both γ's replaced by a conversion electron, so that, from the correlation for any one of the cases $\gamma-\gamma$, $\gamma-c$ (or $c-\gamma$), $c-c$, the correlation for the other two cases may be obtained immediately. In general, the factor f_k (Rose's b_ν) by which a_k must be multiplied to allow for the replacement of a γ-quantum by a conversion electron depends on the multipole order l and character (electric or magnetic), on the atomic number Z, and the energy $K(=h\nu/mc^2)$ released by the nuclear transition.[5] However, in the approximation used by Lloyd[1] (ejection of electron from an s-shell by electric multipole transition, treated in the nonrelativistic limit), the Z and K dependence vanishes and the expression for f_k (Lloyd's z_L) becomes,

$$f_k(l)=2l(l+1)/[2l(l+1)-k(k+1)]. \quad (2)$$

Now Lloyd's approximation is just that used in reference 2, and the point it is desired to make here is the following: By a double application of the approximate f_k (or rather $1/f_k$) to the approximate $c-c$ correlation of reference 2, one arrives at the exact $\gamma-\gamma$ correlation. This is because, as explained by Rose *et al.*,[5] the $\gamma-\gamma$ correlation is independent of Z, K and the multipole character. Thus from (2) above and (18) of reference 2 we have, in the notation of that paper, the following completely general and exact correlation formula for the $\gamma-\gamma$ cascade $J_A(l_1)J_B(l_2)J_C$,

$$I(\theta)\sim\sum_k\left(1-\frac{k(k+1)}{2l_1(l_1+1)}\right)\left(1-\frac{k(k+1)}{2l_2(l_2+1)}\right)\times C_{l_1 0 l_1 0}{}^{k0}C_{l_2 0 l_2 0}{}^{k0}W(J_BJ_Akl_1;l_1J_B)\times W(J_BJ_Ckl_2;l_2J_B)P_k(\cos\theta). \quad (3)$$

(This of course checks with alternative formulas for the same correlation, given elsewhere.[6,7]) Moreover, on dividing Table I of reference 2 by $f_k(l_1)f_k(l_2)$, we have immediately a generic table for all $\gamma-\gamma$ correlations in the physically important case where l_1, l_2 have the lowest values allowed by the angular momentum and parity selection rules. Lloyd's "basic" correlation and numerical tables are derivable, as particular cases, from such a generic table.

A comparison was made in reference 2 between the departure from spherical symmetry in $I(\theta)$ for certain $c-c$ cascades and the corresponding $\gamma-\gamma$ cascades given by Hamilton.[3] The anisotropy, A, was defined as $(I_{\text{greatest}}-I_{\text{least}})/I_{\text{greatest}}$. Such a definition is preferable to one in terms only of $I(\pi)$ and $I(\pi/2)$, which would take no account of the possibility that $I(\theta)$ may have maxima or minima for $\pi>\theta>\pi/2$; thus $\cos^2\theta-\cos^4\theta$ has a maximum at $\theta=3\pi/4$ but is not distinguishable from a spherically symmetric distribution by its values at $\theta=\pi$ and $\pi/2$. It may be objected that our definition takes no account of the width of a peak and so does not distinguish, for example, between a cosine curve and a delta-function, giving $A=1$ for both. However, incorporation of peak-width into the definition of anisotropy would give a formula whose application would be limited not only by its algebraic complexity, but also by the fact that experimentally these angular distributions cannot often be observed with sufficient detail to give a reliable estimate of peak-width. The comparison of A_{c-c} with $A_{\gamma-\gamma}$ was limited to the dipole-dipole tables (Table II of reference 2 and Table I of reference 3) plus a few examples from the quadrupole-dipole tables. For every case tested the result was found $A_{c-c}\geqslant A_{\gamma-\gamma}$, and in particular no example was found of $A_{\gamma-\gamma}=1$, although there were several of $A_{c-c}=1$. This naturally led one to speculate whether it could be a universal, or at least general rule that the correlation for a cascade involving conversion electrons is stronger than that for the corresponding cascade with conversion electrons replaced by γ-quanta. In other words could it be generally true that, for a given cascade $J_A(l_1)J_B(l_2)J_C$, we have $A_{c-c}\geqslant A_{c-\gamma}\geqslant A_{\gamma-\gamma}$?

The question of anisotropy has now been thoroughly investigated for the general angular distributions, $1+a_2P_2(\cos\theta)$ and $1+a_2P_2(\cos\theta)+a_4P_4(\cos\theta)$, by examining algebraically the effect on A of multiplying a_k by some arbitrary factor F_k. The purely algebraic investigation showed that, invariably, the "A" of $1+a_2P_2(\cos\theta)$ is reduced if $|a_2|$ is reduced, and increased if $|a_2|$ is increased. No such general rule was discovered, however, for the $1+a_2P_2(\cos\theta)+a_4P_4(\cos\theta)$ distribution; for example, the reduction of both $|a_2|$ and $|a_4|$ does not necessarily mean a reduction in A, it depends on the relative signs and magnitudes of the a_k and F_k. In physical examples F_k is of course related to f_k; thus it is $(1/f_k)^2$ for converting $I(\theta)_{c-c}$ to the corresponding $I(\theta)_{\gamma-\gamma}$. When f_k is given by Eq. (2) above, a reduction in $|a_k|$ always occurs for the changes $c-c\to c-\gamma\to\gamma-\gamma$; in the exact treatment of Rose *et al.* a reduction usually occurs, the exception being when the substitution $c\to\gamma$ is made in a dipole transition (electric or magnetic) for which $h\nu>mc^2$. The exact treatment also shows that all the F_k approach asymptotically to unity in the high energy limit, so that in this limit there is no distinction between the $c-c$, $c-\gamma$, and $\gamma-\gamma$ angular distributions for the same cascade.

