tensor range is somewhat greater. The phenomena which this potential fits well are: deuteron binding energy; triton binding energy; low energy NP and PP scattering; low energy deuteron photodisintegration; deuteron quadrupole moment.

The singlet potential is[27]

$$V_S=-(\hbar^2K_S^2/M)(e^{-\mu r}/\mu r),$$
$$K_S=10.75\times10^{12}\text{ cm}^{-1},\quad \mu=8.58\times10^{12}\text{ cm}^{-1}.$$

As to the wave functions derived from these potentials: singlet state S and D functions are obtained by numerical integration of the Schroedinger equation, and are used in subsequent numerical integration to compute J_0 and J_2 for σ_m.

For the ^3S ground-state wave the best Hulthén function, an excellent approximation, is fitted to the Feshbach-Schwinger curve. It is

$$u=N(e^{-\gamma r}-e^{-\zeta r}),$$

where $\gamma=2.316\times10^{12}$ cm^{-1}, $\zeta=13.36\times10^{12}$ cm^{-1}, and $N=(7.76\times10^{12}\text{ cm}^{-1})^{\frac{1}{2}}$.

For the ^3D ground state wave several different procedures are used. First, for magnetic calculations, the Feshbach-Schwinger curve is used directly in numerical integration. Two approximate analytic expressions are also used, which are fitted to the numerical curve. A low energy approximation is

$$w\approx M'(1-e^{-\alpha r})e^{-\gamma r}[1+(3/\gamma r)+(3/\gamma^2r^2)],$$

which has the exact asymptotic form. $\alpha=3.60\times10^{12}$ cm^{-1}.

$$M'=(0.0100\times10^{12}\text{ cm}^{-1})^{\frac{1}{2}}.$$

A high energy approximation is

$$w\approx M\{(1-Ar)e^{-\xi r}-Be^{-\eta r}\},$$

where $\xi=3.06\times10^{12}$ cm^{-1}, $A=0.1385\times10^{12}$ cm^{-1}, $B=1.140$, $\eta=11.04\times10^{12}$ cm^{-1}, and $M=(0.455\times10^{12}\text{ cm}^{-1})^{\frac{1}{2}}$.

[27] J. M. Blatt and J. D. Jackson, Phys. Rev. **76**, 18 (1949).

Phase Shifts

The phase shifts δ_S and δ_D of Sec. II were determined first by approximate formulas, and the approximations then improved in the subsequent numerical integration of the wave equation. It may be of some interest to present the Born approximation formula for δ_D, if the potential is V_S, above. It is

$$\delta_D=\frac{K_S^2}{4k\mu}\left\{\left[1+\frac{3}{2}\left(\frac{\mu}{k}\right)^2+\frac{3}{8}\left(\frac{\mu}{k}\right)^4\right]\log\left(\frac{\mu^2+4k^2}{\mu^2}\right)-3\left[1+\frac{1}{2}\left(\frac{\mu}{k}\right)^2\right]\right\}.$$

PHYSICAL REVIEW VOLUME 85, NUMBER 2 JANUARY 15, 1952

The Absorption of Gamma-Rays from Co^{60}

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(Received September 7, 1951)

Measurements of the absorption of gamma-rays from Co^{60} (1.17 and 1.33 Mev) have been made in 27 elements. In order to exclude the errors due to secondary radiations which might be produced in neighboring objects and in the absorbers, particular precautions were taken with respect to the geometrical arrangement of apparatus. The absorption coefficients measured for the elements whose atomic numbers are less than $_{52}$Te show reasonable agreement with those calculated according to existing theories. However, it is noted that the results with $_{73}$Ta, $_{74}$W, $_{78}$Pt, $_{79}$Au, $_{80}$Hg, and $_{81}$Tl are 3 percent to 5.5 percent less than the theoretical values. It seems improbable that the disagreements observed in these elements may be assigned to experimental causes. If the entire deviation were assigned to inaccuracy in theoretical knowledge, it would be reasonable to attribute it to some insufficiency in the Klein-Nishina theory of the Compton effect for this energy of gamma-rays. But further investigation should be undertaken to ascertain the fact.

