(b) The energy for enhanced secondary emission corresponds to the energy required to liberate an electron from the filled band to the vacuum. This indicates secondary electrons originate from the filled band and that the energy required is the threshold energy of secondary emission.

(c) The drifting in secondary emission yields noticed with thicker surfaces may be due to field enhanced secondary emission produced by positive charges at the surface of the material.

VIII. ACKNOWLEDGMENTS

We wish to thank Mr. Herbert Nelson of the Radio Corporation of America for his suggestions pertaining to the design of the second experimental tube and for his discussions during the course of this work. We should also like to acknowledge suggestions from Dr. J. B. Johnson of the Bell Telephone Laboratories, Professor A. van der Ziel of the University of Minnesota, Dr. J. E. Gorham, and Mr. G. Ross Kilgore of the Signal Corps Engineering Laboratories.

PHYSICAL REVIEW VOLUME 85, NUMBER 3 FEBRUARY 1, 1952

# Nuclear Disintegration by Positron-$K$ Electron Annihilation

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(Received October 17, 1951)

A new process for the annihilation of fast positrons is discussed, in which a positron with insufficient energy to excite or disintegrate a nucleus by collision, annihilates a $K$ electron of an atom with subsequent excitation or disintegration of its nucleus. If the positron energy is close to threshold for the process, competition from two-quanta annihilation does not occur. The process is of first order and, apart from the occurrence of negative energy states, is the reverse of internal conversion. The cross section can be factored into a cross section for annihilation with emission of a photon converging on the nucleus times a probability for nuclear disintegration. In the electric dipole case, the latter is just the ratio of the photodisintegration cross section to the $P$-wave blackbody absorption cross section of the nucleus. The photodisintegration cross section is taken from experiment.

The annihilation cross section in light elements has been calculated in the Born approximation using the complete retarded interaction corresponding to converging spherical waves of electric dipole radiation (the nucleus acts as a sink for these waves, in addition to conserving momentum). For large incident energy of the positron ($\gg mc^2$) the difference between positive and negative energy states can be neglected approximately, and the cross section obtained by detailed balancing from the internal conversion coefficient. Insofar as accurate values of the latter are known in the proper energy range for high atomic numbers $Z$, the annihilation cross section for very fast positrons can be obtained to a good approximation for the same values of $Z$.

Numerical estimates have been made for the disintegration of $Be^9$ with emission of a neutron and also for the disintegration of $U^{238}$ resulting in nuclear fission. The total annihilation-disintegration cross sections near the threshold in these two cases are $\sim 10^{-34}$ cm$^2$ and $\sim 10^{-31}$ cm$^2$, respectively. The total cross section for an annihilation-excitation of $In^{115}$ into an activation level for the metastable state, resulting in the formation of a nuclear isomer, is found to be $\sim 10^{-26}$ cm$^2$.

## I. INTRODUCTION

THE purpose of this note is to discuss a new type of annihilation process for positrons.[1] Consider a positron incident on an atom with less kinetic energy than would be needed to disintegrate the nucleus. If the positron annihilates an orbital electron, the energy released, which is greater by $\sim 2\ mc^2$ than the initial kinetic energy, may be sufficient to produce a nuclear disintegration in the same atom. A possible competitive process is the two-quanta annihilation of the positron in which one of the quanta produces photodisintegration of a nucleus of the same kind. This competitive process will not occur, however, if the positron energy is sufficiently close to threshold, since one of the quanta in two-quanta annihilation must take away a minimum energy $\sim mc^2/2$.

The annihilation-disintegration process may be described as a transition of an electron from an orbital state into a vacant negative energy state in the continuum corresponding to the incident positron, accompanied by a nuclear transition from the ground state into an excited continuum state corresponding to disintegration. The mechanism for the process and the perturbation that induces the transitions is the complete retarded electromagnetic interaction between the electron and the nucleus. Clearly our process would be the reverse of internal conversion were it not for the negative energy states. The close relation to internal conversion will be taken advantage of in the following discussion.

