obtained by using the Q-value 5.094 ± 0.010 Mev for the $Mg^{24}(d,p)Mg^{25}$ reaction,[11] and 2.230 ± 0.007 Mev for the binding energy of the deuteron.[12] If one takes level XII as corresponding to the average of the first six neutron levels, XIII and XIV to the next two doublets, respectively, then the agreement would seem satisfactory. It would appear that at least for the higher excitation of Mg^{25} that the levels obtained in this work may be composed of closely spaced unresolved groups rather than distinct levels.

An energy level diagram comparing the present work with the groups reported by Schelberg[2] and the neutron resonances[8–10] is given in Fig. 2.

[11] Strait, Van Patter, Buechner, and Sperduto, Phys. Rev. **81**, 747 (1951).

[12] R. E. Bell and L. G. Elliot, Phys. Rev. **79**, 282 (1950).

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Angular Distribution in the High Energy Deuteron Photoeffect*

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Consideration is given to the possible use of the electric dipole isotropic part of the deuteron photodisintegration cross section as an indicator of weak odd state noncentral forces. This cross section is not very sensitive to weak tensor forces of the usual type, but is strikingly affected by a singular spin-orbit coupling. While for the usual tensor forces the ratio, a/b, of the isotropic to the $\sin^2\theta$ term in the cross section varies slowly over a broad energy range, it rises rapidly with energy if the noncentral force is strongly singular. Calculation is done in first Born approximation. A detailed estimate of the magnetic isotropic cross section is included, for this is an unexpectedly large "background" to the electric term which is of interest.

I. INTRODUCTION

THE ground state is the only even parity state of the NP system which is important in deuteron photodisintegration, and its wave function is fairly well known through studies[1] of the many low energy phenomena into which it enters. Considerably less is known about the important states of odd parity. For these, high energy scattering studies[2] have led to a belief in the "even theory," that is, that the exchange dependence of the nuclear forces is such that two nucleons in an odd parity state of relative motion do not interact. The data do not, however, justify the complete exclusion of odd state noncentral interactions, as these are found not to have strong influence on the NP scattering cross section.[2,3] More definite information about them can in principle be obtained from photodisintegration measurements; therefore careful photodisintegration calculations with odd state noncentral forces are desirable. These would supplement the high energy photodisintegration calculations which were recently done for pure central forces.[4,5]

Odd state noncentral interactions do not much affect the total cross section for photodisintegration, but it has long been known from numerical calculation in special cases[6,7] that they do cause one striking qualitative change in the angular distribution. This change consists of the introduction of an isotropic term in the *photoelectric* cross section, which appears if noncentral interactions are effective in the ^3P continuum states. It is produced by interferences between the three ^3P_J states, and is basically a high energy effect, since nucleons in a relative P state cannot interact when the energy is low.

The present work is intended to determine with what sensitivity the isotropic term might actually measure the strength of a weak ^3P state noncentral force. It was begun after accurate low energy measurements of the photodisintegration angular distribution had become available,[8] and when some attempts were being made to perform such experiments at higher energies.[9–13] Energies up to $\hbar\omega=100$ Mev are considered. This is a range in which it is possible to disregard relativistic and free meson effects[4] and to use the usual phenomenological formalism.[14]

* This work is a portion of the author's doctoral thesis.

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[1] For a recent study of this type see Feshbach and Schwinger, Phys. Rev. **84**, 194 (1951). A copy of their work was supplied to the author by Dr. J. Eisenstein.

[2] R. S. Christian and E. W. Hart, Phys. Rev. **77**, 441 (1950).

[3] K. M. Case and A. Pais, Phys. Rev. **80**, 203 (1950).

[4] L. I. Schiff, Phys. Rev. **78**, 733 (1950).

[5] J. F. Marshall and E. Guth, Phys. Rev. **78**, 738 (1950).

[6] W. Rarita and J. Schwinger, Phys. Rev. **59**, 436 and 556 (1941).

[7] T. M. Hu and H. S. W. Massey, Proc. Roy. Soc. (London) **A196**, 135 (1949).

[8] For a summary of recent work see Bishop, Collie, Halban, Hedgran, Siegbahn, du Toit, and Wilson, Phys. Rev. **80**, 211 (1950); Bishop, Halban, Shaw, and Wilson, Phys. Rev. **81**, 219 (1951); Bishop, Beghian, and Halban, Phys. Rev. **83**, 1052(L) (1951).

[9] E. G. Fuller, Phys. Rev. **79**, 303 (1950).

[10] P. V. C. Hough, Phys. Rev. **80**, 1069 (1950).

[11] Phillips, Lawson, and Kruger, Phys. Rev. **80**, 326 (1950).

[12] G. Goldhaber, Phys. Rev. **81**, 930 (1951).

[13] Gibson, Green, and Livesey, Nature **160**, 534 (1949).

[14] See, for example, R. G. Sachs and N. Austern, Phys. Rev. **81**, 705 (1951).

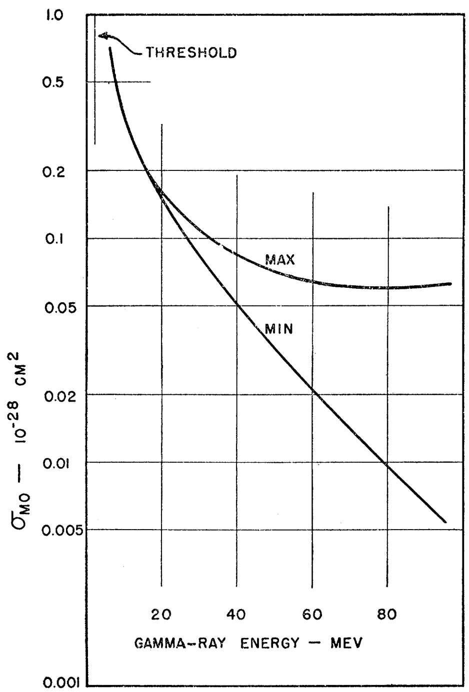

FIG. 1. Photomagnetic disintegration cross section, including only the dipole ${}^3S\rightarrow{}^1S$ transition, and showing the uncertainty caused by the interaction moment.