I. INTRODUCTION

SINCE the application of Co^{60} has rapidly increased in various fields of science, it becomes important to know with greater accuracy the absorption coefficients of the gamma-rays from this radioisotope (1.17 and 1.33 Mev) in various elements. The absorption of gamma-rays in matter may be attributed to the combination of four separate effects, namely the photoelectric effect, the Compton effect, pair production, and the photonuclear reaction. Photonuclear reactions seem to be generally improbable in the energy range below several Mev except for a few nuclei. The absorption due to the photoelectric effect has been theoretically estimated by many workers,[1] and that due to the Compton effect has been formulated by Klein and Nishina,[2] while pair production has been theoretically discussed by Dirac and others.[3] A summary of most of these theories, which give the knowledge of absorp-

[1] F. Sauter, Ann. Physik **9**, 217 (1931); **11**, 454 (1931); H. Hall, Phys. Rev. **45**, 620 (1934); H. Hall and W. Rarita, Phys. Rev. **46**, 143 (1934); J. G. Jaeger and H. R. Hulme, Proc. Roy. Soc. (London) **148**, 708 (1935); Hulme, McDougall, Buckingham, and Fowler, Proc. Roy. Soc. (London) **149**, 131 (1935).
[2] O. Klein and Y. Nishina, Z. Physik **52**, 853 (1928).
[3] P. Dirac, Proc. Cambridge Phil. Soc. **30**, 150 (1934); W. Heisenberg, Z. Physik **90**, 209 (1934); H. Bethe and W. Heitler, Proc. Roy. Soc. (London) **146**, 83 (1934); W. Furry and J. R. Oppenheimer, Phys. Rev. **45**, 245 (1934).

tion of gamma-rays in matter, has been given by Heitler.[4]

Several measurements of absorption of gamma-rays from some radioisotopes and comparisons with the theoretical values have already been published. Among these reports it is noted that Cork and Pidd[5] found apparent discrepancies between observed and computed values of absorption coefficients in lead and copper using gamma-rays from Zn^{65} (1.14 Mev), Na^{24} (2.76 Mev), and Co^{60}. However, the result of the absorption measurement of Na^{24} gamma-rays in lead obtained by Groetzinger and Smith[6] substantiated the theoretical value. Furthermore, Davisson and Evans[7] have recently measured accurate absorption coefficients of gamma-rays from I^{131} (0.367 Mev and others), Cu^{64} (annihilation radiation), Mn^{54} (0.835 Mev), Co^{60}, Zn^{65}, and Na^{24}, and found that the values obtained by their experiments showed agreement within 0.5 percent to 2 percent with theory. However, an anomalous absorption coefficient 5 percent less than the theoretical value with tantalum absorber and Zn^{65} and Co^{60} sources found by their measurements seems to need further investigation.

In the present work, undertaken to obtain more information on the absorption of gamma-rays in many elements, we made absorption measurements for 27 elements using the gamma-rays from Co^{60} with a special arrangement of the apparatus; and we compared the results obtained with theoretical values.

II. APPARATUS AND EXPERIMENTAL PROCEDURE

The source of Co^{60} used in the present experiment was prepared from radioactive cobaltous chloride solution, which was supplied from Oak Ridge last year by courtesy of the United States Atomic Energy Commission. Evaporation residuum of this solution of about 2 millicuries equivalent intensity, sealed in a thin glass sphere of 3 mm in diameter, was used as a gamma-ray source hung from the ceiling of the room in which the experiment was performed (Fig. 1).

As a gamma-ray detector we used an end-window G-M counter, 2 cm in diameter with an effective length of about 3 cm, filled with a mixture of argon and alcohol vapor. In practice, however, we used this counter with a lead plate of 1.2 mm thickness placed just upon a mica window for the reason described below. The counter was connected to a high speed scale-of-100 recording circuit.