To conclude, then, we may say that $A_{c-c}\geqslant A_{c-\gamma}\geqslant A_{\gamma-\gamma}$ is by no means the universal rule one might have expected from the more limited investigation of reference 2. It does, however, apply to all cascades in which one transition is magnetic or electric dipole with $h\nu<mc^2$, and the other transition is also dipole with $h\nu<mc^2$, or is any higher multipole of any energy. For $1+a_2P_2(\cos\theta)+a_4P_4(\cos\theta)$ distributions no general rule can be formulated, although it appears that on the average A is about as often increased as decreased by replacing a conversion electron by a γ-quantum. In view of this it was not thought worthwhile to extend the investigation to more complicated angular distributions, particularly as the algebra here becomes very heavy on account of the numerous alternative analytic expressions for A, depending on the number of maxima and minima in $I(\theta)$, and whether $I(\pi)$ is greater or less than $I(\pi/2)$.

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Search for Long-Lived Nb^{94}

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A 6.6-min activity produced by (n,γ)[1] and (d,p)[2] reactions on niobium has been reported by a number of investigators. This short-lived activity assigned to Nb^{94} emits x-rays[3] which have been characterized as the K-radiation of niobium.[4] Further K, L, and M conversion electron groups, characteristic of niobium, have been observed from a highly converted 41.5-kev x-ray.[5] It is reported[6] that 99.9 percent of the 6.6-min activity decays by the isomeric transition process and the remainder by β^- emission (1.3 Mev) to Mo^{94}. Radiations from a long-lived isomer of Nb^{94} have not been detected and a half-life value of greater than 100 years is indicated on the basis of previous neutron irradiations of niobium.[4]

Since information on the long-lived Nb^{94} is meager, an investigation of this activity seemed desirable. Accordingly, a 500-mg sample of niobium metal was bombarded for 4 months in the Argonne reactor. A spectrographic analysis made by the supplier indicated that less than 0.001 percent each of zirconium, titanium, and iron were present as impurities in the sample. Our own spectrographic studies indicate that tantalum is probably present to a greater extent than 0.001 percent. Hafnium, however, was not detectable spectrographically.

TABLE I. Purification of irradiated niobium.

Purification step	1	2	3	4	5	6
Sp. act. (counts/min/mg)	950	25	11	4	0.7	1

Although the 6.6-min activity was too short to be observed in these laboratories, one or more long-lived activities were present in the irradiated sample. No positrons could be detected in the emitted radiations using standard magnetic deflection techniques. Based on aluminum absorption curves a large part of the activity present could be due to Ta^{182}. This qualitative observation was checked by observing the electron conversion spectrum with a fixed magnetic field camera. Photographs were taken at three different field values, and 30 conversion groups were observed ranging in energy from 30 kev to 415 kev. All but three of these lines matched those reported for Ta^{182}.[7] The remaining three lines, 120±1, 276±2, and 415±4 kev, were initially interpreted as K-conversion lines for a niobium triad ($\gamma=140$, 296, 436 kev). That interpretations based only upon evidence of this type can be erroneous, is seen from the fact that these lines are actually conversion groups arising from the Hf^{181} decay.[8] Corroborative evidence for the statement was provided through the separation of Hf^{181}, macro amounts of niobium, and Ta^{182} in that order from the irradiated niobium on anion exchange resin columns.[9] The activity which preceded the niobium off the column exhibited a delay state of approximately 20 μsec, which again points toward Hf^{181}. On the basis of these experimental results, all of the conversion electron groups observed in the irradiated niobium can be accounted for by Ta^{182} and Hf^{181} activities.

Since a low intensity niobium activity could be masked by the tantalum and hafnium activities, a large sample (220 mg) of the irradiated niobium was decontaminated from these extraneous activities. Six decontamination steps with anion exchange resins reduced the specific activity to a constant value for the niobium. In each step the first fractions off the column (Hf^{181}) and the last fraction (Ta^{182}) were discarded. The specific activity of the niobium after each column separation step is given in Table I. From aluminum absorption curve evidence, the activity still remaining in the niobium is probably Hf^{181}.

On the basis of this investigation a minimum half-life of 5×10^4 years is proposed for Nb^{94}.

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The Equation of Motion of the Landé Electron*

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THE electron, in Landé's modification of classical electrodynamics,[1] is a point mass with an associated charge density function ρ, where

$$\rho=(3ea^2/4\pi)(r^2+a^2)^{-5/2},$$

a is a fundamental length, r is the distance from the point mass, and e the total electronic charge.