In a systematic discussion of possible high energy noncentral forces effects it is important to recognize that for gamma-rays of energy less than 100 Mev the electric dipole radiative interaction always leads to the dominating transitions of the system.[4] These are ${}^3S_1$ and ${}^3D_1\rightarrow{}^3P_{0,1,2}$, and ${}^3D_1\rightarrow{}^3F_2$. All other transitions are much weaker and are observable only in so far as they produce interferences with the strongly excited ${}^3P$ waves. The ${}^3F_2$ state can be disregarded entirely; not only is it excited weakly but it also does not contribute any such unique interferences as would be detectable in the presence of other, larger effects.

With pure central forces, electric dipole transitions from the ${}^3S$ deuteron ground state lead to ${}^3P$ states, and to an angular distribution of the form $\sin^2\theta$. With noncentral forces the angular distribution still cannot possibly[15] have terms of greater complication than $\sin^2\theta$. A term of lesser complication does, however, appear, giving the *electric dipole* angular distribution its most general form, $a_e+b\sin^2\theta$. To the term a_e there must be added the more familiar term, a_m, which comes from photomagnetic transitions to ${}^1S$ waves. The entire dipole-induced angular distribution then becomes $a+b\sin^2\theta$, where $a=a_e+a_m$. a_m plays the role of a "background" to the interesting a_e, so its value must be known for the purposes of this paper.

Transitions to continuum triplet waves of even parity can interfere with the ${}^3P$ waves and produce fore-aft antisymmetric parts in the angular distribution. Electric quadrupole and magnetic dipole transitions are of this type. With pure central forces only the quadrupole transition ${}^3S\rightarrow{}^3D$ can occur. The angular distribution becomes $a+b\sin^2\theta(1+q\cos\theta)$, the disintegration protons favoring the forward hemisphere. The quantity q is given by Schiff,[4] and by Marshall and Guth.[5] The introduction of tensor forces permits the quadrupole transition ${}^3D\rightarrow{}^3S$, and also magnetic dipole transitions to the coupled continuum ${}^3S_1+{}^3D_1$ waves. As it happens, interferences of the altered even waves with the ${}^3P_J$ waves do not change q significantly from its value under central forces. These effects will not be discussed further.

In subsequent sections the magnetic cross section a_m is computed first, and then the electric isotropic cross section a_e is computed in three special cases:

(a) No ${}^3P$ state nuclear interaction.
(b) A triplet state nuclear interaction of the form

$$V=-(\hbar^2\kappa^2/M)\{\varphi_1[\tfrac{1}{2}(1-x)+\tfrac{1}{2}(1+x)P] \\ +\Gamma S_{12}\varphi_2[\tfrac{1}{2}(1-y)+\tfrac{1}{2}(1+y)P]\}. \quad (1)$$

This interaction is so chosen that for even states it reduces to the expression used by Feshbach and Schwinger in their analysis of the low energy data. P is the Majorana exchange operator. Variation of the parameters x and y gives all possible exchange dependences of V. φ_1 and φ_2 have Yukawa shape, $\varphi=(e^{-\mu r}/\mu r)$. $S_{12}\equiv(3/r^2)(\boldsymbol{\sigma}_1\cdot\mathbf{r})(\boldsymbol{\sigma}_2\cdot\mathbf{r})-1$, is the tensor operator. Values of the parameters used in V are given in the appendix.

(c) A very singular $(\mathbf{L}\cdot\mathbf{S})$ interaction. For explicit calculation the expression of Case and Pais[3] is used.

II. PHOTOMAGNETIC CROSS SECTION

The isotropic part of the photomagnetic cross section, a_m, is computed in this section. Effects of the deuteron D function are taken into account, as well as the influence of the spin antisymmetric interaction moment[16] which gives a phenomenological representation of some meson current effects. An $(\mathbf{L}\cdot\mathbf{S})$ interaction of the type of Case and Pais[3] leads to other interaction moments, but these do not have matrix elements with the deuteron ground state.

The photomagnetic matrix element cannot be so easily approximated as the photoelectric matrix element,[17] and this would remain true even were interaction effects not present. Its integrand vanishes less strongly at the origin, by two powers of r, than does that of the electric matrix element. The magnetic matrix element thus tends to be influenced by those parts of the wave function which are most affected by the details of the nuclear potential; and our lack of knowledge of a correct nuclear potential consequently makes the cross section somewhat uncertain. Those other uncertainties which are contributed by the interaction moment are so much greater, however, that the calculations of the present section are only performed with Yukawa well eigenfunctions.[18]

[15] C. N. Yang, Phys. Rev. **74** 764 (1948).

[16] N. Austern and R. G. Sachs, Phys. Rev. **81**, 710 (1951); N. Austern, Phys. Rev. **81**, 307(A) (1951).

[17] H. A. Bethe and C. Longmire, Phys. Rev. **77**, 647 (1950); E. E. Salpeter, Phys. Rev. **82**, 60 (1951).

