarty, Phys. Rev. **74**, 1352 (1948).
[4] H. Messel, Phys. Rev. **82**, 259 (1951).

values. Although the latter reason for the appearance of fewer electrons is unrelated to the experimental one based upon absorption in the plates, it strengthens our assumption in estimating the energies of the showers. We make no effort to distinguish between electron-initiated and photon-initiated showers.

The calculations of reference 3 show that about three radiation units are required to produce ten prongs with a minimum initial energy. All flights contained more than 3 r.u. at normal incidence except that of February 9, 1950, which contained 2.4 r.u. only (two Pb plates). Consequently, a slight correction must be applied to observations on horizontal plates. Since we retain the ten-prong criterion in the study of sprays, the correction will be expressed as a fictitious addition to the observed number of events, ΔN_H.

The method of estimating the energy of each shower is as follows: let i be the order number of the lead plate, increasing as the shower develops, N_i the number of observed prongs, and $\sigma_i(E_0)$ the calculated mean value expected beneath the ith plate (taking obliquity into account) and for an energy E_0 of the incident ray. We assume the prong number obeys a Poisson distribution and determine the best value of E_0 by maximizing $\ln P(E_0)$ with respect to E_0:

$$\ln P(E_0)=\sum_{i=1}^{p}\left[N_i \ln\sigma_i(E_0)-\sigma_i(E_0)\right], \qquad (1)$$

where p is the total number of lead plates. In this way, it was found that all showers entered in Table I have a most probable E_0 such that

$$\mathrm{lin}(E_0/\beta)\geqslant 4.5, \quad E_0\geqslant 90\beta. \qquad (2)$$

The minimum energy required to produce at least ten prongs can be deduced more directly from the calculations, of course, but the preceding method provides a basis for determining the likelihood that the observed event was an electron shower, and each of the recorded events was so tested and accepted. Moreover, arranging the estimated average energies (E_0/β) in descending order and plotting the logarithm of the order number against $\ln(E_0/\beta)$ yields a crude integral spectrum. This plot is shown in Fig. 3. The dashed line corresponds to an integral number distribution varying as $E_0^{-1.5}$ and fits the observations at large-order numbers very well.† Estimated standard deviations in $\ln(E_0/\beta)$ are represented by horizontal lines the lengths of which are determined from individual graphs of $\ln P(E_0)$ as 0.71 times the difference in $\ln(E_0/\beta)$ required to reduce $\ln P(E_0)$ by unity. The estimated uncertainty in energy varies according to the number of lead plates used in the determination and is quite consistent among events involving a given number of plates. The standard deviations correspond to uncertainty factors on the mean energy of 1.45, 1.27, and 1.15 for 2, 3, and 4 plates, respectively.

The least energy that can produce ten prongs, on the the average, beneath 2.4 r.u. is determined from the results of reference 3 to to be E', when $\ln(E'/\beta)=4.63$. Since the integral spectrum varies as $E_0^{-1.5}$, the flux of incident radiation necessary to produce an acceptable event in the flight of February 9, 1950, is lower by $e^{-0.195}=0.823$. Since 0.307 of the pictures used occur in that flight, the fraction of the flux with $E_0\geqslant 90\beta$ actually effective is $0.693+0.307\times 0.823=0.946$. The total number of showers observed is 16 so that an estimate of what should have been observed if all chambers had 3 or more r.u. in lead is 16.9. Hence $\Delta N_H=+0.9$.

In order to estimate the value of β, and thereby obtain E_0 in energy units, we assume that the most important electron energy, $\bar{E}$, to consider is the average energy in the radiation unit preceding the maximum development of a shower with $E_0=90\beta(\bar{E})$. The function $\beta(\bar{E})$ varies slowly with E_0 and thus does not influence

TABLE I. Results on showers.

Date of balloon flight	Av press. millibars	Type geometry	No. frames	No. Pb plates	No. showers $\geqslant 10$ at max
7/16/48	23.3	1	63	5	3
7/23/48	13.8	1	67	5	1
4/2/49	13.2	1	185	5	2
5/24/49	13.8	2	99	3	1
7/10/49	16.7	2	279	3	5
2/9/50	19.3	3	307	2	4
10/12/50[a]	19.3	4	148	4	5

[a] Vertical plate flight.

the deductions about the integral spectrum significantly. If a shower has 10 prongs at maximum (3 r.u.), we estimate that there will be 9 electrons and 7 gamma-rays at 2 r.u. and that, therefore, $\bar{E}=E_0/16$. The desired value of E_0 is the root of the equation

$$E_0-90\beta(E_0/16)=0.$$

We used the calculations of Halpern and Hall[5] as applied to condensed materials for the determination of $\beta(\bar{E})$. The result for E_0 is

$$E_0\geqslant 0.73 \text{ Bev} \qquad (3)$$

corresponding to $\beta=8.1$ Mev/r.u.