In the geometrical arrangement of apparatus involved in the measurement of gamma-ray absorption, it is essential to minimize secondary photons and electrons from neighboring bodies and from the absorber itself as far as possible, since such scattered radiations may reach the detector. In work so far published, various precautions were taken in this respect; namely, the adequate collimation of gamma-rays was generally made by placing the source in a deep hole situated in a solid lead block, the detector was surrounded on all sides by lead except for the side facing the source, and the solid angle subtended by the detector at the source and absorber was minimized. However, it should be noted that the lead block holding the source and the collimation lead may produce scattered radiations.

Taking into account these conditions, we arranged the apparatus as shown schematically in Fig. 1. A gamma-ray source without any lead block was hung from the ceiling of the room at 80 cm above the detector, an end-window G-M counter; to define the path of the gamma-rays falling upon the detector, a lead cylinder, *B*, 2.7 cm in inner diameter and 9.8 cm in outer diameter, with a height of 11.2 cm, was placed just above the counter. In order to prevent the secondary electrons. which might be produced at the absorber and the inner wall of the lead cylinder, from reaching the counter, and to count only the gamma-ray quanta

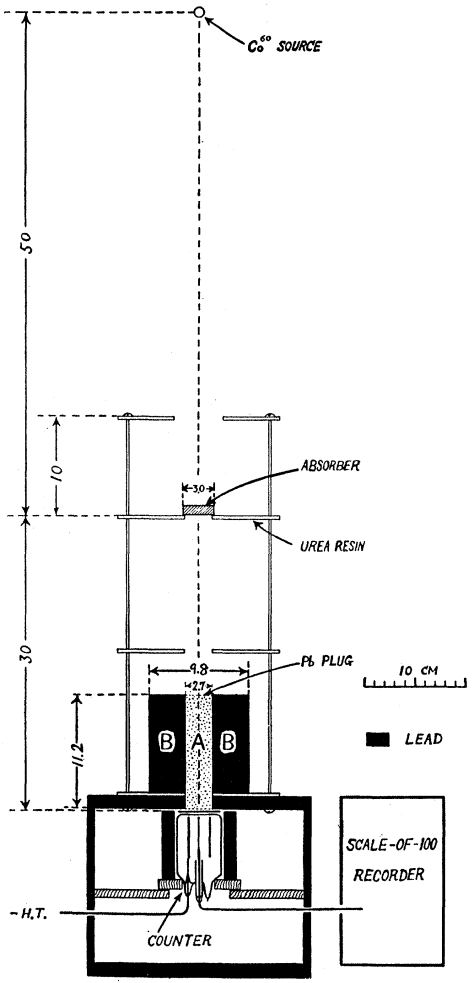

FIG. 1. Geometrical arrangement of apparatus to measure absorption coefficients.

[4] W. Heitler, *The Quantum Theory of Radiation* (Oxford University Press, London, 1936).
[5] J. M. Cork and R. W. Pidd, Phys. Rev. **66**, 227 (1944); J. M. Cork, Phys. Rev. **67**, 53 (1945).
[6] G. Groetzinger and L. Smith, Phys. Rev. **67**, 53 (1945).
[7] C. M. Davisson and R. D. Evans, Phys. Rev. **81**, 404 (1951).

TABLE I. Absorption coefficient of gamma-rays of Co^{60} in lead, as measured with different distances between source, absorber, and counter.

Distance between source and counter (cm)	Distance between absorber and source (cm)	Absorption coefficient $\mu(cm^{-1})$
80	30	0.6426±0.0028
80	40	0.6376±0.0031
80	50	0.6382±0.0047
150	30	0.6382±0.0058
150	40	0.6410±0.0047
150	50	0.6375±0.0044

passing through the canal of the cylinder, the mica window of the counter was covered with a thin lead plate of 1.2 mm thickness and the side wall of the counter was sufficiently surrounded by lead as shown in Fig. 1.