[18] Descriptions of the various wave functions used in this paper are collected in the appendix.

The magnetic dipole transitions ${}^3S_1 \rightarrow {}^1S_0$ and ${}^3D_1 \rightarrow {}^1D_2$ must both be considered. Although the D transition may not be large in itself, it does interfere with the S transition at all energies in such a way as to reduce the apparent value of the magnetic cross section. This can be seen from the differential magnetic dipole photodisintegration cross section:

$$\sigma_m(\theta) = \sigma_{m0}\{1 + (5 - 3\cos^2\theta)J_2^2/4J_0^2 - (3\cos^2\theta - 1)\cos(\delta_S - \delta_D)J_2/\sqrt{2}J_0\}, \quad (2)$$

where σ_{m0} is the simple S wave cross section, δ_S and δ_D are the scattering phase shifts, and

$$J_0 \equiv \int_0^\infty uu_S\{\tfrac{1}{2}(\mu_N - \mu_P) + \Phi\}dr,$$
$$J_2 \equiv \int_0^\infty ww_S\{\tfrac{1}{2}(\mu_N - \mu_P) + \Phi\}dr. \quad (3)$$

The functions u and w are the usual ground-state "radial" wave functions, while u_S and w_S are the corresponding singlet state functions. Φ is the shape function of the spin-antisymmetric interaction moment, which affects both the S and D wave transitions. Equation (2) can be separated into a part with $\sin^2\theta$ angular dependence, and an isotropic part. The former part is then lost experimentally beneath the large $\sin^2\theta$ electric dipole cross section. The latter part, which is the apparent magnetic cross section, is given by

$$a_m = \sigma_{m0}\{1 + J_2^2/2J_0^2 - \sqrt{2}\cos(\delta_S - \delta_D)J_2/J_0\}. \quad (4)$$

This is smaller than the value σ_{m0}.

In calculating upper and lower bounds on σ_{m0} the spin-antisymmetric interaction moment,

$$\Delta\mathbf{M} = (e\hbar/2Mc)(\boldsymbol{\sigma}_N - \boldsymbol{\sigma}_P)\Phi(r), \quad (5)$$

is assumed as the only correction which must be added to the free nucleon spin magnetic moments, and the form

$$\Phi(r) = \Phi_0(t)\delta(r - t) \quad (6)$$

is once again adopted, $\Phi_0(t)$ being taken from the work of reference 16. σ_{m0} is then found as a function of t, and of photon energy. Figure 1 shows the bounds which are subsequently obtained on σ_{m0} by choosing, at each energy, the largest and smallest values of $\sigma_{m0}(t)$ in the region $t \leqslant 2\times10^{-13}$ cm. The upper limit on the values for t is comparable with the usual ranges for nuclear forces and is probably a little large.

Figure 2 shows a_m as a function of energy. This quantity is derived from σ_{m0} by the prescription of Eq. (4). For the integrals J_0 and J_2 in (4) it is more convenient to employ some definite function Φ rather than the form of Eq. (6). A very reasonable function is $\Phi = \Phi_0 e^{-\mu r}$. For $\mu = 8.5\times10^{12}$ cm^{-1}, the value of Φ_0 which gives the correct three-body moment anomaly is $\Phi_0 = -0.56$. With this, the ratio a_m/σ_{m0} is computed as a unique function of energy and is used to pass from Fig. 1 to Fig. 2.

A remark is in order about the determination of Φ, the shape function of the interaction moment. Figure 1 shows only that uncertainty which is produced in σ_{m0} because the detailed shape of Φ is unknown. Additional large uncertainties come through lack of exact knowledge of the H^3 wave function and through the extreme idealization involved in the description of the three-body moment anomaly by means of the simple spin-antisymmetric moment. While the H^3 wave function of Pease and Feshbach[19] is certainly much better, the present work is based on the wave function of Avery and Adams.[20] The status of the interaction moment theory does not really seem to justify recalculating all the two-body results on the basis of the better H^3 wave function.

Magnetic quadrupole-magnetic dipole interference might be important at higher energies, but cannot be computed reliably. It would appear through a small fore-aft antisymmetric term in the cross section, which cancels completely when the experimental data are folded about 90° to find a.

III. *P* WAVE INTERFERENCE—FREE *P* WAVES

The D wave admixture in the deuteron ground state insures some effect of noncentral forces, even when the ${}^3P_J$ waves are entirely free. In particular, the isotropic component, a_e, does not quite vanish for free P waves.

It is convenient to use the differential electric dipole

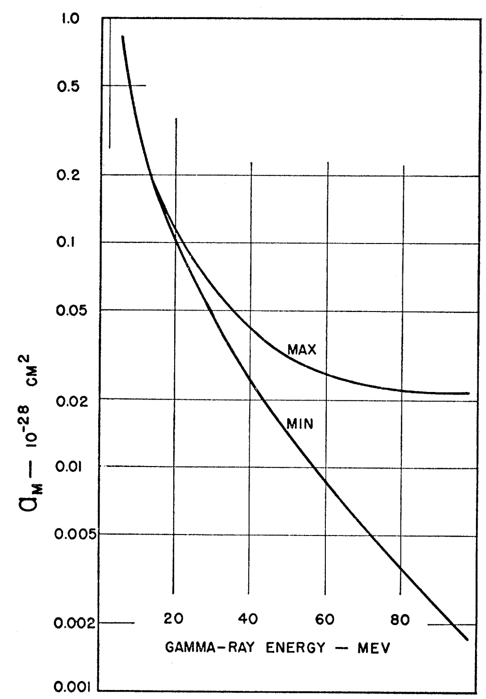

FIG. 2. Isotropic part of the photomagnetic disintegration cross section, including both the dipole transitions ${}^3S \rightarrow {}^1S$ and ${}^3D \rightarrow {}^1D$.

