Decay of the π-Meson*†

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THE decay of the charged π-meson through virtual nucleon pairs and the annihilation of a pair with the creation of a neutrino and an electron or μ-meson is permitted by the accepted couplings between these particles.[1] Conservation of angular momentum and parity together with Furry's theorem[2] forbid the decay for most choices of the meson field and the β-decay coupling. The forbiddenness is independent of perturbation theory.[3] Assuming the π to be a pseudoscalar meson at rest in order that the final spinor pair have the same transformation properties under reflection and rotation as the π, the β-coupling must be $\alpha_1\alpha_2\alpha_3$ or $\alpha_1\alpha_2\alpha_3\alpha_4$. Therefore, only the axial vector or pseudoscalar β-decay theories can lead to the pseudoscalar $\pi\rightarrow e, \nu$ decay.[4]

If electrons and π-mesons are similarly coupled to nucleons, a pseudoscalar matrix element in β-decay leads to an electron decay five times as often as a μ-decay. For axial vector it has been shown[1] that the matrix element involves

$$1-[(p_\nu)^2/E_\nu E_{e,\mu}].$$

$\pi\rightarrow\mu, \nu$ is then 10^4 times as probable as $\pi\rightarrow e, \nu$. Any mixture of scalar, vector, axial vector, and tensor is not in contradiction with experiment.[5]

The decay of the pseudoscalar π into a photon and electron (or a μ-meson) and a neutrino is not limited by such rigid selection rules. For axial vector β-decay this mode is more probable than $\pi\rightarrow e, \nu$ because of the latter's singularly small matrix element. The emission of a photon by the π^+ or the electron in Figs. (1b) and (1d) does not alter any of the selection rules forbidding the nonradiative decay. Only the graph of Fig. (1c) can contribute. For vector β-decay and direct meson coupling the matrix element for $\pi\rightarrow e+\nu+$ a photon of momentum $\mathbf{k}$ and polarization $\mathbf{e}$ is

$$-i\sum\int\frac{d^4p}{(2\pi)^4}\frac{4\pi egg_\beta}{(4m_\pi E_\gamma)^{\frac{1}{2}}}\times\text{Spur}\left(\gamma_5\frac{1}{\mathbf{p}-M}\mathbf{e}\frac{1}{\mathbf{p}-\mathbf{k}-M}\gamma_\mu\frac{1}{\mathbf{p}-\mathbf{q}-M}\right)\cdot\langle\psi_e{}^+\gamma_\mu\psi_\nu\rangle \quad (1)$$

$\mathbf{q}$ is the meson 4-vector. The decay rate is

$$\tau^{-1}=(e^2/4\pi)(g^2/4\pi)(m_\pi/M)^2(g_\beta{}^2m_\pi{}^5)(2/15\pi^3) = 4\times10^2g^2\ \text{sec}^{-1} \quad (2)$$

for

$$g_\beta=10^{-49}\ \text{erg cm}^3.$$

The lifetime is between 6×10^{-4} and 6×10^{-5} sec, corresponding to a g^2 of 4 or 40. Gradient coupling gives a divergent matrix element for (1), spoiling the equivalence theorem. Tensor β-decay diverges for both couplings. Conservation of angular momentum and parity still forbid a decay through scalar β-decay.

For axial vector β-coupling the emission of a photon by the electron as in Fig. (1b) is expected to be more probable than emission by the heavy nucleon or π. (Emission from the π does not alter the very small matrix element associated with the β-decay.) If the μ and electron are assumed to have the same axial vector coupling with nucleons, the ratio $(\pi\rightarrow e+\nu+\gamma):(\pi\rightarrow\mu+\nu)$ is again independent of the perturbation treatment of the meson field and the divergent integrals arising in the separate calculation of each lifetime. The ratio of the probability for radiative $\pi-e$ decay to nonradiative $\pi-\mu$ decay is

$$(e^2/4\pi)(3.1)=5.7\times10^{-3}. \quad (3)$$

A symmetrical coupling scheme with axial vector coupling predicts one nonradiative electron decay and over 10 radiative electron decays for every 10^4 $\pi-\mu$ decays. The ambiguities arising from the divergences and the pseudoscalar direct coupling constant preclude conclusions for vector and tensor β-coupling.

I wish to thank Professor R. P. Feynman for helpful discussion.

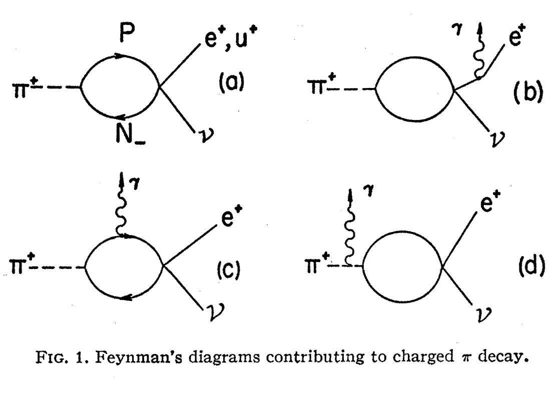

FIG. 1. Feynman's diagrams contributing to charged π decay.

