

Bipolar Expansion of Coulombic Potentials. Addenda*

ROBERT J. BUEHLER AND JOSEPH O. HIRSCHFELDER
University of Wisconsin Naval Research Laboratory,
Madison, Wisconsin

(Received November 9, 1951)

IN our recent article¹ the general formula for $B_{n_1, n_2}^{|m|}(r_1, r_2; R)$ was not given in Region II where $|r_1 - r_2| \leq R \leq r_1 + r_2$. Instead, Table I gave the coefficients to be used for all those cases where n_1 and n_2 are equal to 3 or less. By using r_2' as the independent variable in the integrations indicated in Eq. (34) and expanding the integrands in powers of r_2' , it was possible to obtain the following general expression for $B_{n_1, n_2}^{|m|}(r_1, r_2; R)$ in Region II.

$$B_{n_1, n_2}^{|m|}(r_1, r_2; R) = H_{n_1, n_2}^{|m|} \sum_{j=0}^{2m} \sum_{w=0}^{n_2 - |m| - n_1 - |m|} \sum_{v=0}^{n_1 - |m|}$$

$$K(j)^{|m|} W_{n_2}^{|m|}(w) S_{n_1}^{|m|}(v) X_{n_1}(\beta).$$

Here $\beta = j + v + w$ and

$$H_{n_1, n_2}^{|m|} = \frac{\binom{2n_2+1}{2} (n_2 - |m|)! (n_1 - |m|)! (-1)^{n_2 - |m|}}{(n_2 + |m|)! (n_1 + |m|)! 2^{n_1 + n_2} R}.$$

$$K(j)^{|m|} = \frac{(-1)^j (|m|)!}{j!} \sum_{k=k_1}^{|m|} \frac{(-1)^k (2k)!}{k! (2k-j)! (m-k)!} \left(\frac{R^2 + r_2^2}{2Rr_2} \right)^{2k}.$$

$$k_1 = j/2 \text{ or } (j+1)/2.$$

$$W_{n_2}^{|m|}(w) = \frac{(-1)^w}{w!} \sum_{q=0}^{n_2} \frac{(-1)^q (2n_2 - 2q)! [(R^2 + r_2^2)/2Rr_2]^{n_2 - |m| - 2q}}{(n_2 - q)! (n_2 - |m| - 2q - w)! q!}.$$

$$q_2 = \frac{1}{2}(n_2 - |m| - w) \text{ or } \frac{1}{2}(n_2 - |m| - w - 1).$$

$$S_{n_1}^{|m|}(v) = \left(\frac{2R}{r_2} \right)^{n_1 - |m|} \left(\frac{R^2 + r_2^2}{4R^2} \right)^v \sum_{p=p_1}^{n_1 - |m| - v} \frac{(-1)^{n_1 - |m| - v - p} (2|m| + 2v + 2p)! [(R^2 - r_2^2)/4R^2]^p}{(n_1 - |m| - v - p)! (p + 2v - n_1 + |m|)! p! (m + v + p)!}.$$

$p_1 = n_1 - |m| - 2v$ when $v \leq \frac{1}{2}(n_1 - |m|)$ or $\frac{1}{2}(n_1 - |m| - 1)$; otherwise $p_1 = 0$.

$$X_{n_1}(\beta) = \left(\frac{r_2}{r_1} \right)^{n_1 - 1} \left(\frac{r_1^2}{R^2 + r_2^2} \right)^\beta \left\{ \frac{1}{2\beta + 2} \left[1 - \left(\frac{R - r_2}{r_1} \right)^{2\beta + 2} \right] + \frac{1}{2\beta + 1 - 2n_1} \left[-1 + \left(\frac{R + r_2}{r_1} \right)^{2\beta + 1 - 2n_1} \right] \right\}.$$

The running summation variables only assume integer values so that the choice of k_1 , q_2 , and p_1 is unique.

The authors would like to thank George Gioumoussis for checking these formulas.

