Decay of the π-Meson*†

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THE decay of the charged π-meson through virtual nucleon pairs and the annihilation of a pair with the creation of a neutrino and an electron or μ-meson is permitted by the accepted couplings between these particles.[1] Conservation of angular momentum and parity together with Furry's theorem[2] forbid the decay for most choices of the meson field and the β-decay coupling. The forbiddenness is independent of perturbation theory.[3] Assuming the π to be a pseudoscalar meson at rest in order that the final spinor pair have the same transformation properties under reflection and rotation as the π, the β-coupling must be $\alpha_1\alpha_2\alpha_3$ or $\alpha_1\alpha_2\alpha_3\alpha_4$. Therefore, only the axial vector or pseudoscalar β-decay theories can lead to the pseudoscalar $\pi\rightarrow e, \nu$ decay.[4]

If electrons and π-mesons are similarly coupled to nucleons, a pseudoscalar matrix element in β-decay leads to an electron decay five times as often as a μ-decay. For axial vector it has been shown[1] that the matrix element involves

$$1-[(p_\nu)^2/E_\nu E_{e,\mu}].$$

$\pi\rightarrow\mu, \nu$ is then 10^4 times as probable as $\pi\rightarrow e, \nu$. Any mixture of scalar, vector, axial vector, and tensor is not in contradiction with experiment.[5]

The decay of the pseudoscalar π into a photon and electron (or a μ-meson) and a neutrino is not limited by such rigid selection rules. For axial vector β-decay this mode is more probable than $\pi\rightarrow e, \nu$ because of the latter's singularly small matrix element. The emission of a photon by the π^+ or the electron in Figs. (1b) and (1d) does not alter any of the selection rules forbidding the nonradiative decay. Only the graph of Fig. (1c) can contribute. For vector β-decay and direct meson coupling the matrix element for $\pi\rightarrow e+\nu+$ a photon of momentum **k** and polarization **e** is

$$-i\Sigma\int\frac{d^4p}{(2\pi)^4}\frac{4\pi egg_\beta}{(4m_\pi E_\gamma)^{\frac{1}{2}}}\times\text{Spur}\left(\gamma_5\frac{1}{\mathbf{p}-M}\mathbf{e}\frac{1}{\mathbf{p}-\mathbf{k}-M}\gamma_\mu\frac{1}{\mathbf{p}-\mathbf{q}-M}\right)\cdot\langle\psi_e^+\gamma_\mu\psi_\nu\rangle \quad (1)$$

q is the meson 4-vector. The decay rate is

$$\tau^{-1}=(e^2/4\pi)(g^2/4\pi)(m_\pi/M)^2(g_\beta^2m_\pi^5)(2/15\pi^3)$$
$$=4\times10^2g^2\ \text{sec}^{-1} \quad (2)$$

for

$$g_\beta=10^{-49}\ \text{erg cm}^3.$$

The lifetime is between 6×10^{-4} and 6×10^{-5} sec, corresponding to a g^2 of 4 or 40. Gradient coupling gives a divergent matrix element for (1), spoiling the equivalence theorem. Tensor β-decay diverges for both couplings. Conservation of angular momentum and parity still forbid a decay through scalar β-decay.

For axial vector β-coupling the emission of a photon by the electron as in Fig. (1b) is expected to be more probable than emission by the heavy nucleon or π. (Emission from the π does not alter the very small matrix element associated with the β-decay.) If the μ and electron are assumed to have the same axial vector coupling with nucleons, the ratio $(\pi\rightarrow e+\nu+\gamma):(\pi\rightarrow\mu+\nu)$ is again independent of the perturbation treatment of the meson field and the divergent integrals arising in the separate calculation of each lifetime. The ratio of the probability for radiative $\pi-e$ decay to nonradiative $\pi-\mu$ decay is

$$(e^2/4\pi)(3.1)=5.7\times10^{-3}. \quad (3)$$

A symmetrical coupling scheme with axial vector coupling predicts one nonradiative electron decay and over 10 radiative electron decays for every 10^4 $\pi-\mu$ decays. The ambiguities arising from the divergences and the pseudoscalar direct coupling constant preclude conclusions for vector and tensor β-coupling.