The second point in evaluation of the experimental results is that of the effective solid angle times cross-sectional area presented by the plates in the cloud chambers. The rectangular illuminated areas on the plates are of equal dimensions, length L and width D, and it is assumed that the solid angle is defined by a pair of parallel plates, the position of the top plate being obtained from that of the bottom plate by displacing

† *Note added in proof:* Dr. Phyllis Freier has suggested inserting the $E^{-2.5}$ differential spectrum thus obtained into the probability equation and computing the spectrum and E_0 to a higher order of approximation. This may be done readily for showers produced in three or more plates and yields $E_0=66\beta$, or about 0.52 Bev. The spectrum is unchanged, as are other results of this paper, except that the disparity between gamma-ray and mu-meson flux is increased.

[5] O. Halpern and H. Hall, Phys. Rev. 73, 477 (1947).

the latter vertically a distance h without rotation. Actually, of course, the plates in the cloud chamber are rotated slightly so as to present only their edges to the camera. The distance h embraces three illuminated spaces, as mentioned previously. The effectiveness of the geometry, thus defined, for detection of showers depends upon the sensitive time, the zenith angle dependence of the incident radiation, and upon whether the plate is to be considered "thick" or "thin." In our interpretation of showers we consider the plates as "thick," i.e., something is certain to happen in the first plate that is struck, so that the effective area decreases with angle of incidence. In general, the averages over various powers of $\cos Z$, where Z is the zenith angle, are required in

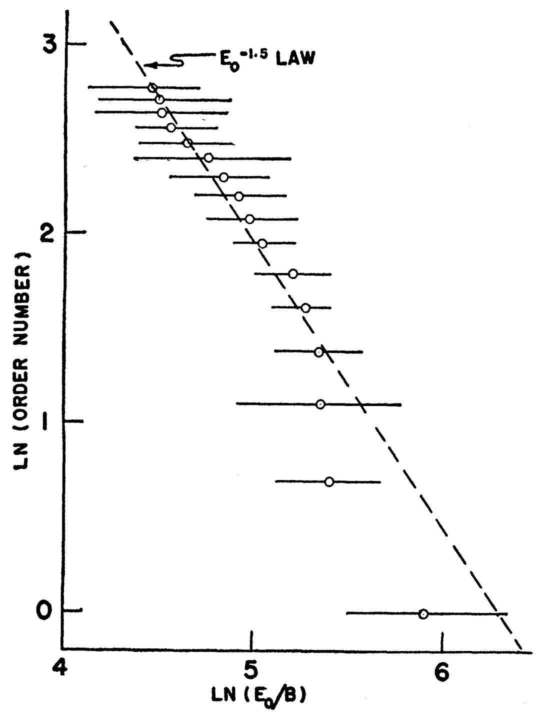

FIG. 3. Integral number distribution of electron showers in horizontal plates. The estimated energies are arranged in descending order and the natural logarithm of the order number plotted against that of the energy.

computing the geometry. Since the integrals involved do not appear in the literature, at least not collected, we present them in Appendix I, together with convenient expansions. The average of $\cos^n Z$ is denoted by G_n when the plates are horizontal.

In this section, and the following one, the zenith angle dependence will be approximated by assuming an average vertical flux f_v and horizontal flux f_h, both isotropic but not necessarily equal. Refinements on this model will be considered in the last section. Since the assumed fluxes are isotropic, the appropriate average is over $\cos Z$ (horizontal plates) and the geometric factor is $G_1(h)$ for plates spaced a distance h. Since the $\cos Z$ represents the effect of obliquity, and not a property of the flux, the same factor applies to vertical plates. The factor $G_1(h)$ does not apply directly to all flights, however, since in most cases there were four illuminated spaces, whereas only three were required for acceptance of a shower. Since h corresponds to three spaces, the geometric factor for four spaces is of the form

$$\bar{G}=2G_1(h)-G_1(4h/3). \tag{4}$$

The results are given in Table II.