* This work was carried out under a contract between the United States Navy Bureau of Ordnance and the University of Wisconsin.

¹ R. J. Buehler and J. O. Hirschfelder, Phys. Rev. **83**, 628 (1951).

Disintegration of the Ground-State Sr^{85} and Metastable Energy Level of 514 KeV in Rb^{85}

W. S. EMMERICH AND J. D. KURBATOV
Ohio State University, Columbus, Ohio
(Received November 13, 1951)

IT is accepted that the disintegration of the ground state of strontium 85 to rubidium 85 takes place by orbital electron capture of 65 days half-life followed by emission of a gamma-ray of 513 kev.¹ Electrons have also been found.² The radiations of strontium 85 were reinvestigated in an effort to compare excited states in Rb^{85} observed in disintegration of Sr^{85} and Kr^{85} .

Strontium 85 was produced by deuteron bombardment of spectroscopically pure rubidium and was separated from the latter without addition of carrier. Adsorption procedure was employed for the separation. An analysis of the radiations emitted by the

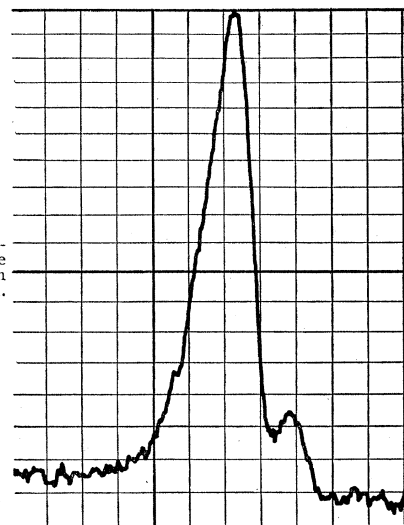


FIG. 1. Internal conversion electrons of the 514-kev gamma-ray in the Sr^{85} disintegration.

65-day period showed that electrons, x-rays, and gamma-rays are present. The electrons were measured by spectrometers and found to consist of internal conversion electrons corresponding to a 514+3-kev gamma-transition. An energy determination of this gamma-ray from photoelectrons with a lead radiator agreed with the aforementioned value. In the region from 120 to 200 kev, where an energy level of Rb^{85} was reported from the disintegration of Kr^{85} , neither internal conversion nor photoelectrons were observed. The limit of measurement extended to 0.5 percent of monochromatic electronic intensity of the 514-kev transition.

For the 514-kev transition, the internal conversion coefficient in the K-shell was found to be 0.7 percent from comparative measurements with the well-known gamma-transition in the decay of Cs^{137} .³ This internal conversion was determined using a spectrometer with sources prepared, from Sr^{85} and Cs^{137} , suitable for measurement of internal conversion electrons and photoelectrons at the same geometry. Comparison of the intensity ratios yielded the foregoing value of the experimental conversion coefficient after a correction had been applied for difference in radiator efficiency because of difference in gamma-ray energies. An alternate method consisted of measuring the ratio of electrons to gamma-rays in a calibrated Geiger counter. In this measurement, a correction was made for Compton electrons and L-shell conversion electrons as found with a spectrometer. The ratio of K- to L-shell conversion was obtained, as shown in Fig. 1, from photographic plates in a permanent magnet spectrograph. From these data, the K to L ratio is estimated to be 12 ± 3 . The internal conversion tables of Rose *et al.*⁴ give $\beta_2 = 6.6 \times 10^{-3}$ and $\alpha_3 = 8.2 \times 10^{-3}$ for 514 kev in rubidium as the two closest values to the experimental results of 7×10^{-3} . Using a conventional coincidence counter with a resolving time of \sim one microsecond, the only coincidences observed could be traced to electrons in coincidence with \sim one percent of the total number of x-rays, because of internal conversion. No coincidences were observed between x-rays and gamma-rays, indicating that the 514-kev level has a measurable lifetime.