In Section II we show that the annihilation-disintegration cross section ($\sigma_{ad}$) can be factored into a cross section for annihilation with emission of a spherical wave of photons converging on the nucleus ($\sigma_{aq}$) times a probability for nuclear disintegration. The latter is just the ratio of the nuclear photodisintegration cross section to the maximum (blackbody) absorption cross section of the nucleus for the photon. In the absence of an adequate nuclear theory, the cross section for the nuclear photoeffect is to be taken from experiment. In Section III we make use of a theorem that for large

[1] A preliminary abstract appeared in Phys. Rev. **83**, 238 (1951).

incident energy ($\gg mc^2$) of the positron, the difference between positive and negative energy states can be neglected in the annihilation calculation, i.e., the negative energy wave functions replaced by positive energy functions for the same energy. In this limit the annihilation cross section $\sigma_{aq}$ can be obtained by detailed balancing from the internal conversion coefficient. In Section IV the cross section $\sigma_{aq}$ is calculated relativistically in the Born approximation using the retarded interaction corresponding to converging spherical waves of electric dipole radiation. Finally, in Section V, some estimates are made of $\sigma_{ad}$.

## II. FACTORING THE CROSS SECTION

Although the annihilation-disintegration process is a radiation process of the first order in the sense of perturbation theory, it can be thought of as occurring in two steps because of the factorization of the cross section previously mentioned. This is quite similar to the case of internal conversion in which the nucleus is treated as a source of diverging electromagnetic waves. For simplicity we assume that only electric dipole radiation is involved. Since the absolute square of the matrix element of the perturbation is the same for a given process as for the statistically reverse process (assuming the same normalization), we shall substitute for our matrix element, the appropriate matrix element for the reverse process, which is simpler to calculate because of the connection with internal conversion. The reverse process is this: the nucleus in a dissociated continuum state makes an electric dipole transition to the ground state; an unobservable electron in a continuum negative energy level absorbs the nuclear radiation, going to the $K$-shell and leaving behind an unoccupied negative energy state (positron). The matrix element of the perturbing energy is

$$H_{fi}'=\int\psi_f{}^\dagger H'\psi_i d\tau=-e\int\psi_f{}^\dagger(\Phi+\boldsymbol{\alpha}\cdot\mathfrak{A})\psi_i d\tau \qquad (1)$$

where $\boldsymbol{\alpha}$ is the Dirac matrix vector, $\psi_f$ is the Dirac function for an electron in the $K$-shell, $\psi_i$ is the Dirac function for a free electron in a negative energy state (normalized in unit volume), and $\Phi$ and $\mathfrak{A}$ are the electrodynamic scalar and vector potentials representing diverging waves of electric dipole radiation coming from nuclear sources of transition charge and current densities

$$\begin{aligned}\rho&=\epsilon(\chi_f{}^*\chi_i e^{-i\omega t}+\chi_f\chi_i{}^*e^{i\omega t}),\\ \mathbf{j}&=(\epsilon\hbar/2Mi)(\chi_f{}^*\nabla\chi_i-\chi_i\nabla\chi_f{}^*)e^{-i\omega t}+\text{c.c.}\end{aligned} \qquad (2)$$

Here $\chi_f$ represents the ground-state nuclear wave function and $\chi_i$ the wave function for the dissociated nucleus. For simplicity we represent the nuclear transition as if only one nuclear particle of charge $\epsilon$ and mass $M$ were involved. The electric dipole case is obtained by taking $\chi_f$ and $\chi_i$ to represent $S$ and $P$ states. Proceeding by well-known methods,[2,3] the solutions for the retarded potentials are found to be

$$\begin{aligned}&\Phi(\mathbf{r})=\mathfrak{M}_{fi}A_0(\mathbf{r}), \quad A_0(\mathbf{r})=(r^{-2}-ikr^{-1})\cos\theta e^{i(kr-\omega t)},\\ &\mathfrak{A}_z(\mathbf{r})=\mathfrak{M}_{fi}A_z(\mathbf{r}), \quad A_z(\mathbf{r})=-ikr^{-1}e^{i(kr-\omega t)},\\ &\mathfrak{A}_x=\mathfrak{A}_y=0, \qquad \mathfrak{M}_{fi}=\int d\tau'\chi_f{}^*r'\cos\theta'\chi_i,\end{aligned} \qquad (3)$$

where the conjugate complex terms have been dropped as noncontributing to the radiation process. Here $k=\omega/c$ and $\mathfrak{M}_{fi}$ denotes the nuclear electric dipole matrix element. The scalar and vector potentials in (3) are just the usual expressions for an electric dipole oscillating in the $z$ direction at the origin in the conventional gauge, the power radiated being $4\omega^4|\mathfrak{M}_{fi}|^2/3c^3$. The use of the conventional gauge avoids the complication of singularities at the origin.[4] Inserting (3) in (1), one obtains for the annihilation-disintegration cross section (for one $K$-electron)