The sensitive time of the chambers was determined by counting tracks from a Ra D+E source placed in the chamber. The tracks were required to have a vertical range of at least 5 mg of argon. The number found per second was then compared with the rate obtained with a thin-walled counter under similar geometrical and range conditions. The value obtained is 0.0040 ± 0.0002 minute.

Combining the results of Table I, Table II, ΔN_H, and that for the sensitive time we obtain values of f_v and f_h in units of shower producing radiation per steradian-cm^2-minute:

$$f_v=(16+0.9)/129.2=0.13\pm0.03. \tag{5}$$

$$f_v=5/22.9=0.22\pm0.10. \tag{6}$$

The result that $f_h > f_v$ is understandable, at the altitudes involved, because shower producing radiation probably arises from collisions between primary protons and the air nuclei and that in a mean path much longer than 18 $\mathrm{g/cm^2}$. The intensity of secondaries must therefore increase with zenith angles, at least up to a point. As mentioned previously, shower producing radiation generated in the chamber was ignored.

RESULTS ON SPRAYS

A relatively high intensity of soft radiation at large zenith angles entails the possibility that an entire electron shower may be developed in one lead plate by an incident electron or gamma-ray near grazing incidence. The result will be a "spray" of relatively low energy soft radiation, i.e., incapable of further multiplication or of penetration of the neighboring plates. These events, therefore, are of the same nature as those reported by Oppenheimer and Ney,[2] and the equation arises as to what extent the incidence of one-plate showers accounts for "spray" phenomena. This question has been studied by Rau[6] and by Rau and Wightman,[7] who conclude that all spray events of moderate size are cascade showers. In the following, we shall show that this conclusion is substantiated by considerations of the measured flux.

Although "sprays" have been discussed by previous writers, it is worthwhile to repeat the criteria used in their determination. Sprays are defined as events formed of minimum—to 2 or 3 times minimum—ionizing particles which, on the whole, do not penetrate

[6] R. R. Rau, Phys. Rev. **80**, 914 (1950).
[7] R. R. Rau and A. S. Wightman, Phys. Rev. **80**, 914 (1950).

another lead plate. Their appearance shows great variety. Some are narrow angled, tight bundles of prongs, others are wide angled. Some appear to be formed at a point, others are formed over an area extending a centimeter or two. The number of prongs within a spray can vary from three to enormous sprays having more than fifty prongs. In the flights with horizontal plates there are occasional upward prongs. In the vertical plate flight the prongs often emerge from both sides of a lead plate in comparable numbers. Sprays have never been seen in carbon or aluminum plates and have been seen only very infrequently at lower than balloon altitudes.

A number of difficulties beset the determination or recognition of sprays. One is simply that there is always a rather complex background of events at high altitude. Another is that their possible association with a nuclear event in the chamber is difficult to ascertain. This difficulty is more serious in the vertical plate flight for which there is no background of experience in identifying such events. Moreover, since high energy primaries come in much more frequently near the vertical than near the horizontal, there is a greater chance of mistaking a hard shower created in the plates for a true spray, when the plates are vertical.

The observations on sprays were made under the same conditions as for the showers. A spray is rejected if it is evidently associated with another event in the chamber. It is assumed that a spray of 10 or more prongs may be a cascade with 10 or more prongs at maximum development. The minimum energy of an acceptable event is, therefore, presumed to be the same as that pertaining to the shower events.

Results on sprays appearing in three balloon flights are given in Table III. The detection of sprays requires a somewhat clearer picture than that sufficient for the detection of showers so that the number of frames is less, in two of the flights, than the number appearing in Table I. Moreover, spraylike events occurring in the bottom lead plate of July 10, 1949, are not included because it is not possible to test their penetration. Similarly, events in the outer plates of October 12, 1950 (vertical plates) and in which all prongs go outward cannot be tested completely. There is so little data on vertical plates that we have presented it in two ways in Table III. One way is to consider each outer plate as one-half plate and accept all events that show all prongs going inward and one-half of the events in which prongs go both ways. The result of this selection is entered in Table III under October 12, 1950—3 plates. The alternate way is to accept all events that look like sprays and count the end plates to be as effective as the internal ones. This is entered under 4 plates.