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3 If Furry's theorem forbids the Feynman graph of Fig. (1a), it will also forbid all more complex graphs. Except for the original meson line, for every meson emitted from a closed loop, one is also absorbed so that the "evenness" or "oddness" of the matrix element is unchanged. For exact cancellation of graphs with an odd number of "even" operators it is necessary to assume that the square of the coupling constant between neutral mesons and protons is equal to that between neutrals and neutrons. All of the forbidden cases of reference (1) except scalar mesons with gradient coupling and vector β-decay vanish to all orders.
4 If one adopts the spinor reflection rule of Yang and Tiomno, Phys. Rev. **79**, 496 (1950), their interactions (12) and (13) are the appropriate ones. Referring the β-decay labeling to the proton-neutron coupling our results are independent of the μ, e, and ν inversion property.
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Some Comments on the Mechanism of Fission

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A NUMBER of spontaneous fission rates are now known,[1–4] and a study of the relation of these to Z and A should make it possible to come to a better understanding of this process, which in turn should lead to a better understanding of the slow neutron fission mechanism as well. A number of spontaneous fission rates are summarized in Table I.

Our attempts to correlate these rates with the existing theoretical expectations[5,6] have not been successful, and therefore it seems worthwhile to attempt to study the data from the point of view of finding their empirical relationship.

Figure 1 shows a plot of the logarithm of the "half-life" for spontaneous fission *versus* the fissionability parameter, Z^2/A, and leads to some very interesting conclusions. The points for the even-even nuclides, with some exceptions, seem to indicate that the rate for this nuclear type depends exponentially in a simple way on the parameter Z^2/A. It is tempting to assume that

TABLE I. Summary of spontaneous fission rates.

Nuclide	Fissions/gram/hour	Half-life (years)	References
Th^{230}	$\leqslant 1.4$	$\geqslant 1.5\times10^{17}$	1
Th^{232}	0.15	1.4×10^{18}	1
	1.2	1.7×10^{17}	2
Pa^{231}	$\leqslant 20$	$\geqslant 10^{16}$	1
U^{232}	$\leqslant 25{,}000$	$\geqslant 8\times10^{12}$	1
U^{233}	<0.7	$>3\times10^{17}$	1
U^{234}	<30	$>7\times10^{15}$	1
U^{235}	1.2	1.9×10^{17}	1
U^{238}	24.8 ± 0.9	8.0×10^{15}	3, 1
Np^{237}	$\leqslant 5$	$\geqslant 4\times10^{16}$	1
Np^{239}	$\leqslant 40{,}000$	$\geqslant 5\times10^{12}$	1
Pu^{238}	5.1×10^{6}	5.4×10^{10}	1
Pu^{239}	36	5.5×10^{15}	1
Am^{241}	$\leqslant 14{,}000$	$\geqslant 1.4\times10^{13}$	1
Cm^{242}	2.7×10^{10}	7.2×10^{6}	4

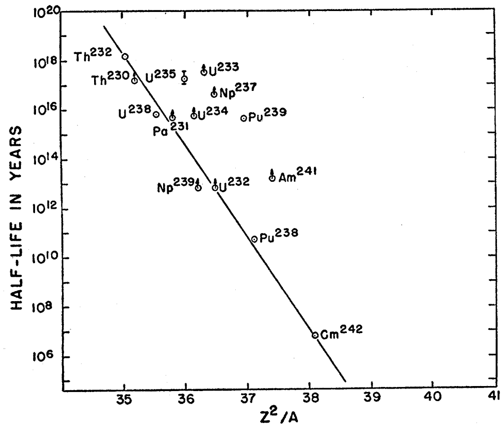

FIG. 1. Plot of spontaneous fission rates (ô signifies lower limit to half-life).

the rate of spontaneous fission is controlled by a Boltzmann, type factor in which the required activation energy for fission depends on Z^2/A; the form of the plot would suggest that this might be a linear dependence with a negative coefficient for Z^2/A. However, another type such as an inverse dependence on Z^2/A in the exponential term also fits nearly as well over the range of data plotted. In any case it is interesting to note that extrapolation of the line in Fig. 1 to the region of instantaneous rate of spontaneous fission (that is, half-life of order of 10^{-20} seconds) gives a value of about 47 for Z^2/A, which corresponds with the predicted limiting value[1] for Z^2/A.