$$\sigma_{ad}=(2\pi/\hbar v)|\mathfrak{M}_{fi}|^2 \cdot\left\langle\left|\int d\tau\psi_f{}^\dagger(-eA_0-e\alpha_z A_z)\psi_i\right|^2\right\rangle\cdot\rho_n, \qquad (4)$$

where $v$ is the velocity of the incident positron and $\rho_n$ the density per unit energy of the final states of the dissociated nucleus. Equation (4) is averaged over the initial spin states of electron and positron, and also over angles between the incident direction of the positron and the direction of the oscillating dipole.

The cross section $\sigma_{aq}$ for annihilation of the positron and orbital ($K$) electron, with emission of a spherical wave of electric dipole radiation converging on the origin (center of the nucleus), is normalized to one photon passing per unit time through a sphere about the origin. The previous expression for $|H_{fi}'|^2$ can be used if $|\mathfrak{M}_{fi}|^2$ is replaced by $3\hbar/4k^3$. The number of final states per unit energy corresponding to one photon emitted per unit time is $2/2\pi\hbar$, since there are two photon states with different polarizations allowed in the elementary cell of area $2\pi\hbar$ in the energy-time phase space. Hence for one $K$-electron

$$\sigma_{aq}=(2\pi/\hbar v)(3\hbar/4k^3) \times\left\langle\left|\int d\tau\psi_f{}^\dagger(-eA_0-e\alpha_z A_z)\psi_i\right|^2\right\rangle(2/2\pi\hbar). \qquad (5)$$

The nuclear matrix element $\mathfrak{M}_{fi}$ enters the cross section for the nuclear photoeffect in the case of an electric

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[2] H. M. Taylor and N. F. Mott, Proc. Roy. Soc. (London) A138, 665 (1932). The electric quadrupole case is here worked out in detail.

[3] A more rigorous discussion than the one given by Taylor and Mott has just been published: N. Tralli and G. Goertzel, Phys. Rev. 83, 399 (1951).

[4] S. M. Dancoff and P. Morrison, Phys. Rev. 55, 122 (1939).

dipole transition. This cross section may be written

$$\sigma_{ph}=(2\pi/\hbar c)(2\pi\hbar\omega)|\mathfrak{M}_{fi}|^2\rho_n, \quad (6)$$

where the radiation potentials are normalized to one photon per unit volume and the normalization of the dissociated states is the same as in (4). Assuming the nucleus to act as a blackbody, the maximum absorption cross section for a photon of $2^l$ pole radiation would be $(2l+1)\pi k^{-2}$ or $3\pi k^{-2}$ for electric dipole radiation. Combining Eqs. (4), (5), and (6) we obtain

$$\sigma_{ad}=\sigma_{aq}\cdot(\sigma_{ph}/3\pi k^{-2}). \quad (7)$$

From the definition of $\sigma_{aq}$ it is clear that the nucleus, in addition to conserving momentum in the annihilation process, acts as a sink for the annihilation radiation. The ratio $\sigma_{ph}/3\pi k^{-2}$ represents the probability that an incoming photon will cause a photodisintegration of the nucleus. Hence the cross section $\sigma_{ad}$ can be factored as mentioned in the introduction.