[19] R. L. Pease and H. Feshbach, Phys. Rev. **81**, 142(L) (1951); also private communications from Dr. Pease.

[20] R. Avery and E. N. Adams, Phys. Rev. **75**, 1106(L) (1949).

photodisintegration cross section in a slightly different form from that of Rarita and Schwinger:[6]

$$\sigma_e(\theta)d\Omega=\frac{\pi}{3}\frac{e^2}{\hbar c}\frac{M\omega}{\hbar k}\frac{d\Omega}{4\pi}\bigg[\bigg\{\frac{1}{9}|e^{i\delta_0}I_0-e^{i\delta_2}I_2|^2+\frac{1}{4}|e^{i\delta_1}I_1-e^{i\delta_2}I_2|^2\bigg\}+\sin^2\theta\bigg\{\frac{1}{4}|e^{i\delta_1}I_1+3e^{i\delta_2}I_2|^2-\frac{1}{15}|e^{i\delta_0}I_0-5e^{i\delta_2}I_2|^2+\frac{I_0^2}{15}\bigg\}\bigg],\quad(9)$$

where

$$I_0=\int_0^\infty rdrv_0[u-w\sqrt{2}],$$
$$I_1=\int_0^\infty rdrv_1[u+(w/\sqrt{2})],\quad(10)$$
$$I_2=\int_0^\infty rdrv_2[u-(w/5\sqrt{2})].$$

Here $\hbar\omega$ is the gamma-ray energy, $\hbar k$ is the relative momentum of neutron and proton in the continuum state, and the v_J are the ^3P_J state "radial wave functions."

A convenient notation is now introduced. Denote

$$U(f)\equiv\int_0^\infty rdruf,\quad W(f)\equiv\int_0^\infty rdrwf.\quad(11)$$

Then, for example, $I_0=U(v_0)-\sqrt{2}W(v_0)$.

For the "even theory" there is no ^3P wave interaction, so the $\delta_J=0$, and the v_J reduce to the spherical Bessel function, F_1. I_0 reduces to $I_0{}^0=U(F_1)-\sqrt{2}W(F_1)$, etc., so only the ground-state wave functions, u and w, must be known to compute σ_e. σ_e reduces to

$$\sigma_e(\theta)d\Omega=\frac{\pi}{3(137)}\frac{k^2+\gamma^2}{k}\frac{d\Omega}{4\pi}\bigg[\frac{9}{25}W^2(F_1)+\frac{3}{2}\sin^2\theta\times\bigg\{U^2(F_1)+\frac{1}{25}W^2(F_1)\bigg\}\bigg].\quad(12)$$

$\hbar^2\gamma^2/M$ is the deuteron binding energy, so $\gamma=2.32\times10^{12}$ cm^{-1}.[17] The total cross section is just

$$\sigma_{e0}=(\pi/3(137))(k^2+\gamma^2/k)[U^2(F_1)+(2/5)W^2(F_1)],\quad(13)$$

and

$$(a_e/b)_0=(6W^2(F_1)/25U^2(F_1)+W^2(F_1)).\quad(14)$$

Here the subscript "0" indicates that the results are the ones appropriate to no ^3P wave interaction.

Numerical results have been computed for σ_{e0} and for $(a_e/b)_0$ using the ground-state wave functions which are described in the appendix. $(a_e/b)_0$ is graphed as Fig. 3. Note especially the near-constancy of $(a_e/b)_0$ for $\hbar\omega\gtrsim20$ Mev. This result occurs because u and w have nearly the same shape at small r (as a result of the importance of the S–D cross term in the expectation value of the deuteron potential energy).

Brief consideration is desirable for the effect that noncentral forces in the ground-state configuration might have on the effective range treatment of the low energy photoelectric cross section. There is no room here for a full discussion; that would involve setting up and carrying through an effective range formalism for noncentral forces.[21] The main points of such an analysis can be mentioned: It is only necessary to consider the ground-state S wave, since at low energy the D wave contributes a negligible amount to σ_{e0}. This is illustrated by Fig. 3, $(a_e/b)_0$ being a measure of the D wave contribution. The easily computed zero range photodisintegration cross section is corrected for the nonzero range of forces merely by a renormalization of the ground-state S function. For central forces Bethe and Longmire[17] have shown that the effective range derived from low energy NP scattering experiments can be used very simply to compute the necessary renormalization of the S function. A recheck of the Bethe and Longmire method and of the definition of the effective range in scattering discloses that, while each has to be modified for the presence of the D wave, still, so long as the ***experimentally determined*** scattering effective range is used to renormalize the S wave, the noncentral forces produce no change in the method. This agrees with a similar remark by Bethe and Longmire.

Thus at very low energy Eq. (13) gives identically the simple central forces result. At high energy the D wave term increases the cross section. For the wave functions used here this increase is roughly constant for energies above about 18 Mev and has the value

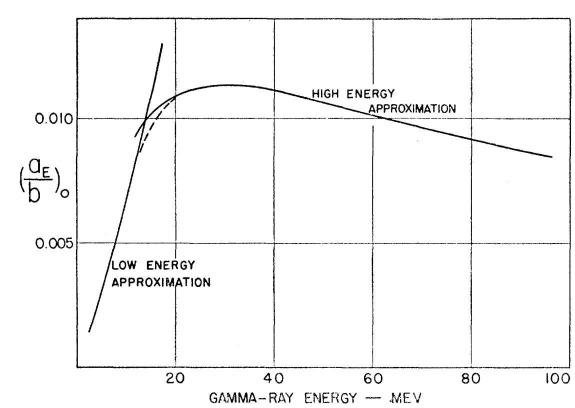

FIG. 3. The ratio of electric isotropic cross section to $\sin^2\theta$ cross section for the case of vanishing ^3P wave forces.