An isolated, horizontal plate presents a certain solid angle times area to radiation that can produce a spray if the radiation impinges at a zenith angle greater than $\sec^{-1}n$, where n is the number of thicknesses of plate necessary to generate the required number of prongs. According to the results of the last section, if a spray is an electron shower produced in a single plate with at least 10 prongs at maximum, at least 3 radiation units are required. Since the normal thickness is 1.2 r.u., the required value of n is $n=2.5$.

The geometric factor for sprays arising from an isotropic flux of soft radiation is

$$g=LD\int_0^{\pi/2}\sin Z\,\cos Z dZ\int_0^{2\pi}d\psi$$
$$=\pi LD/n^2=42.1 \text{ per plate.} \qquad (7)$$

The shielding of one plate by a neighboring lead plate is negligible. Using g, the number of plates, the sensitive time and f_h, Eq. (6), we can calculate the number of sprays expected to occur in flights with horizontal plates. A similar calculation is made for vertical plates assuming the flux is $\frac{1}{2}(f_v+f_h)$. The results are entered in Table III as $\geqslant 10$ calc. It is apparent that cascades developed in one plate are of the proper frequency to account for the "spray" phenomenon. In horizontal plates 37 are expected from showers and 40 found.

TABLE II. Geometric factors for showers.

Type	No. frames	D	L	h	$\bar{G}_H$	$\bar{G}_V$
1	315	4.13	19.4	12.9	33.0	
2	379	4.13	20.3	12.8	34.5	
3	307	4.13	20.3	12.0	28.9	
4	148	4.13	20.3	12.0		38.6
Weighted average of 1, 2, 3					32.3	

ANALYSIS OF THE SOFT RADIATION

It appears, from the foregoing results, that all spray events recorded could have been showers that developed in a single plate. If we assume that all sprays are electron showers, the horizontal and vertical plate results on both showers and sprays can be used to obtain the best values of f_v and f_h. There are four independent determinations, which yield the most probable values:

$$f_v=0.130, \quad f_h=0.236. \qquad (8)$$

The probability of obtaining values different from those in Eq. (8) is lower by about a factor of two along a circle with its center on the most probable values and with a radius of 0.035. This gives a measure of the reliability of the results for the flux considering sprays as well as showers.

The value of the f's can be related to the cross sections for meson production and cascade processes on the basis of plausible assumptions. We shall derive the flux and angular distribution of soft radiation from the known flux of μ-mesons. The latter has been analyzed by Sands[8] on the assumption that the energy spectrum

[8] M. Sands, Phys. Rev. 77, 180 (1950).

of the created mesons is independent of the path length in air traversed by the primary radiation. Let the unit of energy be the rest mass of the μ-meson ($\sim$110 Mev) and the unit of path length 100 g/cm^2; then, Sand's results for the integral flux, per steradian cm^2 minute and per 100 g/cm^2 may be expressed

$$y_\mu(U)=94U^{-1.5}, \quad U\geqslant 10, \tag{9}$$

where U is the minimum energy of mu-meson being considered. These mesons are presumed to originate in the decay of pi-mesons which are created directly in nuclear collisions in a mean path of 125 g/cm^2. It has been found that approximately one-third of all pi-mesons created in high energy collisions in air are uncharged[9] and these disintegrate into two gamma-rays. It is possible, therefore, to derive the total number of gamma-rays expected, $y_\gamma(U)$, from $y_\mu(U)$.

Let the ratio of pi-meson mass to mu-meson mass be ρ. By relativistic kinematics it is readily shown that the energy of mu-mesons created by the disintegration of pi-mesons, of energy ϵ and velocity v, is equally distributed on an interval of energy, $(1-\rho^{-2})(v/c)\epsilon$, if the other product of disintegration is a neutrino. Let $F(\epsilon)d\epsilon$ be the differential number spectrum of pi-mesons as created in the atmosphere (per 100 g/cm^2). The differential number spectrum of mu-mesons, $Y(U)$, will then be

$$Y(U)=[\rho^2/(\rho^2-1)]\int_{\epsilon_1(U)}^{\epsilon_2(U)}\frac{cF(\epsilon)}{v\epsilon}d\epsilon, \tag{10}$$

where ϵ_1 and ϵ_2 are the lower and upper roots, respectively, of the equation

$$2\rho^2U=(\rho^2+1)\epsilon+(\rho^2-1)(\epsilon^2-\rho^2)^{\frac{1}{2}}. \tag{11}$$