## III. THE CROSS SECTION IN THE HIGH ENERGY LIMIT RELATED TO THE INTERNAL CONVERSION COEFFICIENT

In the limiting case of positron energy greatly in excess of $mc^2$, it is possible to obtain the annihilation-converging photon cross section $\sigma_{aq}$ directly from the internal conversion coefficient. This result will be seen to follow from a theorem which was first used by Solomon[5] to relate the cross section for one-quantum annihilation of a very fast positron with a $K$-electron to the cross section for the photoelectric effect in the $K$-shell. The theorem may be stated in general form as follows: In a radiation process involving the annihilation of a free positron with a bound electron, provided that the positron energy is large compared to $mc^2$, the negative energy state solutions of the Dirac equation may be replaced by positive energy solutions for the same energy, and the cross section will be the same as for a corresponding process involving the capture of a free electron of the same energy as the positron into the same bound state. The theorem is readily seen to be valid in the Born approximation, i.e., if plane wave functions are used to represent the continuum states. That the theorem cannot be strictly true for a finite nuclear charge seems evident from the lack of symmetry with respect to the nuclear charge in the two processes which are compared. Nevertheless, in the particular application made by Solomon, the theorem proves to be much more accurate for large values of the atomic number $Z$ than would be expected from the Born approximation, since the condition $Ze^2/\hbar v\ll 1$ is not fulfilled.[6] The substantial error incurred through the use of plane wave functions appears to be nearly the same in the two related processes in spite of the lack of symmetry with respect to the nuclear charge. In view of this result, we may expect the theorem to have greater accuracy in the present application than would be anticipated on the basis of the Born approximation.

It follows, then, that for positron total energy $E\gg mc^2$, the cross section $\sigma_{aq}$ must be approximately the same as for capture of a free electron of energy $E$ into a vacancy in the $K$-shell, accompanied by the emission of a spherical wave of electric dipole radiation converging on the nucleus. The latter is precisely the statistical reverse of an internal conversion process, and $\sigma_{aq}$ can thus be obtained in this limit from the internal conversion coefficient by using the principle of detailed balancing. According to the latter one must have

$$\rho_e w_{aq}=\rho_q w_{ic} \quad (E\gg mc^2), \quad (8)$$

where $w_{aq}$ is the transition probability per unit time corresponding to $\sigma_{aq}$, $w_{ic}$ is the transition probability per unit time for the reverse process (internal conversion), $\rho_e$ is the number of electron states per unit energy per unit volume, and $\rho_q$ is the number of photon states per unit energy per unit time. Taking into account both spin directions of the electron and both polarization directions of the photon, one obtains

$$\rho_e=pE/\pi^2c^2\hbar^3 \quad \text{and} \quad \rho_q=1/\pi\hbar. \quad (9)$$

Since the free electron states are normalized in unit volume, the incident flux is $v=c^2p/E$ and $\sigma_{aq}$ is given by

$$\sigma_{aq}=\pi\hbar^2c^2(E^2-m^2c^4)^{-1}w_{ic}\cong\pi(\hbar/mc)^2\xi^{-2}w_{ic}, \quad (10)$$

where $\xi\equiv E/mc^2\gg 1$. Equation (10) is probably a good approximation even for large $Z$. Now $w_{ic}$ represents the number of electronic transitions per unit time from the $K$ shell to the continuum induced by a nuclear electric dipole radiating one photon per unit time; i.e., just the internal conversion coefficient for one $K$ electron. A relativistic calculation of the internal conversion coefficient in the Born approximation, due to Dancoff and Morrison,[4] gives in the high energy limit

$$w_{ic}\cong Z^3\alpha^4\xi^{-1} \quad (\xi\gg 1) \quad (11)$$

where $\alpha=e^2/\hbar c$. Equation (11) is independent of multipole type and order. Substituting (11) into (10) one finds

$$\sigma_{aq}=\pi\alpha^2(Z/\xi)^3(e^2/mc^2)^2 \quad (\xi\gg 1). \quad (12)$$

Hence, if the Born approximation is used, Eq. (5) of the preceding section must reduce to Eq. (12) in the limit of very fast positrons. Equation (12) is accurate only for low atomic numbers ($Z\ll 137$) because of the use of the Born approximation. The $Z^3$ dependence of $\sigma_{aq}$ is to be contrasted with the $Z^5$ dependence of the cross section for ordinary one-quantum annihilation. In the latter process the emitted photon, which goes to infinity, is represented by a plane wave and the nucleus serves only to take up momentum; in the former process the emitted photon is represented by a spherical wave

[5] J. Solomon, J. phys. 6, 114 (1935).

[6] This has been pointed out by J. C. Jaeger and H. R. Hulme, Proc. Cambridge Phil. Soc. 32, 158 (1936).

converging on the nucleus and, in this case, the nucleus takes up momentum and absorbs the energy of the photon. Equations (10) and (12) are not quite adequate for our purposes, since the positron energy is not always greatly in excess of $mc^2$, being restricted to values close to the annihilation-disintegration threshold to avoid competition with direct disintegration and two-quanta annihilation.