The data seem also to indicate that on the average for a given value of Z^2/A, the rate is greater for an even-even nuclide than one with an odd number of nucleons. Since Z^2/A is a representation of Z^2/r^3, where r is the nuclear radius, the slower rates for the odd-nucleon nuclides may be related to their expected larger radii; on this basis the largest departure of an odd-nucleon nuclide from the line in Fig. 1 corresponds to the order of one percent larger radius than for the "hypothetical" corresponding even-even nuclide. On this picture an important contributing factor to the slower rates may result from the lower zero-point energy of the modes of vibration which lead to fission associated with the nuclei with the larger radii.

Similar considerations may be useful in interpreting some of the results from the study of slow neutron fission probabilities. The slow neutron fission probabilities of the even-even nuclides in the transuranium region seem to be lower than expected on the simple theory.[1] For example, a nucleus like Cm^{242} has a slow neutron fission cross section of less than 5 barns[7] in spite of the fact that the critical fission energy of the intermediate Cm^{243} is of the order of 4 Mev, much less than the estimated 6 Mev of neutron binding energy. It is possible that the time for fission is lengthened for such an odd-nucleon intermediate nucleus to the point where the (n, γ) reaction is able to compete more successfully than is the case for even-odd nuclides like U^{235}, Pu^{239}, etc., where the intermediate fissioning nuclei are of the even-even type.

The effect of an odd nucleon in slowing the fission process may also explain the photofission results of Koch, McElhinney, and Gasteiger[8] who found, for example, higher effective energetic thresholds for U^{235}, U^{233}, and Pu^{239} than for U^{238}, contrary to expectations from existing theory.[5,6]

It will be interesting to see whether even-even nuclei with abnormally small nuclear radii due to closed sub-shells will have especially high rates of spontaneous fission. Thus, a nucleus such as 100^{248} which would have two closed sub-shells on the Mayer picture,[9] 100 protons and 148 neutrons, might be expected to exhibit such an abnormally high rate. Similarly, the large slow neutron fission cross section[7] of a nuclide like Am^{242} might be connected with the sub-shell of 148 neutrons in the intermediate Am^{243}.

The above considerations may make it possible to predict with a fair degree of confidence, especially for the even-even nuclides, the spontaneous fission rates for undiscovered nuclides and hence make it possible to plan experiments more intelligently for their detection.

It is a pleasure to acknowledge helpful discussions with K. Street, Jr., A. Ghiorso, J. O. Rasmussen, and I. Perlman.

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The Melting Pressure of He^3

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THE previously reported measurements[1] on the melting pressure of He^3 between 1.51° and 1.02°K have been extended to 0.16°K. The lower temperatures were obtained by adiabatic demagnetization of ferric ammonium alum, which surrounded the capillary (0.16 mm i.d. cupro-nickel) containing the He^3. In order to enhance the thermal contact between the capillary and the salt, copper vanes were soldered to the capillary, and the salt chamber was filled with He^4 to a pressure of one atmosphere at liquid nitrogen temperature. The He^3 apparatus external to the cryostat was the same as in the earlier measurements.

The melting pressures were again obtained by the blocked capillary technique. Magnetic temperature measurements were made with a ballistic galvanometer and a secondary coil around the salt as described elsewhere.[2] Corrections were made for the difference between the magnetic and thermodynamic temperatures on the basis of the results of Kurti, Simon, and Squire.[3]

The results of all the measurements are shown in Fig. 1. The solid circles represent the previous results,[1] obtained with the capillary immersed directly in the He^4 bath, and the open circles represent the present results, obtained with the capillary im-

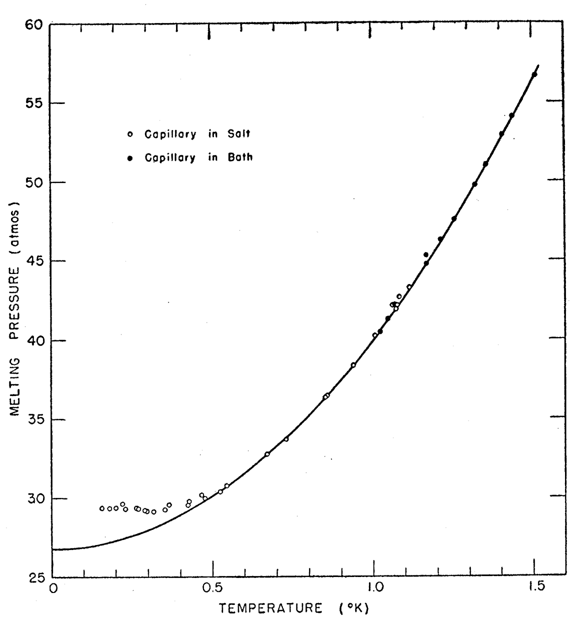

FIG. 1. Melting pressure of He^3.