[21] See, for example, J. Schwinger, notes on a course in nuclear physics.

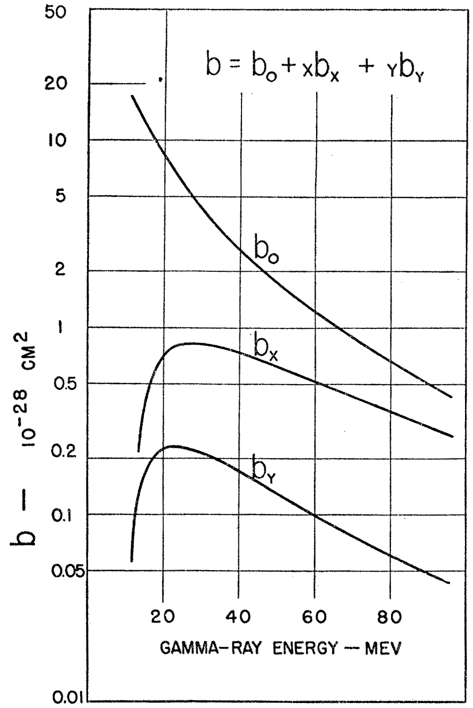

FIG. 4. Coefficient of $\sin^2\theta$ part of electric dipole photodisintegration cross section, showing the first derivatives of the cross section with respect to the exchange parameters for the central and tensor forces. Divide by 4π to get the cross section per steradian.

two percent. If Fig. 3 is again used as a measure of the D wave contribution to σ_{e0} the increase of σ_{e0} is seen to enter approximately linearly with energy for energies up to 18 Mev.

IV. *P* WAVE INTERFERENCE—WEAK TENSOR FORCES

In this case Eq. (9) does not reduce to the simple form of (12), for the v_J are affected by the forces. The even theory is considered as the basis for departure, and small deviations from the even theory are computed in first Born approximation. The expression (1) is used for the nuclear potential, where the even theory is obtained if $x=y=0$. Evidently the Born approximation is rigorously correct for very small x and y, and may be expected to give an indication of the effects if x and y become large.

For the odd parity ${}^3P_J$ states V becomes

$$V=(\hbar^2\kappa^2/M)\{x\varphi_1+c_J\Gamma y\varphi_2\}, \qquad (15)$$

$c_J=-4,\ 2,\ -2/5$, for $J=0,\ 1,\ 2$; and the Schroedinger equation takes the form[22]

$$L_1(v_J)=\kappa^2\{x\varphi_1+c_J\Gamma y\varphi_2\}v_J, \qquad (16)$$

where

$$L_1(v_J)=v_J''-(2/r^2)v_J+k^2v_J,\quad L_1(F_1)=0. \qquad (17)$$

This differential equation for v_J is solved in terms of the kernel[23]

$$K_1(r,r')=F_1(r_<)G_1(r_>), \qquad (18)$$

where $r_<$, $r_>$ are the lesser and greater, respectively, of r, r'; F_1 is the regular solution of $L_1(F_1)=0$, while G_1 is the irregular solution. They are normalized so that the Wronskian $F_1'G_1-F_1G_1'=1$; thus K_1 satisfies

$$L_1(K_1)=-\delta(r-r'). \qquad (19)$$

Now to Born approximation

$$\delta_J=-(\kappa^2/k^2)\int_0^\infty F_1^2\{x\varphi_1+c_J\Gamma y\varphi_2\}dr \qquad (20)$$

$$\begin{aligned} v_J&=F_1+x\partial v_J/\partial x]_{x=0}+y\partial v_J/\partial y]_{y=0}\\ &=F_1+\delta v_J, \end{aligned} \qquad (21)$$

and

$$\delta v_J=-\kappa^2\int_0^\infty K_1(r,r')dr' \times\{x\varphi_1(r')+c_J\Gamma y\varphi_2(r')\}F_1(r'). \qquad (22)$$

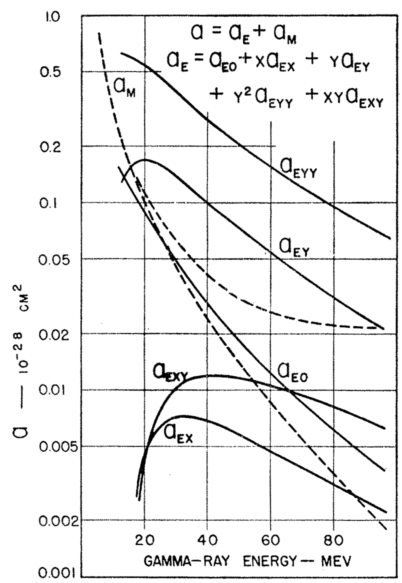

FIG. 5. Coefficients of the quantities which go to make up the isotropic part of the photodisintegration cross section. The dotted curves show the uncertainty in the photomagnetic part of this cross section. Divide by 4π to get the cross section per steradian.