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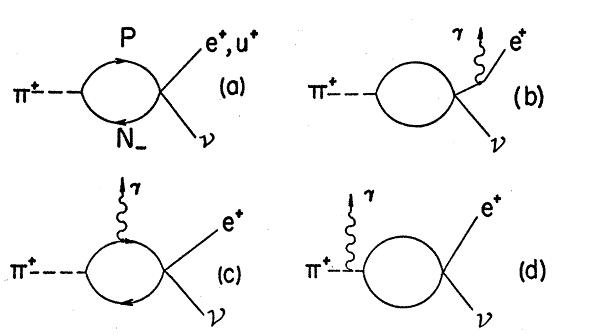

FIG. 1. Feynman's diagrams contributing to charged π decay.

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1 M. Ruderman and R. Finkelstein, Phys. Rev. **76**, 1458 (1949).
2 W. H. Furry, Phys. Rev. **51**, 125 (1937).
3 If Furry's theorem forbids the Feynman graph of Fig. (1a), it will also forbid all more complex graphs. Except for the original meson line, for every meson emitted from a closed loop, one is also absorbed so that the "evenness" or "oddness" of the matrix element is unchanged. For exact cancellation of graphs with an odd number of "even" operators it is necessary to assume that the square of the coupling constant between neutral mesons and protons is equal to that between neutrals and neutrons. All of the forbidden cases of reference (1) except scalar mesons with gradient coupling and vector β-decay vanish to all orders.
4 If one adopts the spinor reflection rule of Yang and Tiomno, Phys. Rev. **79**, 496 (1950), their interactions (12) and (13) are the appropriate ones. Referring the β-decay labeling to the proton-neutron coupling our results are independent of the μ, e, and ν inversion property.
5 H. L. Friedman and J. Rainwater, Phys. Rev. **82**, 334 (1951).

Some Comments on the Mechanism of Fission

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A NUMBER of spontaneous fission rates are now known,[1–4] and a study of the relation of these to Z and A should make it possible to come to a better understanding of this process, which in turn should lead to a better understanding of the slow neutron fission mechanism as well. A number of spontaneous fission rates are summarized in Table I.

Our attempts to correlate these rates with the existing theoretical expectations[5,6] have not been successful, and therefore it seems worthwhile to attempt to study the data from the point of view of finding their empirical relationship.

Figure 1 shows a plot of the logarithm of the "half-life" for spontaneous fission *versus* the fissionability parameter, Z^2/A, and leads to some very interesting conclusions. The points for the even-even nuclides, with some exceptions, seem to indicate that the rate for this nuclear type depends exponentially in a simple way on the parameter Z^2/A. It is tempting to assume that

TABLE I. Summary of spontaneous fission rates.

Nuclide	Fissions/gram/hour	Half-life (years)	References
Th^{230}	$\leqslant 1.4$	$\geqslant 1.5\times10^{17}$	1
Th^{232}	0.15	1.4×10^{18}	1
	1.2	1.7×10^{17}	2
Pa^{231}	$\leqslant 20$	$\geqslant 10^{16}$	1
U^{232}	$\leqslant 25{,}000$	$\geqslant 8\times10^{12}$	1
U^{233}	<0.7	$>3\times10^{17}$	1
U^{234}	<30	$>7\times10^{15}$	1
U^{235}	1.2	1.9×10^{17}	1
U^{238}	24.8 ± 0.9	8.0×10^{15}	3, 1
Np^{237}	$\leqslant 5$	$\geqslant 4\times10^{16}$	1
Np^{239}	$\leqslant 40{,}000$	$\geqslant 5\times10^{12}$	1
Pu^{238}	5.1×10^{6}	5.4×10^{10}	1
Pu^{239}	36	5.5×10^{15}	1
Am^{241}	$\leqslant 14{,}000$	$\geqslant 1.4\times10^{13}$	1
Cm^{242}	2.7×10^{10}	7.2×10^{6}	4