Assuming a neutral pi-meson, of essentially the same mass as that of a charged pi-meson, emits two quanta and occurs one-half as frequently as a charged meson we obtain the number distribution of gamma-rays of energy γ to be

$$W(\gamma)=\int_\gamma^\infty\frac{dF(\epsilon)}{v\epsilon}d\epsilon$$

$$=[(\rho^2-1)/\rho^2]\sum_{n=0}^{\infty}Y(U_n) \quad \gamma>\rho, \tag{12}$$

with

$$\begin{aligned}U_0&=[(\rho^2+1)\gamma+(\rho^2-1)(\gamma^2-\rho^2)^{\frac{1}{2}}]/2\rho^2,\\ U_n&=[(\rho^4+1)U_{n-1}+(\rho^4-1)(U_{n-1}^2-1)^{\frac{1}{2}}]/2\rho^2.\end{aligned} \tag{13}$$

In the limit of large n, $U_n\rightarrow\rho^2U_{n-1}$.

In the approximation which we shall apply $Y(U)\simeq AU^{-2.5}$, $U_0\simeq\gamma$, and

$$W(\gamma)\simeq A(\rho^2-1)\gamma^{-2.5}/[\rho^2(1-\rho^{-5})]=0.568Y(\gamma). \tag{14}$$

The integral number spectrum of the γ-rays should be, therefore

$$w(\gamma)=53\gamma^{-1.5}. \tag{15}$$

The flux of soft component at depth d owing to the (immediate) disintegration of neutral pi-mesons of energy $\geqslant\gamma_0$ and produced by nuclear collisions in a free path of 125 g/cm^2 is

$$f(\gamma_0, d)=53\int_0^d e^{-0.8(d-x)+x/l}(\gamma_0e^{x/l})^{-1.5}dx. \tag{16}$$

In Eq. (16) it has been assumed that the soft radiation multiplies by $e^{x/l}$ in the path length x in air. The length l is therefore the radiation unit in air and equal to 0.43 (100 g/cm^2). At 0.73 Bev, $\gamma_0=6.6$, and the integration yields

$$f(d)=8.7(e^{-0.8d}-e^{-1.16d}) \tag{17}$$

as the flux of soft radiation of energy $\geqslant$0.73 Bev and as a function of zenith angle through

$$d=d_0\sec Z, \quad d_0=0.18. \tag{18}$$

The vertical flux predicted by Eq. (17), with $d=d_0$, is 0.47 sterad^{-1} cm^{-2} min^{-1} as compared with 0.13 observed from the vertical over a range of zenith angles from 0 to about 45°. In order to effect a more direct comparison we average $f(d)$ over Z from 0 to 45° to obtain f_v and from 45° to 90° to obtain f_h. The results are

$$\bar{f}_v=0.53, \quad \bar{f}_h=0.84, \tag{19}$$

$$\bar{f}_h/\bar{f}_v=1.6, \tag{20}$$

$$f_h/f_v=1.8^{+0.7}_{-0.5}. \tag{21}$$

The absolute fluxes predicted from the simple theory of meson production are too high, by about a factor of four, in both vertical and horizontal directions. It is satisfactory, however, that the ratios Eqs. (20) and (21) agree as closely as they do.

Inasmuch as it is very improbable that a cascade event would not be observed in the cloud chamber, it must be concluded that pi-mesons of energy of the order 1 Bev are not created at the expected rate in the upper atmosphere. As Sands has pointed out, his analysis would not apply if, for example, lower energy mesons are produced in significant amounts in secondary processes or if the energy of meson and length of path in air traversed by the primary are not independent.

TABLE III. Results on sprays.

Date	No. Pb plates	No. frames	No. of prongs 10–15	>15	≥10	≥10 calc
7/10/49	2	200	9	4	13	15
2/9/50	2	298	22	5	27	22
10/12/50[a]	3	148	12	1	13	13
10/12/50[a]	4	148	20	3	23	17

[a] Vertical plates.

[9] Carlson, Hooper, and King, Phil. Mag. 41, 701 (1950).

Presumably such an effect exists, and the main source of the slower mesons lies deeper in the atmosphere than that for faster mesons.