[22] Equation (16) is not correct. It is written as if the ${}^3P_2$ wave were an eigenstate of S_{12}, whereas actually S_{12} couples ${}^3P_2$ with ${}^3F_2$. But to the approximation used in this work it is correct to use (16) as given. The complete coupled differential equations for ${}^3P_2$ and ${}^3F_2$ are

$$\left\{\begin{aligned} 0&=v_2''-(2/r^2)v_2+[k^2-\kappa^2x\varphi_1+(2/5)\kappa^2\Gamma y\varphi_2]v_2\\ &\qquad-[(6\sqrt{6})/5]\kappa^2\Gamma y\varphi_2\epsilon f_2\\ 0&=\epsilon f_2''-(12/r^2)\epsilon f_2+[k^2-\kappa^2x\varphi_1+(8/5)\kappa^2\Gamma y\varphi_2]\epsilon f_2\\ &\qquad-[(6\sqrt{6})/5]\kappa^2\Gamma y\varphi_2 v_2 \end{aligned}\right\}.$$

In these equations f_2 is that ${}^3F_2$ radial function which is strongly coupled to ${}^3P_2$, and normalized to be $\cos(kr+\delta)$ at $r=\infty$. ϵ is the amplitude with which it properly enters the calculation. $\epsilon\rightarrow 0$ as the coupling vanishes.

The first equation above reduces to (16) when the coupling term is not present and reduces rigorously to (16) in the first Born approximation. The coupling term contains as a factor the product of two infinitesimals ϵ and y; thus it vanishes with y to one higher order than the rest of the nuclear potential.

[23] For this formalism see, for example, F. Rohrlich and J. Eisenstein, Phys. Rev. **75**, 705 (1949).

With these expressions the remainder of the calculational procedure is to insert (20) and (21) into (9); perform the necessary integrals; and evaluate numerically. The results of calculation are graphed as Figs. 4, 5, and 6.

Figure 4 shows that the magnitude of the $\sin^2\theta$ part of the cross section, which is nearly the total cross section, depends very strongly on the exchange character of the central potential but hardly at all on the exchange character of the tensor potential. The magnitude of the total cross section is thus a measure of x, the exchange parameter of the central potential. This was indicated before by Marshall and Guth,[5] as well as by other authors.

Figure 5 shows the isotropic cross section, with the photomagnetic background, a_m, included. Study of Fig. 5 shows that a_e is not much influenced by the central potential but does depend strongly on y, the exchange parameter of the tensor force. Above about 15 Mev the tensor forces isotropic component competes strongly with the photomagnetic cross section and dominates it if y is large, i.e., if $y \gtrsim 0.5$. A very favorable energy region for experiments designed to detect the tensor forces isotropic component lies from about 15 Mev to 60 Mev, the lower part of this energy region being the more desirable. At higher energy the photomagnetic cross section becomes unmanageably uncertain.

Figure 6 is an approximate graph of the ratio a_e/b for the case $x=0$ and is obtained by using the even forces value, b_0, in place of b. It is seen that a_e/b is not a rapidly varying function of energy. An experiment to measure a_e/b would, therefore, not need to be performed with monochromatic gamma-rays, and might be well adapted for a betatron laboratory.

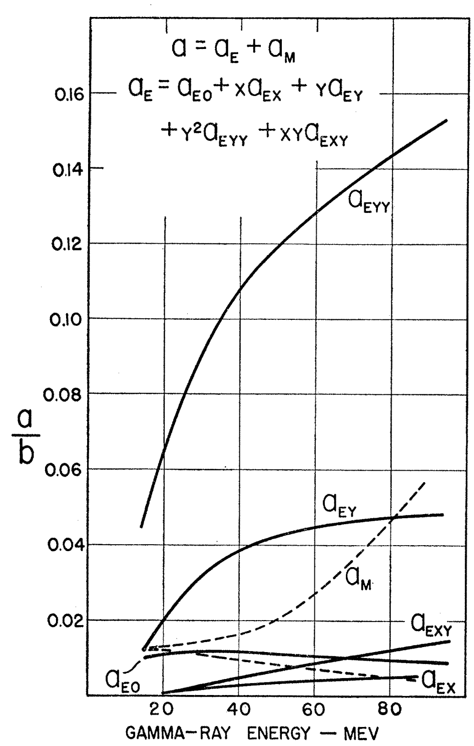

FIG. 6. Ratio of the curves of Fig. 5 to the curve, b_0, of Fig. 4.

Figures 4–6 are not quantitatively reliable for large values of x and y. Exact calculations of some special cases[6,7] have shown, for example, that very much larger tensor isotropic effects sometimes result from a "neutral" theory ($x=y=-1$) than from a "charged" theory ($x=y=1$). This occurs because there is an essential asymmetry between attractive and repulsive potentials, in that an attractive potential has greater influence on the wave function than a repulsive potential of the same strength. While this type of asymmetry is less important for long tailed potentials than for the square wells which have been emphasized before, it should appear to some degree for the cases treated here. That it does not results from the use of first Born approximation and illustrates the inadequacies of the numerical results. Thus Figs. 5 and 6 indicate more or less symmetric changes of a_e for equal positive and negative changes of x or y, so evidently should not be used for large x or y. This is not an unreasonable restriction, as very large deviations from the "even theory" are not expected.