The four-force on the electron is given, following Lorentz, by

$$\int \kappa dv,$$

where κ is the four-vector of force density due to external and self fields acting upon the charge density, and dv is an element of three-volume. The region of integration is a sphere of radius R where R increases without limit.

The change of mechanical energy-momentum of the electron between two time intervals is equated to

$$\int \kappa d\tau,$$

where $d\tau$ is an element of four-volume.

Following Dirac,[2] this volume integral is transformed into a surface integral. Then, using the methods of Dirac,[2] the inversion of infinite series, and Landé's signal law,

$$r^2-c^2t^2+a^2=0,$$

the equation of motion of the electron is obtained as an infinite series:

$$m_0\ddot{z}_\mu=\lim_{R\to\infty}\frac{Q(\alpha+\beta)}{4\pi}\int\int f_{\mu\nu}\dot{z}^\nu(1+R\dot{v}\cos\vartheta)^{\frac{1}{2}}\sin\vartheta d\vartheta d\varphi$$
$$+\frac{eQ\ddot{z}_\mu(\alpha+\beta)^2}{16R\dot{v}(a^2+R^2)^{\frac{1}{2}}}\left[3\ln\left(\frac{1-R\dot{v}}{1+R\dot{v}}\right)+(1+R\dot{v})^{-1}-(1-R\dot{v})^{-1}\right]$$
$$+\tfrac{1}{3}(\beta-\alpha)(\alpha+\beta)eQ(R\dot{v})^{-1}[\dddot{z}^2\dot{z}_\mu+d\ddot{z}_\mu/ds]$$
$$\times[(1-R\dot{v})^{-\frac{1}{2}}-(1+R\dot{v})^{-\frac{1}{2}}]$$

plus terms of higher order in $R\dot{v}$. Here, m_0 is the mechanical rest mass of the electron, z is a positional coordinate in four-space and dots indicate differentiation with respect to proper time, $s(c=1)$, α and β are arbitrary pure numerics such that the effective field of the electron is given as α (advanced field) plus β (retarded field), $f_{\mu\nu}$ is a component of the external field six-vector, $\dot{v}$ is the magnitude of the three-acceleration in a coordinate system in which the electron is instantaneously at rest, Q is the total charge enclosed by the sphere of radius R about the point mass,

$$Q=\int 4\pi\rho r^2dr=eR^3(a^2+R^2)^{-\frac{3}{2}},$$

where $Q\approx e$ for $R\gg a$. This series converges, if and only if, $R\dot{v}<1$.

If R is arbitrarily restricted to a finite value, in other words, if the effective radial extension of the charge of the electron is defined as R, then it can be shown that the above series will reduce (under the proper limiting conditions) to the equations of (a) Lorentz-Dirac,[1] (b) Eliezer,[3] and (c) Groenewald.[4]

The conditions to be introduced into the series are: R is finite and much larger than a, $\dot{v}R\ll1$ (quasistationary motion), and $f_{\mu\nu}$ to be of negligible variation over the distance $2R$ (the wavelength of the external field is to be much larger than $2R$, the effective diameter of the electron).

Under these conditions the series assumes the form,

$$m_0\ddot{z}_\mu=(\alpha+\beta)Qf_{\mu\nu}\dot{z}^\nu-\tfrac{1}{2}(\alpha+\beta)^2Qe(R^2+a^2)^{-\frac{1}{2}}\ddot{z}_\mu$$
$$+\tfrac{2}{3}(\beta-\alpha)(\alpha+\beta)eQ(\dddot{z}^2\dot{z}_\mu+d\ddot{z}_\mu/ds),$$

plus negligible terms of higher order in $R\dot{v}$. Then:

(a) When we define $m=m_0+\frac{1}{2}Qe(R^2+a^2)^{-\frac{1}{2}}$ and allow only retarded fields (i.e., $\alpha=0$, $\beta=1$), the Lorentz-Dirac equation is obtained

$$m\ddot{z}_\mu=Qf_{\mu\nu}\dot{z}^\nu+\tfrac{2}{3}eQ(\dddot{z}^2\dot{z}_\mu+d\ddot{z}_\mu/ds).$$

(b) By using m as an (a) and taking $\alpha=-\kappa$ and $\beta=\kappa+1$, Eliezer's equation results

$$m\ddot{z}_\mu=\tfrac{2}{3}eQ(2\kappa+1)(\dddot{z}^2\dot{z}_\mu+d\ddot{z}_\mu/ds)+Qf_{\mu\nu}\dot{z}^\nu.$$

(c) Groenewald's equation was obtained by assuming, first of all, that the charge of the electron is concentrated on its world line and secondly that the self fields are the half-sums of advanced and retarded fields.

To obtain similar results from the above series, Q is replaced by e independently of R, α and β are each equated to $\frac{1}{2}$, and then R is allowed to approach zero as a limit. The resulting equation is

$$m_0\ddot{z}_\mu=ef_{\mu\nu}\dot{z}^\nu-(e^2/2a)\ddot{z}_\mu$$

plus higher order terms.