## IV. CALCULATION OF THE CROSS SECTION FOR INTERMEDIATE ENERGIES

Assuming that $Z \ll 137$, we proceed to a direct evaluation of Eq. (5) in the Born approximation—i.e., using plane waves for the continuum eigenfunctions. The matrix element to be calculated is

$$H_{fi}'' = -e(3\hbar/4k^3)^{\frac{1}{2}} \int \psi_f^\dagger (A_0 + \alpha_z A_z)\psi_i d\tau, \tag{13}$$

where $\psi_f$ and $\psi_i$ are given in transposed form by

$$\begin{aligned} \psi^T{}_f &= (\pi a^3)^{-\frac{1}{2}}[-\tfrac{1}{2}i\alpha Z\cos\theta,\ -\tfrac{1}{2}i\alpha Z\sin\theta e^{i\phi},\ 1,\ 0]e^{-r/a}, \quad (m_s)_f = \tfrac{1}{2} \\ &= (\pi a^3)^{-\frac{1}{2}}[-\tfrac{1}{2}i\alpha Z\sin\theta e^{-i\phi},\ \tfrac{1}{2}i\alpha Z\cos\theta,\ 0,\ 1]e^{-r/a}, \quad (m_s)_f = -\tfrac{1}{2} \end{aligned} \tag{14}$$

$$\begin{aligned} \psi^T{}_i &= \left(\frac{E+mc^2}{2E}\right)^{\frac{1}{2}}\left[1,\ 0,\ \frac{-cp_z}{E+mc^2},\ \frac{-c(p_x+ip_y)}{E+mc^2}\right]e^{-i\mathbf{K}\cdot\mathbf{r}}, \quad (m_s)_i = \tfrac{1}{2} \\ &= \left(\frac{E+mc^2}{2E}\right)^{\frac{1}{2}}\left[0,\ 1,\ \frac{-c(p_x-ip_y)}{E+mc^2},\ \frac{cp_z}{E+mc^2}\right]e^{-i\mathbf{K}\cdot\mathbf{r}}, \quad (m_s)_i = -\tfrac{1}{2}. \end{aligned} \tag{15}$$

Equations (14) give the Dirac eigenfunctions for an electron in the $K$ shell neglecting $\alpha^2 Z^2 = (Z/137)^2$; the effect of screening can also be neglected ($a = \hbar^2/me^2Z$). Equations (15) give the eigenfunctions for a free electron in a negative energy state normalized in unit volume. The propagation vector $\mathbf{K} = \mathbf{p}/\hbar$, the momentum $\mathbf{p}$ and the energy $E$ in Eq. (15) all refer to the positron instead of the negative energy electron. On substituting (3), (14), and (15) into (13) and carrying out the matrix multiplications neglecting small terms of order $\alpha Z$, one obtains for $(m_s)_i = \frac{1}{2}$

$$\begin{aligned} H_{fi}'' &= -ieG(E)\{cp_z(E+mc^2)^{-1}(J+ik^{-1}L) - I\}, \quad (m_s)_f = \tfrac{1}{2} \\ &= -ieG(E)\{c(p_x+ip_y)(E+mc^2)^{-1}(J+ik^{-1}L)\}, \quad (m_s)_f = -\tfrac{1}{2} \end{aligned} \tag{16}$$

where

$$\begin{aligned} G(E) &= [(3\hbar/4k)\cdot(1/\pi a^3)\cdot(E+mc^2/2E)]^{\frac{1}{2}}, \\ I &= \int d\tau r^{-1} e^{-r/a+ikr-i\mathbf{K}\cdot\mathbf{r}}, \\ J &= \int d\tau r^{-1}\cos\theta e^{-r/a+ikr-i\mathbf{K}\cdot\mathbf{r}}, \\ L &= \int d\tau r^{-2}\cos\theta e^{-r/a+ikr-i\mathbf{K}\cdot\mathbf{r}}. \end{aligned} \tag{17}$$