The measurement procedure with this geometry was as follows: for each absorber first we measured the count, c_1, with a lead plug, A, which filled exactly the canal of the lead cylinder, B, and then the count, c_2, without this plug. The difference of counts with and without the plug, c_2-c_1, should be proportional to the number of gamma-ray quanta which penetrate the absorber and pass through the canal, since the count c_1 seems practically to be due only to the scattered radiation reaching the counter sidewards. Therefore, we took c_2-c_1 as the true count of the gamma-rays for each absorber. In practice, when no absorber was used, c_1 was only 1.2 percent of c_2.

With this geometry and procedure[8] we could exclude the effect of scattered photons produced in the surrounding matters in the room, and of secondary electrons produced in the absorber and in the lead block, B. However, it was feared that the Compton singly scattered photons produced in the absorber itself and the lead block might cause counts. To clarify this point we made some absorption measurements with the procedure above mentioned and with different relative distances between the source, absorber, and detector, which resulted in different scattering angles with respect to the detector. The experimental values of the absorption in lead obtained by these test measurements are shown in Table I. The results agreed within the experimental error, showing that the effect of singly scattered photons falling on the counter was negligible with this geometry. For this reason we adopted the geometry shown in Fig. 1.

The absorbers of most elements, 3 cm in diameter, were placed 30 cm above the detector. However, in the cases of a few elements such as Mn, Ta, W, Pt, Au, and Tl, for which 3 cm absorbers were not obtainable, we used plates of about 2.5×2.5 cm as absorbers. These were placed 10 cm above the normal position so that their projections from the source covered the window of the counter. This alternation of position of absorbers would not affect the final results as shown by test measurements above mentioned. The absorption in metallic mercury was measured by filling a glass cylinder whose absorption when empty was known. For each element, several absorbers of different thicknesses were used, so that we might be able to obtain several points on the absorption curve until the count of gamma-rays decreased to about one-third of that without absorber. Generally, we paid no attention to the presence of impurities in absorber elements, since the quantitative chemical analysis of the absorbers used showed that the purities of most elements were at least more than about 99 percent. Densities of most absorbers were precisely determined from their weights measured in both air and pure water, while those of a few samples such as carbon and calcium were determined from measurements of volume and weight in air.

The absorption coefficients were calculated from the observed transmission and measured absorber densities, using the method of least squares.

[8] Similar geometry and procedure were used by Dr. Y. Uemura for the absorption measurements of high energy gamma-rays from Li(p,γ) and F(p,γ) reactions [Bull. Inst. Chem. Research, Univ. of Kyoto **22**, 18 (1950)].

III. RESULTS AND DISCUSSION

Experimental results obtained are summarized in Table II. In the fifth column of the table are listed atomic absorption coefficients expected theoretically.

In the computation of theoretical values, for the energies of the gamma-rays from Co^{60} we used the precise values measured by Lind, Brown, and DuMond[9]

TABLE II. Measured absorption coefficients of gamma-rays from Co^{60} in 27 elements compared with those computed theoretically.