It is evident from the form of $f(d)$, Eq. (17), that a good approximation to the zenith angle dependence, near $Z=0$, is

$$f(d)\simeq 0.47 \sec Z, \text{ small } Z.$$

Hence, in computing the geometric factor for horizontal plates, the average should be taken over the zero power of $\cos Z$. This means using G_0 in Eq. (4) in place of G_1. The vertical flux is reduced to 0.115 when G_0 is used. If the soft radiation were of primary origin, the vertical component would have multiplied by a factor 1.5, and the incident flux required to produce the observed showers would be 0.076 sterad^{-1} cm^{-2} min^{-1} with energy $\geqslant 1.1$ Bev. Thus, the soft radiation in this range of energy cannot constitute more than 0.6 percent of the primary flux as determined by Winckler *et al.*[10] This differs from the estimate of reference 1 (i.e., 0.2 percent) for the following reasons: in the present paper, showers are counted whether accompanied by other, penetrating tracks or not, the accepted solid angle is more accurately defined and proves to be smaller by a factor 0.8, and the measured sensitive time is 1.2 times the estimate in reference 1. In view of the "deficit" of soft radiation obtained by comparing with charged mu-mesons it seems most likely that no appreciable part of the primary radiation is composed of electrons or gamma-rays.

It must be concluded, also, that events customarily classified as "sprays" are adequately accounted for as cascade events in a single plate in agreement with Rau's conclusion.

APPENDIX I. SHOWER GEOMETRY

The most suitable coordinate system for the calculation is shown in Fig. 4 where O–L is the x-axis and O–D the y-axis for the lower plate. The ray is then projected upon a vertical plane parallel to the x axis and makes the angle ϕ with that plane. The projected ray makes an angle θ with the vertical. The angles θ and φ constitute a polar coordinate system about the y axis with θ as azimuth and φ the complement of the polar angle. The differential solid angle is then $\cos\varphi d\theta d\varphi$ and the cosine of the zenith angle is given by $\cos Z = \cos\theta \cos\varphi$. The expression for G_n is, therefore

$$G_n=\int_0^L dx\int_0^D dy\int_{-\tan^{-1}x/h}^{\tan^{-1}(L-x)/h}\cos^n\theta d\theta \times\int_{-\tan^{-1}(y\cos\theta)/h}^{\tan^{-1}(D-y)\cos\theta/h}\cos^{n+1}\varphi d\varphi. \quad (22)$$

[10] Winckler, Stix, Dwight, and Sabin, Phys. Rev. **79**, 656 (1950).

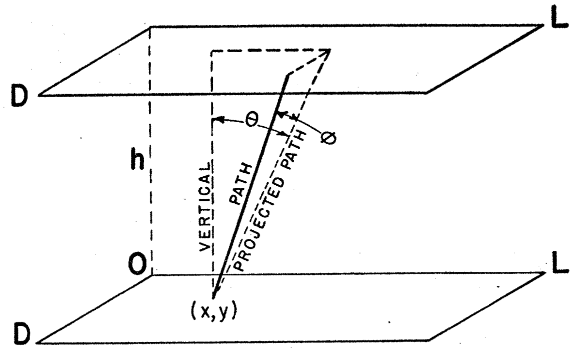

FIG. 4. Angles used in calculating geometrical formulas. The x axis is O-L; the y axis is O-D.

Let

$$a=D/h,\quad b=L/h. \quad (23)$$

Then

$$\begin{aligned}G_0=4h^2\{&ab\tan^{-1}[ab/(1+a^2+b^2)^{1/2}]+(1+a^2)^{1/2}\\&+(1+b^2)^{1/2}-1-(1+a^2+b^2)^{1/2}\\&+\tfrac{1}{2}a\ln[(1+a^2)^{1/2}-a][(1+a^2+b^2)^{1/2}+a]\\&-\tfrac{1}{2}a\ln[(1+a^2)^{1/2}+a][(1+a^2+b^2)^{1/2}-a]\\&+\tfrac{1}{2}b\ln[(1+b^2)^{1/2}-b][(1+a^2+b^2)^{1/2}+b]\\&-\tfrac{1}{2}b\ln[(1+b^2)^{1/2}+b][(1+a^2+b^2)^{1/2}-b]\}\\=D^2\{&2[(1+b^2)^{1/2}-1]\\&-(a^2/6)[(2b^2+1)/(1+b^2)^{1/2}-1]+0(a^4)\}, \quad (24)\end{aligned}$$