The integrals are readily evaluated by using the familiar expansion of $e^{i\mathbf{K}\cdot\mathbf{r}}$ in terms of Legendre polynomials and spherical Bessel functions, together with the addition theorem for the Legendre functions. The evaluation of the radial part of the last integral requires some care at the lower limit. Both $k$ and $|\mathbf{K}|$ are very much greater ($\sim 137$) than $a^{-1}$; hence $e^{-r/a}$ plays the role of a convergence factor in the integral and $a$ appears in the final result only through the normalization factor for the $K$ shell wave function. This accounts for the $Z^3$ dependence of $\sigma_{aq}$ which was noted in Sec. III. The integration gives

$$\begin{aligned} I &= -4\pi(k^2-K^2)^{-1}, \quad K = |\mathbf{K}| \gg a^{-1}, \quad k \gg a^{-1}, \\ J + ik^{-1}L &= -4\pi\cos\theta' K k^{-1}(k^2-K^2)^{-1}, \end{aligned} \tag{18}$$

where $\theta'$ denotes the polar angle of $\mathbf{K}$—i.e., the angle between the incident direction of the positron and the direction of the oscillating dipole. $|H_{fi}''|^2$ is averaged over spins and also over the angle $\theta'$. Neglecting the $K$-shell binding energy we set $k = (E+mc^2)/\hbar c$. After some elementary calculation it is found that

$$\sigma_{aq} = \pi\alpha^2 Z^3(e^2/mc^2)^2\cdot(\xi^2+2\xi+3)\cdot(\xi+1)^{-9/2}\cdot(\xi-1)^{-1/2}. \tag{19}$$

This reduces to Eq. (12) in the limit $\xi \gg 1$. Because of the use of the Born approximation, Eq. (19) is accurate only for $Ze^2/\hbar v \ll 1$. The effect of replacing the plane wave functions (15) by the exact Coulomb wave functions for the continuous spectrum would be to decrease the cross section. The order of magnitude of the correction to Eqs. (12) and (19) can be seen from the reverse process: internal conversion in the $K$ shell of electric dipole radiation. The Born approximation formula[4] for the conversion coefficient, when compared with the results of numerical calculations[7] with continuous spectrum Coulomb functions, is found to be too large in the case of the heaviest elements in the energy region of interest (a few Mev) by about a factor of two. It is to be noted that the interaction between the incident positron and the orbital electron has been ignored, in accordance with an approximation customarily followed in the theory of creation and annihilation processes.[8]

[7] H. R. Hulme, Proc. Roy. Soc. (London) **A138**, 643 (1932); B. A. Griffith and J. P. Stanley, Phys. Rev. **75**, 534 (1949); Rose, Goertzel, Spinrad, Harr, and Strong, Phys. Rev. **83**, 79 (1951).

[8] W. Heitler, *The Quantum Theory of Radiation* (Oxford University Press, London, 1936 and 1944), p. 198.

## V. CONCLUSION

Consider the application of formulas (7) and (19) to the disintegration of $Be^9$ with emission of a neutron. The binding energy of the neutron is 1.67 Mev[9] and the cross section for photodisintegration has been measured[10] in the neighborhood of the threshold. If the positron has an energy $E$ of 2.85 $mc^2$ and a two-quanta annihilation process[11] takes place, one of the photons must carry away a minimum energy of 0.59 $mc^2$ and the greatest energy that the other photon can have is just barely in excess of the $Be^9$ threshold. Accordingly, possible competition from two-quanta annihilation cannot occur for energies $E<2.85\ mc^2$. No direct disintegration of the nucleus without annihilation is possible for positron energies $E<4.26\ mc^2$. Guth and Mullin's calculations[12] of the photodisintegration cross section of $Be^9$, based on a potential well interaction between the neutron and residual nucleus, show that the important transitions at low energies are electric dipole in character. Hence formula (19), developed for the electric dipole case, is directly applicable. For $E=2.6$ $mc^2$, Eq. (19) gives: $\sigma_{aq}=3.1\times10^{-29}$ cm$^2$. The experimental photodisintegration cross section $\sigma_{ph}$ is $6\times10^{-28}$ cm$^2$ at $\hbar\omega=3.6\ mc^2$. From Eq. (7), $\sigma_{ad}=1.7\times10^{-35}$ cm$^2$, and this must be doubled, because of the two electrons in the $K$ shell, to give a total annihilation-disintegration cross section of $3.4\times10^{-35}$ cm$^2$. Nearer to the threshold ($E=2.3\ mc^2$), the total cross section is $6.9\times10^{-35}$ cm$^2$.