Element	Linear abs. coef. μ (cm^{-1})	Mass abs. coef. μ_m (cm^2/g)	Atomic abs. coef. Experimental σ_{exp} (barn)	Atomic abs. coef. Theoretical σ_{th} (barn)
$_{6}C$	0.0884±0.0005	0.0589±0.0003	1.175±0.006	1.133
$_{12}Mg$	0.0994±0.0008	0.0572±0.0005	2.307±0.020	2.267
$_{13}Al$	0.1433±0.0011	0.0529±0.0004	2.378±0.018	2.457
$_{16}S$	0.1121±0.0004	0.0567±0.0002	3.021±0.011	3.026
$_{20}Ca$	0.0895±0.0011	0.0584±0.0007	3.887±0.046	3.788
$_{22}Ti$	0.2286±0.0025	0.0527±0.0006	4.189±0.046	4.171
$_{25}Mn$	0.3792±0.0096	0.0513±0.0013	4.675±0.119	4.750
$_{26}Fe$	0.4076±0.0037	0.0519±0.0005	4.826±0.044	4.943
$_{27}Co$	0.4469±0.0020	0.0512±0.0002	5.013±0.022	5.137
$_{28}Ni$	0.4881±0.0022	0.0546±0.0002	5.324±0.024	5.333
$_{29}Cu$	0.4638±0.0064	0.0516±0.0007	5.506±0.076	5.529
$_{30}Zn$	0.3662±0.0027	0.0513±0.0004	5.565±0.041	5.726
$_{34}Se$	0.2154±0.0017	0.0501±0.0004	6.572±0.053	6.523
$_{42}Mo$	0.5268±0.0038	0.0514±0.0004	8.192±0.059	8.179
$_{47}Ag$	0.5305±0.0032	0.0505±0.0003	9.046±0.055	9.283
$_{48}Cd$	0.4382±0.0049	0.0505±0.0006	9.418±0.104	9.512
$_{50}Sn$	0.3621±0.0035	0.0496±0.0005	9.779±0.096	9.971
$_{51}Sb$	0.3349±0.0026	0.0500±0.0004	10.11 ±0.08	10.21
$_{52}Te$	0.3042±0.0023	0.0487±0.0004	10.33 ±0.08	10.46
$_{73}Ta$	0.8990±0.0202	0.0535±0.0012	16.06 ±0.36	16.56
$_{74}W$	1.011 ±0.005	0.0527±0.0003	16.10 ±0.08	16.90
$_{78}Pt$	1.167 ±0.023	0.0543±0.0011	17.61 ±0.34	18.39
$_{79}Au$	1.074 ±0.005	0.0555±0.0003	18.17 ±0.09	18.78
$_{80}Hg$	0.7493±0.0095	0.0553±0.0007	18.41 ±0.23	19.18
$_{81}Tl$	0.6322±0.0017	0.0542±0.0001	18.53 ±0.05	19.60
$_{82}Pb$	0.6426±0.0028	0.0566±0.0002	19.46 ±0.09	19.97
$_{83}Bi$	0.5658±0.0025	0.0579±0.0003	20.10 ±0.09	20.42

[9] Lind, Brown, and DuMond, Phys. Rev. **76**, 1838 (1949).

with the use of the two-meter focusing curved-crystal gamma-ray spectrometer, namely 1.3316±0.0010 Mev and 1.1715±0.0010 Mev with equal intensities.

For the photoelectric absorption, we used the theory developed by Hulme, McDougall, Buckingham, and Fowler,[10] who gave their results in the form of a graph showing a calculated function of the photoelectric absorption coefficient per atom over a range of atomic number and quantum energies. Since the procedure of their theoretical calculation was very complicated, values of the photoelectric absorption coefficient per atom were deduced by graphical interpolation from their results for the elements and gamma-ray energies used by us.

For the computation of the atomic cross section due to Compton scattering we used the well-known Klein-Nishina formula:

$$\sigma_{\text{Comp}}=2\pi r_0^2 Z\left[\left(\frac{1+\alpha}{\alpha^2}\right)\left(\frac{2(1+\alpha)}{1+2\alpha}-\frac{1}{\alpha}\ln(1+2\alpha)\right)+\frac{1}{2\alpha}\ln(1+2\alpha)-\frac{1+3\alpha}{(1+2\alpha)^2}\right], \quad (1)$$

where $r_0=e^2/m_0c^2$, the classical electron radius, Z is the atomic number, and α is the gamma-ray energy in unit of m_0c^2.

The cross section per atom for the pair production for gamma-ray energies less than $\alpha=10m_0c^2$ was expressed by Hirschfelder and Adams[11] in the following relation,

$$\sigma_{\text{pair}}=0.2545r_0^2(Z^2/137)(\alpha-2.332). \quad (2)$$

We applied this relation to the computation of the cross section for the pair production, because this expression gave good agreement with the numerical values cited by Heitler.[12] However, since this relation was not valid for the gamma-ray energies less than $\alpha=2.332m_0c^2$, we took the cross section for the lower energy component of gamma-rays from Co^{60} ($\alpha=2.2933m_0c^2$) to be negligibly small for any element used, namely to be $\sigma_{\text{pair, 1.17}}=0.0000\times10^{-24}$ cm^2.