In order to determine whether orbital electron capture takes place directly to the ground state of Rb^{85} , a relative intensity measurement of x-rays and gamma-rays was made. For this purpose, the counter efficiency for x-rays of rubidium was determined from the coincidences between electrons and x-rays as mentioned in the previous paragraph. For the particular counter used, the ratio of x-ray to gamma-ray efficiency was thus found to be 3.6 ± 0.4 , whereas the actual counting rate showed a ratio of x-rays to gamma-rays of 3.8, from which it can be concluded that disintegration directly to the ground state and to excited

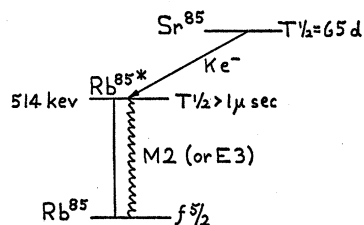


FIG. 2. Decay scheme for the ground state of Sr^{85} .

states other than the 514-kev level in Rb^{85} does not occur with appreciable intensity. A partial decay scheme of Sr^{85} summarizing these results is shown in Fig. 2.

The authors wish to thank Major J. E. Duval for the chemical separation.

Support by grants from the Ohio State University Research Foundation through the Graduate School and from the O. S. U. Development Fund are gratefully acknowledged by the authors.

¹ M. Ter-Pogossian and F. T. Porter, *Phys. Rev.* **81**, 1057 (1951).

² L. S. Cheng and J. D. Kurbatov, *Phys. Rev.* **79**, 237 (1950).

³ M. A. Waggoner, *Phys. Rev.* **82**, 906 (1951).

⁴ Rose, Goertzel, Spinrad, Harr, and Strong, *Phys. Rev.* **83**, 79 (1951).

Studies on the Linearity of a Scintillation Gamma-Ray Spectrometer

V. O. ERIKSEN AND G. JENSSEN

Joint Establishment for Nuclear Energy Research, Kjeller, Norway
(Received November 13, 1951)

THE linearity between pulse-height and absorbed energy in NaI(Tl) crystal has been carefully investigated during the last two years. Johanson¹ has reported a linear relationship in the region 0.4–6 Mev. Pringle and Standil² have found that the relationship is not strictly linear in the region 0.05–0.4 Mev. We have made some measurements in the region 0.05–0.5 Mev, and are finding a linear relationship in this region.

A single-crystal scintillation spectrometer has been used. One clear NaI(Tl) crystal, supplied from Harshaw Chemical Company, 20 mm thick and 30 mm in diameter was employed in conjunction with an EMI-5311 photomultiplier tube. Mirrors of thin mica coated with silver or aluminium to ensure high reflection were used. Just a slight oxidation of the mirrors caused considerable variation in the peak widths.

The crystal was kept in a glass tube cemented on to the surroundings of the photocathode and filled with paraffin. The crystal had been carefully polished with flannel or velvet in a dry atmosphere. The paraffin was dried with metallic Na.

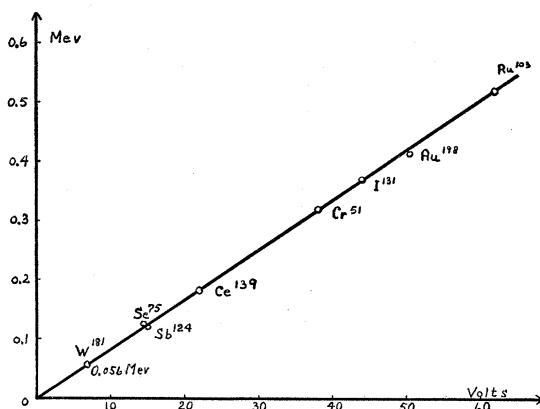


FIG. 1. Pulse-height as a function of gamma-ray energy.