$$\begin{aligned}G_1=2h^2\{&b(1+a^2)^{1/2}\tan^{-1}[b/(1+a^2)^{1/2}]-b\tan^{-1}b\\&+a(1+b^2)^{1/2}\tan^{-1}[a/(1+b^2)^{1/2}]-a\tan^{-1}a\\&+\tfrac{1}{2}\ln[(1+a^2)(1+b^2)]-\tfrac{1}{2}\ln(1+a^2+b^2)\}\\=D^2\{&[1-(a^2/4)]b\tan^{-1}b\\&-a^2b^2/12(1+b^2)+0(a^4)\}, \quad (25)\end{aligned}$$

$$\begin{aligned}G_2=(4/3)h^2\{&ab\tan^{-1}[ab/(1+a^2+b^2)^{1/2}]\\&+(1+a^2+b^2)^{1/2}+1-(1+a^2)^{1/2}-(1+b^2)^{1/2}\}\\=D^2\{&\tfrac{2}{3}[(2b^2+1)/(1+b^2)^{1/2}-1]+a^2/6\\&-a^2(8b^4+12b^2+3)/18(1+b^2)^{1/2}+0(a^4)\}, \quad (26)\end{aligned}$$

$$\begin{aligned}G_3=\tfrac{1}{2}h^2\{&[(1+2a^2)b/(1+a^2)^{1/2}]\tan^{-1}[b/(1+a^2)^{1/2}]\\&+[(1+2b^2)a/(1+b^2)^{1/2}]\tan^{-1}[a/(1+b^2)^{1/2}]\\&-a\tan^{-1}a-b\tan^{-1}b\}\\=\tfrac{1}{4}D^2\{&(3-(5/4)a^2)b\tan^{-1}b+b^2/(1+b^2)\\&-a^2b^2(7b^2+9)/12(1+b^2)+0(a^4)\}. \quad (27)\end{aligned}$$

The second forms given in each case are explicit up to order a^4, $0(a^4)$, for convenience in those calculations in which a^2 is very small. If both a and b are small, the double power series yields in general:

$$G_n=D^2b^2\{1-(n+3)(a^2+b^2)/12+(n+3)(n+5)\times[6(a^4+b^4)+5a^2b^2]/720+\cdots\}. \quad (28)$$

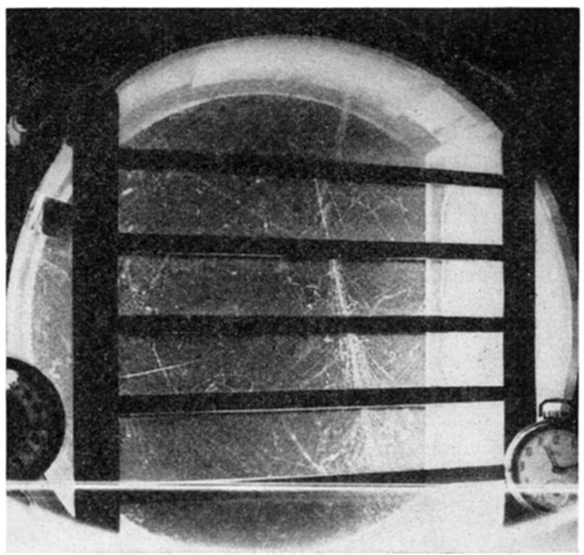

Fig. 1. An electron shower obtained in the flight of July 16, 1948. It has the fifth highest energy of those obtained with horizontal plates. The estimated total energy is 1.6 Bev.

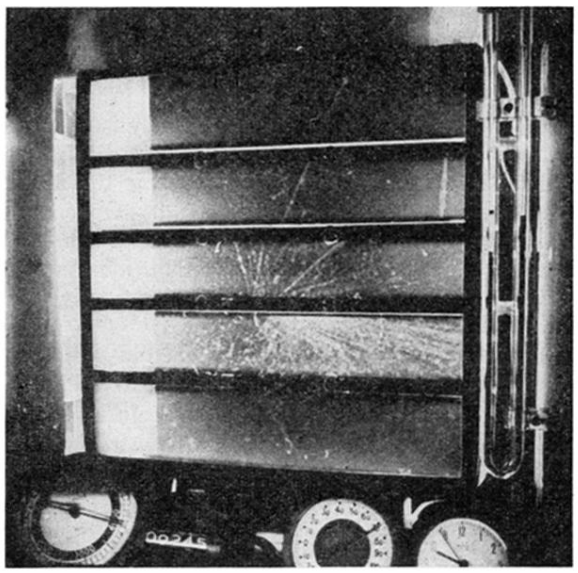

Fig. 2. A large spray event recorded on a flight over Cuba.