Some care is needed to obtain the maximum information from such an approximate calculation as the one described here, without in the process stepping beyond the approximation. Equation (9) gives $\sigma_e(\theta)$ as a sum of squares. The Born approximation gives the linear correction in x and y to the quantities which are squared, i.e., to the outgoing wave amplitudes. Of course, quadratic and higher corrections also exist. It is clear that, after squaring, the linear terms of σ_e will be known exactly. The quadratic terms of σ_e come partly as squares of the linear corrections to the wave function, which are known, and partly as zero-order second-order cross terms, which are unknown. Where the linear correction to an amplitude is considerably larger than the zero-order amplitude, it is expected that the square of the linear term will be the dominant quadratic addition to σ_e. This situation is nearly met for the noncentral force modification of the isotropic cross section, but not at all for the modifications of the $\sin^2\theta$ cross section. For this reason, the only quadratic terms which are carried are the y^2 and xy terms of a_e.

V. *P* WAVE INTERFERENCE—SINGULAR $(L\cdot S)$ COUPLING

For the deuteron the Case and Pais interaction[3] reduces to

$$U=-(2/\hbar)\Lambda(\mathbf{L}\cdot\mathbf{S})\times 6\text{ Mev}, \tag{23}$$

where

$$\Lambda=(1/\lambda r)[d/d(\lambda r)](e^{-\lambda r}/\lambda r), \quad \lambda=9\times 10^{12}\text{ cm}^{-1}, \tag{24}$$

and for ${}^3P_J$ states,

$$(\mathbf{L}\cdot\mathbf{S})=(\hbar/2)\{J(J+1)-4\}. \tag{25}$$

Here, just as for the tensor interaction, the photodisintegration is computed to Born approximation. This approximation does not now have an automatic domain of validity, because this problem does not involve any parameterization which centers conveniently on vanishing ^3P wave interaction. The justification for using Born approximation is that it is easy, and that it should give some indication of how a very short range $(\mathbf{L}\cdot\mathbf{S})$ interaction might influence the electric dipole photodisintegration.

There is no important correction to the cross section which is linear in the amplitude of U. An important contribution of the $(\mathbf{L}\cdot\mathbf{S})$ interaction is a very large quadratic correction in the isotropic cross section—a correction which rises very rapidly with energy until at 100 Mev it has already surpassed by a factor of ten the largest possible tensor forces contribution. Figure 7 shows a_e and a_e/b_0. While there may be large quadratic corrections to the $\sin^2\theta$ cross section, they are, unfortunately, beyond the scope of the present work.

It should be noted that both the magnitude of the results shown in Fig. 7 and their peculiar energy dependences are due more to the radial shape of the interaction than to its $(\mathbf{L}\cdot\mathbf{S})$ nature. Any strongly singular, noncentral ^3P state interaction must clearly give similar results. The tensor interactions introduced for PP scattering by Christian and Noyes[24] and by Jastrow[25] are other examples. An experiment which determines the angular distribution in photodisintegration by 100-Mev gamma-rays may thus decide whether there were any strongly singular noncentral ^3P state interaction in the NP system. The isotropy of the cross section would be unmistakable.

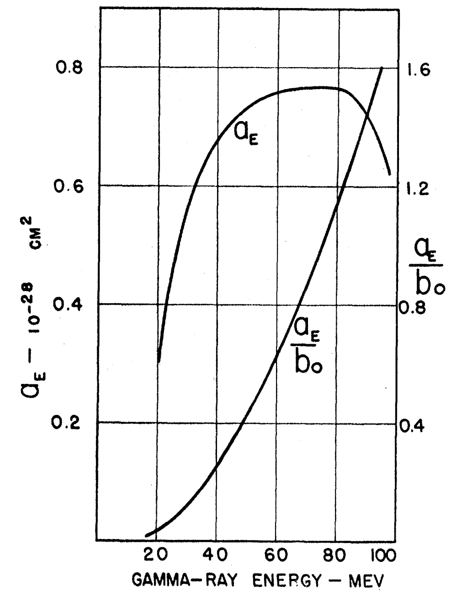

FIG. 7. Isotropic cross section, a_e, if the Case-Pais interaction is present; also its ratio to b_0 of Fig. 4. Divide a_e by 4π to get the cross section per steradian.

VI. CONCLUSIONS

The results of these calculations indicate that a measurement of a_e would be very difficult unless there exist singular noncentral forces in the P states. However, this exception would make the results of such experiments most interesting. In the absence of singular forces, a separation of a_e from the very uncertain a_m becomes nearly impossible unless the odd state strength of the tensor force exceeds 25 percent of its even state strength. The scattering data[2] indicate that the odd state strength is probably not so great.

Aside from the question of magnitude, it is interesting that for nonsingular tensor forces of the usual type there is a broad region of energy in which the magnetic term is not too uncertain and the value of a/b is slowly varying. In this region, a monochromatic source of gamma-rays would not be required. The energy region of interest begins near 20 Mev and may continue as high as 60 Mev, although that upper limit should not be crowded.

The agreement between theory and low energy photodisintegration experiments is well known to be excellent. Measurements of the angular distribution at higher energies have been attempted in several laboratories,[9–13,26] but few have been accurate enough to compare with theory. Goldhaber's result for (a/b) at 7 Mev seems much too large to understand. It is interesting that the experiments have demonstrated the dipole-quadrupole interference effect, and that satisfactory agreement with theory is found. In analyzing any experimental data it should be noted that the very existence of the noncentral forces effects discussed in the present paper vitiates Schiff's[4] remark about the possibility of using the experimental angular distribution for the separation of the electric dipole, electric quadrupole, and magnetic dipole cross sections.

The work was performed with the advice and assistance of Professor R. G. Sachs.

APPENDIX

Wave Functions

All the even state wave functions used in this calculation are eigenfunctions of two nuclear potentials: the triplet potential of Feshbach and Schwinger[1] and a singlet state Yukawa well.