The total cross section for each component of the gamma-rays used is given by the sum,

$$\sigma_{\text{tot, 1.17}}=\sigma_{\text{phot, 1.17}}+\sigma_{\text{Comp, 1.17}}+\sigma_{\text{pair, 1.17}}$$

for 1.1715-Mev photon,

and

$$\sigma_{\text{tot, 1.33}}=\sigma_{\text{phot, 1.33}}+\sigma_{\text{Comp, 1.33}}+\sigma_{\text{pair, 1.33}}$$

for 1.3316-Mev photon.

Further, taking into account the facts that these components have equal intensities and that the gamma-ray counter used has the different counting efficiency for each of these photons, the theoretical total cross section per atom for the Co^{60} gamma-radiation, to be compared with the experimental value obtained by the present experiment, should be calculated by the following relation,

$$\sigma_{\text{th}}=(R\sigma_{\text{tot, 1.17}}+\sigma_{\text{tot, 1.33}})/(R+1), \quad (3)$$

where $R=\epsilon_{1.17}/\epsilon_{1.33}$ is the ratio of counting efficiencies of the counter for each component of the gamma-rays used. In the present computation $\epsilon_{1.17}$ and $\epsilon_{1.33}$ were taken to be 1.00 percent and 1.11 percent respectively, according to the experimental values determined by the work of Bradt *et al.*[13] for a lead counter of 1.0 mm thickness.

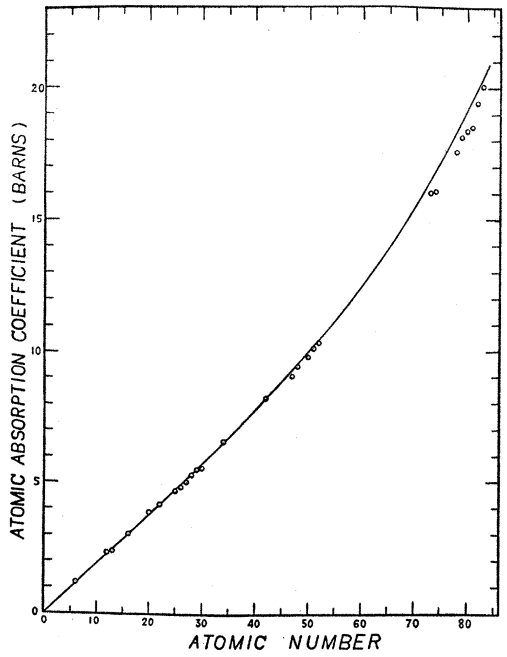

FIG. 2. Experimental and theoretical values of the absorption coefficient per atom plotted against atomic number Z. The solid curve indicates the result expected theoretically.

The atomic absorption coefficients computed theoretically by these procedures are listed in the fifth column of Table II, and measured and theoretical values plotted against atomic number are shown graphically in Fig. 2. The solid curve in this figure shows the result expected theoretically.

The results obtained with the elements whose atomic numbers are less than $_{52}$Te show agreement within 0.2 percent to 2.5 percent with theory; and with some elements, our results are in agreement within the experimental errors with those of other workers who have used the same source.[5,7,14] However, it is noted that the results with $_{73}$Ta, $_{74}$W, $_{78}$Pt, $_{79}$Au, $_{80}$Hg, and $_{81}$Tl are 3 percent to 5.5 percent less than the values

[10] Hulme, McDougall, Buckingham, and Fowler, Proc. Roy. Soc. (London) **149**, 131 (1935).
[11] J. O. Hirschfelder and E. N. Adams, II, Phys. Rev. **73**, 863 (1948).
[12] See reference 4, 2nd edition, 1944, Appendix II, p. 259.
[13] Bradt, Gugelot, Huber, Medicus, Preiswerk, and Scherrer, Helv. Phys. Acta **19**, 77 (1946).
[14] W. V. Mayneord and A. J. Cipriani, Can. J. Research **A25**, 303 (1947).

computed theoretically, as shown in Fig. 2. These disagreements are apparently outside the experimental error of about 2.5 percent. We have checked the possible reasons for these deviations. The error which may be caused by the presence of impurities in absorbers used was checked, but we found immediately that their effect on the final results should be masked by the statistical error involved in the counting measurements, since the impurities were generally less than 0.5 percent. The chemical analysis showed, for instance, 99.98 percent purity for tungsten, 99.99 percent for gold, and 99.91 percent for thallium.