A larger cross section is expected in heavier elements because of the $Z^3$ factor, although this is partly compensated by having larger values of $\xi$. An interesting case would be the annihilation-disintegration of uranium, resulting in nuclear fission. Photofission in uranium has been investigated near the threshold by Haxby *et al.*[13] who found a cross section ($\sigma_{ph}$) of $3.5\times10^{-27}$ cm$^2$ for $\gamma$-rays of energy 6.3 Mev, produced by bombarding fluorine with protons. The same value of $\hbar\omega$ would be obtained in an annihilation of a positron of total energy $E=11.5\ mc^2$ (taking account of the $K$-shell ionization energy). Since $\xi\gg1$, we first apply the high energy formula (12) which is independent of multipole type and order. The result for the total annihilation-disintegration cross section is $5\times10^{-31}$ cm$^2$. Although the energy is high the large value of $Z$ makes the Born approximation very unreliable. A better result is obtained by using formula (10) together with an accurate value of the internal conversion coefficient calculated with continuous spectrum Coulomb functions. We assume the transition to be electric dipole in character.[14] Using the tables of Griffith and Stanley,[7] one finds that the internal conversion coefficient for $Z=92$ and $\hbar\omega=12.3\ mc^2$ and for one $K$ electron is given by: $w_{ic}=1.64\times10^{-4}$. This is smaller by a factor of 2.39 than the Born approximation value.[4] On substituting into Eq. (10), and thence into Eq. (7), doubling to take account of two $K$ electrons, and increasing the result by fifteen percent for the contributions from $L$, $M$, and higher shells, we obtain a total cross section for the annihilation-disintegration process of $2.5\times10^{-31}$ cm$^2$ for positron energy $E=11.5\ mc^2$.

One may also consider a process in which the nucleus is excited instead of being disintegrated. The nuclear excitation might be detected, for example, by the formation of a nuclear isomer.[15] Miller and Waldman[16] have located the principal activation state for the $In^{115}$ isomer at 1.04 Mev above the ground state. In their experiments the nuclear excitation was produced by bremstrahlung from a monoenergetic electron beam incident on a gold target. From the observed over-all cross section and Guth's calculations[17] on the bremstrahlung isochromat, they have estimated the cross section for production of a metastable level by a photon of 1.04 Mev to be of the order of $10^{-22}$ cm$^2$. In considering the activation of $In^{115*}$ by positrons, one sees that no direct excitation of the 1.04-Mev level[18] is possible for positron energies $E<3.04\ mc^2$ and that the same level cannot be excited in a two-quanta annihilation if $E<1.77\ mc^2$. An annihilation-excitation of the type considered in this paper can take place if the positron has an energy $E=1.10\ mc^2$. Assuming the transition to be electric dipole in character and applying formula (19), one obtains for $\sigma_{aq}$ the value $1.12\times10^{-24}$ cm$^2$. Since $Ze^2/\hbar v=0.86$, the Born approximation is unreliable. The $K$-shell electric dipole internal conversion coefficient for $Z=49$ and $\hbar\omega=1.04$ Mev is too large in the Born approximation by a factor of 2.2, according to the tables of Rose, Goertzel, *et al.*[7] A comparable error is expected in the stated value of $\sigma_{aq}$. Taking $\sigma_{ph}=10^{-22}$ cm$^2$, correcting roughly for the Born approximation and doubling to take account of two $K$ electrons, one obtains a total annihilation-activation cross section in $In^{115}$ of $3\times10^{-26}$ cm$^2$. Although this cross section is much larger than those previously calculated, the process can only take place if the positron has just the right energy (within a fraction of an electron volt).

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[11] P. A. M. Dirac, Proc. Cambridge Phil. Soc. **26**, 361 (1930); H. A. Bethe, Proc. Roy. Soc. (London) **A150**, 129 (1935).

[12] E. Guth and C. J. Mullin, Phys. Rev. **76**, 234 (1949).

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[14] M. Goldhaber and E. Teller, Phys. Rev. **74**, 1046 (1948); J. S. Levinger and H. A. Bethe, Phys. Rev. **78**, 115 (1950).

[15] This was suggested to us by P. R. Bell.

[16] W. Miller and B. Waldman, Phys. Rev. **75**, 425 (1949).

[17] E. Guth, Phys. Rev. **59**, 325 (1941).

[18] Miller and Waldman (see reference 16) have found a lower activation level in the neighborhood of 0.87 Mev with a cross section 100 times smaller than the cross section for the 1.04-Mev level. No direct excitation of this level by positrons is possible if $E<2.7\ mc^2$.