The amplifier was type 1008 delivered by Cole, Ltd. No pre-amplifier or cathode-follower was used. A ten-channel pulse-height analyzer was used for the pulse-height analysis. The stability of the apparatus was very good; for the high voltage it was better than ± 0.5 per thousand.

The counter was surrounded by a 5-cm-thick shield of lead.

Nuclides with 7 different gamma-rays in the region 0.1–0.5 Mev were used.³ The activity of the sources varied between 10^{-8} – 10^{-9} curie. The collimated rays had an angular width of about 5° .

Figure 1 shows the linearity between pulse-height and energy of the various gamma-rays.

¹ S. A. E. Johanson, *Arkiv Fysik*, B2 (1950).

² Pringle and Standil, *Phys. Rev.* **80**, 762 (1950).

³ Gamma-ray energies are according to *Nuclear Data* (National Bureau of Standards Circular 499, 1950).

Heat Capacity and Lattice Defects of Silver Chloride

KOICHI KOBAYASHI

Chemical Institute, Faculty of Science, Tohoku University,
Sendai, Japan

(Received October 25, 1951)

THE magnitude of the heat of activation for the formation of lattice defects in AgCl crystal, ΔH , has hitherto been in doubt. Previously, I had measured the heat capacity of AgCl from room temperature to 500°C to calculate the thermodynamic relations on some chemical reaction¹ and found an abnormal increase in the heat capacity curve in the vicinity of the melting point.

Berry² has concluded that the defects in AgBr crystal are of Frenkel type. I assume that Frenkel defects may dominate in AgCl by analogy with the result on the lattice behavior of AgBr and by consideration of the ionic conductivity measurements on AgCl , in spite of Mitchell's conclusion.³

If the concentration of the Frenkel defect, n/N , is expressed by the equation,

$$n/N = C \exp(-\Delta H/2RT), \quad (1)$$

the excess heat capacity ΔC_p per mole should be given by

$$\Delta C_p = [C(\Delta H)^2/2RT^2] \exp(-\Delta H/2RT). \quad (2)$$

So, ΔH may be obtained by the relation between $\log(T^2\Delta C_p)$ and $1/T$. For the heat capacity contributed by the perfect lattice of AgCl , I assume

$$C_p = 11.21 + 6.57 \times 10^{-3}T - 3.2 \times 10^{-6}T^2 \text{ cal/mole } (T:^\circ\text{K}), \quad (3)$$

which was obtained by extrapolation of the heat capacity data in the range 300 – 400°K , as given in the previous paper.¹ ΔC_p was estimated as the deviation from the expression (3) of the measured heat capacity which was shown in Table II of the previous paper.¹ Figure 1 shows a plot of $\log(T^2\Delta C_p)$ versus $1/T$ in the range 600 – 724°K , indicating that the resultant points fall well on a straight line in the range from 600 to 700°K and deviate upwards above this. The slope of this straight line determines ΔH to be $35,200$ cal/mole. This value is 20 percent larger than that for AgBr derived by Teltow.⁴ The contribution of C_p of the perfect lattice to ΔH is quite sensitive, and a variation of 0.1 cal/mole (6 percent in ΔC_p) at 700°K corresponds to 1 kcal in ΔH , but by this treatment the plot at 600°K deviates from the linear relation.

From this ΔH we may estimate n/N at any temperature. We find n/N to be 7.5×10^{-3} at 724°K , 4.3×10^{-4} at 600°K , and 1.6×10^{-10} (4×10^{12} per cc) at 300°K . The integrated value of the excess enthalpy at 724°K is 220 cal/mole and the corresponding entropy is 0.31 eu. The value of C in Eq. (1) is obtained as 1.08×10^3 ; on the other hand, C for AgBr is estimated as 1.6×10^4 from the work of Christy and Lawson.⁵ The fact that C is exceedingly large seems to indicate that the defects in silver halide are of the Schottky type, but I shall not discuss this problem further.

The deviation of the plots above 700°K from the linear relation seems to be associated with an increase in ΔH , or a small con-