The other possible source of errors responsible for the disagreement above mentioned is the Compton singly scattered radiation from the absorber itself and the inner wall of the canalizing lead slit placed before the counter. An analysis of such secondary radiation reaching the detector has been made by Tarrant[15] and Davisson and Evans.[7] Under the geometry here considered, the correction for this secondary radiation is small and may be evaluated using the formula given by Davisson and Evans for the difference between the true and apparent atomic absorption coefficient, namely,

$$\sigma_{\text{true}}-\sigma_{\text{app}}=\pi r_0^2 Z\theta_0^2[1-(\theta_0^2/12)(9\alpha+4)], \quad (4)$$

where θ_0 is the maximum angle of scattering for radiation entering the center of the detector.

From Eq. (4) we can estimate the effect of the scattered radiation from the absorber on the final result. For the Co^{60} radiation α is about 2.4 and the maximum angle of scattering in the present geometry is at most 0.0803 radian, so that the maximum scattering correction is about $0.0016\times10^{-24}Z$ cm^2. The amount of such a small correction for each of the absorber elements is nearly equal to or less than the statistical error due to the counting measurements. Therefore, the effect of the secondary radiation from the absorber may be neglected.

To estimate further the effect of the secondary radiation from the inner wall of the lead canal, we made transmission measurements with the source and absorber in different positions, thus varying the solid angle subtended by the inner wall at the source. The measured values show agreement within the statistical errors as shown in Table I, telling us that such secondary radiation is too minute to have any appreciable effect on the final results. From these considerations it can be affirmed that the deviations from theory observed with some elements should not be assigned to experimental causes, particularly not to the effect of the Compton scattered radiation.

For the gamma-rays from Co^{60} the absorption in the elements with which deviations from theory were found is primarily due to the Compton effect. If the entire deviation observed were assigned to the theoretical insufficiency of the photoelectric absorption, then a reduction of the absorption coefficient by about one-fourth would be required. This seems unlikely since reasonable agreement between the experimental and theoretical values was found in the other elements we examined. And since the absorption due to the pair production is vanishingly small at these photon energies, it is impossible to explain the observed discrepancies even by reducing this absorption to zero. Therefore, if the deviations found in $_{73}$Ta, $_{74}$W, $_{78}$Pt, $_{79}$Au, $_{80}$Hg, and $_{81}$Tl are real, it may be more reasonable to attribute them to some insufficiency in the Klein-Nishina theory of the Compton effect for photons of about 1.2 Mev. However, in order to ascertain this fact, further tests should be made with a stronger source of Co^{60} or Zn^{65}.

In conclusion, the authors wish to express their sincere appreciation for valuable discussions given by Professor K. Kimura and Dr. Y. Uemura. We have to thank sincerely the United States Atomic Energy Commission and Dr. H. C. Kelly, former chief of the Scientific and Technical Division, ESS-GHQ-SCAP, and the Scientific and Technical Administration Committee of Japan for introducing radioisotopes into Japan from Oak Ridge, Tennessee, U.S.A. Thanks are also due to Professor T. Okada and Professor K. Nishihara of the University of Kyoto, and to Kobe Kogyo Company, Ltd., and Mitsubishi Electric Manufacturing Company, Ltd. for the loan and the kind presents of some samples of pure elements, which enabled us to perform the work. This work was financially supported by the special fund of the Ministry of Education for the Advancement of Science.

[15] G. T. P. Tarrant, Proc. Cambridge Phil. Soc. **28**, 475 (1932).