For the triplet potential of Eq. (1) the following parameters are used: $\mu_1=8.45\times10^{12}$ cm^{-1}; $\mu_2=4.71\times10^{12}$ cm^{-1}; $\Gamma=0.224$; $\kappa=11.57\times10^{12}$ cm^{-1}; 2.5 percent D function in the deuteron. That case of Feshbach and Schwinger is chosen which is closest to the set of parameters designated by Pease and Feshbach[19] as giving best results for H^3 binding. The central force range is that derived for low energy PP scattering; the

[24] R. S. Christian and H. P. Noyes, Phys. Rev. **79**, 85 (1950).
[25] R. Jastrow, Phys. Rev. **81**, 165 (1951).
[26] Gibson, Grotdal, Orlin, and Trumpy, Phil. Mag. **42**, 555 (1951).

tensor range is somewhat greater. The phenomena which this potential fits well are: deuteron binding energy; triton binding energy; low energy NP and PP scattering; low energy deuteron photodisintegration; deuteron quadrupole moment.

The singlet potential is[27]

$$V_S = -(\hbar^2 K_S^2/M)(e^{-\mu r}/\mu r),$$
$$K_S = 10.75\times10^{12}\ \text{cm}^{-1},\quad \mu = 8.58\times10^{12}\ \text{cm}^{-1}.$$

As to the wave functions derived from these potentials: singlet state S and D functions are obtained by numerical integration of the Schroedinger equation, and are used in subsequent numerical integration to compute J_0 and J_2 for σ_m.

For the ^3S ground-state wave the best Hulthén function, an excellent approximation, is fitted to the Feshbach-Schwinger curve. It is

$$u = N(e^{-\gamma r} - e^{-\zeta r}),$$

where $\gamma = 2.316\times10^{12}$ cm^{-1}, $\zeta = 13.36\times10^{12}$ cm^{-1}, and $N = (7.76\times10^{12}\ \text{cm}^{-1})^{\frac{1}{2}}$.

For the ^3D ground state wave several different procedures are used. First, for magnetic calculations, the Feshbach-Schwinger curve is used directly in numerical integration. Two approximate analytic expressions are also used, which are fitted to the numerical curve. A low energy approximation is

$$w \approx M'(1-e^{-\alpha r})e^{-\gamma r}[1+(3/\gamma r)+(3/\gamma^2 r^2)],$$

which has the exact asymptotic form. $\alpha = 3.60\times10^{12}$ cm^{-1}.

$$M' = (0.0100\times10^{12}\ \text{cm}^{-1})^{\frac{1}{2}}.$$

A high energy approximation is

$$w \approx M\{(1-Ar)e^{-\xi r} - Be^{-\eta r}\},$$

where $\xi = 3.06\times10^{12}$ cm^{-1}, $A = 0.1385\times10^{12}$ cm^{-1}, $B = 1.140$, $\eta = 11.04\times10^{12}$ cm^{-1}, and $M = (0.455\times10^{12}\ \text{cm}^{-1})^{\frac{1}{2}}$.

Phase Shifts

The phase shifts δ_S and δ_D of Sec. II were determined first by approximate formulas, and the approximations then improved in the subsequent numerical integration of the wave equation. It may be of some interest to present the Born approximation formula for δ_D, if the potential is V_S, above. It is

$$\delta_D = \frac{K_S^2}{4k\mu}\left\{\left[1+\frac{3}{2}\left(\frac{\mu}{k}\right)^2+\frac{3}{8}\left(\frac{\mu}{k}\right)^4\right]\log\left(\frac{\mu^2+4k^2}{\mu^2}\right) - 3\left[1+\frac{1}{2}\left(\frac{\mu}{k}\right)^2\right]\right\}.$$

[27] J. M. Blatt and J. D. Jackson, Phys. Rev. **76**, 18 (1949).

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The Absorption of Gamma-Rays from Co^{60}

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Measurements of the absorption of gamma-rays from Co^{60} (1.17 and 1.33 Mev) have been made in 27 elements. In order to exclude the errors due to secondary radiations which might be produced in neighboring objects and in the absorbers, particular precautions were taken with respect to the geometrical arrangement of apparatus. The absorption coefficients measured for the elements whose atomic numbers are less than $_{52}$Te show reasonable agreement with those calculated according to existing theories. However, it is noted that the results with $_{73}$Ta, $_{74}$W, $_{78}$Pt, $_{79}$Au, $_{80}$Hg, and $_{81}$Tl are 3 percent to 5.5 percent less than the theoretical values. It seems improbable that the disagreements observed in these elements may be assigned to experimental causes. If the entire deviation were assigned to inaccuracy in theoretical knowledge, it would be reasonable to attribute it to some insufficiency in the Klein-Nishina theory of the Compton effect for this energy of gamma-rays. But further investigation should be undertaken to ascertain the fact.

I. INTRODUCTION

SINCE the application of Co^{60} has rapidly increased in various fields of science, it becomes important to know with greater accuracy the absorption coefficients of the gamma-rays from this radioisotope (1.17 and 1.33 Mev) in various elements. The absorption of gamma-rays in matter may be attributed to the combination of four separate effects, namely the photoelectric effect, the Compton effect, pair production, and the photonuclear reaction. Photonuclear reactions seem to be generally improbable in the energy range below several Mev except for a few nuclei. The absorption due to the photoelectric effect has been theoretically estimated by many workers,[1] and that due to the Compton effect has been formulated by Klein and Nishina,[2] while pair production has been theoretically discussed by Dirac and others.[3] A summary of most of these theories, which give the knowledge of absorp-

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