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Eigenvalue Problem in Quantum Electrodynamics*

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The eigenvalue problem in quantum electrodynamics is discussed from the point of view of the Fredholm theory of integral equations. Starting with positron theory—the theory of a quantized Dirac field interacting with an external field only—the external potentials are replaced by bare photon fields. To insure causality the photon operators are ordered in time. Certain integral equations for the Fredholm minors constructed on the Feynman kernel are taken to be the equations for the wave functions of n particle systems. An expansion in interaction patterns rather than the coupling constant is indicated. The one particle problem is treated in the first pattern approximation. Procedures proposed by Snyder and Snow and the mass renormalization scheme are discussed in this connection. Finally a purely formal derivation of the Bethe-Salpeter equation for the two-body problem in the lowest pattern approximation is given.

I. INTRODUCTION

SEVERAL new attempts[1,2] have been made recently to deduce from a field theory the equations for a system of two Dirac particles in interaction. The problem has been approached from the point of view of the S matrix[1] and by a formal extension of the theory of Green's functions.[2] The difficulties of a rigorous deduction of these equations have been emphasized by Nambu.[3] Work in this direction has therefore been either purely formal or largely heuristic.

Using arguments of the same character, it is intended to provide in this note additional motivation for the acceptance of certain equations as approximate descriptions of bound systems. The mathematical imagery employed is that of the Fredholm theory of integral equations. Intuition is relied on to extract from it a workable set of equations for one, two, and many body problems.

In brief outline the procedure is the following. We first investigate an electron-positron field subject to an external electromagnetic field only. The integral equation considered is that of a scattering problem with an inhomogeneous term corresponding to the wave function of a free particle in the absence of the external field. The associated homogeneous integral equation may have nontrivial solutions, if certain relations between the energy, the mass and the charge are satisfied. These represent the bound states of the system. The restrictions on the values of the parameters are expressed by the requirement that the Fredholm determinant constructed on the kernel vanish. The wave function of the system is then given by the first Fredholm minor. If for a particular relation between the energy, the mass and the charge the determinant as well as the first minor are identically zero the solution of the equation is given by the second minor, if this quantity does not vanish. The second minor is antisymmetric in its two indices and satisfies the homogeneous equation independently in both. It is therefore the wave function for a two-particle noninteracting system. The external field is now replaced by a quantized electromagnetic field satisfying Maxwell's equations without sources. If a radiation field of such character is to give rise to a causal interaction between charged particles the photon operators must be chronologically ordered[4] and an expectation value relative to a state without photons taken. An expansion in various patterns of interaction, rather then in the coupling constant is then carried out and the effective potentials that arise out of various modes of interaction are isolated. The Bethe-Salpeter equation is obtained in the lowest pattern approximation.

II. ALGEBRAIC PRELIMINARIES

The Fredholm theory in its conventional form is somewhat clumsy for our purpose. Our first task will

* Research carried out under the auspices of the AEC.
[1] H. A. Bethe and E. E. Salpeter, Phys. Rev. **82**, 309 (1951); M. M. Gellman and F. Low, Phys. Rev. **84**, 350 (1951).
[2] J. Schwinger, Proc. Nat. Acad. Sci. **37**, 452 (1951).
[3] Y. Nambu, Prog. Theor. Phys. **5**, 614 (1950).
[4] M. Fiertz, Helv. Phys. Acta **23**, 731 (1950).

therefore be to find operational expressions for the Fredholm minors. The integral equations that they satisfy will then be immediately evident from these representations.

We consider a set of operators $A_1, A_2, \cdots A_n$. Let a typical matrix element of anyone of these be denoted by $\langle x|A|y\rangle$. The symbol A_1A_2 stands for an operator whose matrix element $\langle x|A_1A_2|y\rangle$ is $\langle x|A_1|\tau\rangle\langle\tau|A_2|y\rangle$, where the repeated variable τ is being summed or integrated over. We similarly define a symbol (a cross product) $A_1\times A_2$ with a matrix element

$$\langle x_1x_2|A_1\times A_2|y_1y_2\rangle=\epsilon^{ij}\langle x_1|A_1|y_i\rangle\langle x_2|A_2|y_j\rangle. \quad (1)$$

The letter ϵ^{ij} is here the familiar alternating symbol. Associated with $A_1\times A_2$ is an operator $A_1\times A_2{}^+$ whose matrix element is given by the expression,

$$\langle x_1x_2|A_1\times A_2{}^+|y_1y_2\rangle=\epsilon^{ij}\langle x_i|A_1|y_1\rangle\langle x_j|A_2|y_2\rangle. \quad (2)$$

In general

$$\langle x_1\cdots x_n|A_1\times\cdots\times A_n|y_1\cdots y_n\rangle = \epsilon^{i(1)\cdots i(n)}\langle x_1|A_1|y_{i(1)}\rangle\cdots\langle x_n|A_n|y_{i(n)}\rangle \quad (1')$$

and

$$\langle x_1\cdots x_n|A_1\times\cdots\times A_n{}^+|y_1\cdots y_n| = \epsilon^{i(1)\cdots i(n)}\langle x_{i(1)}|A_1|y_1\rangle\cdots\langle x_{i(n)}|A_n|y_n\rangle. \quad (2')$$

If $A_1=\cdots=A_n=A$ we shall write

$$A_1\times\cdots\times A_n=[A\times]^n.$$

It is readily seen that

$$[A\times]^{n+}=[A\times]^n.$$

We now define scalar operator multiplication into a cross product of two operators. Thus

$$\langle x_1x_2|A\cdot A_1\times A_2|y_1y_2\rangle=\langle x_1|A|\tau\rangle\langle\tau x_2|A_1\times A_2|y_1y_2\rangle, \quad (3)$$

$$\langle x_1x_2|A.A_1\times A_2|y_1y_2\rangle=\langle x_2|A|\tau\rangle\langle x_1\tau|A_1\times A_2|y_1y_2\rangle. \quad (4)$$

Making use of (1) we then obtain

$$A\cdot A_1\times A_2=AA_1\times A_2, \quad (3')$$

$$A.A_1\times A_2=A_1\times AA_2. \quad (4')$$

Scalar postmultiplication is defined for $A_1\times A_2{}^+$:

$$A_1\times A_2{}^{+}\cdot A=A_1A\times A_2{}^+, \quad (5)$$

$$A_1\times A_2{}^+.A=A_1\times A_2A^+. \quad (6)$$

The extension of these definitions to a cross product of n operators (1′), (2′) is obvious. The Fredholm theory can now be placed in this algebraic framework.

III. OPERATIONAL FORM OF THE FREDHOLM THEORY

The integral equation,

$$\psi=\phi+\lambda K\psi, \quad (7)$$

is solved by

$$\phi=\psi-\lambda[\Delta^{(0)}(\lambda)]^{-1}\Delta^{(1)}(\lambda)\phi, \quad (8)$$

where $\Delta^{(0)}(\lambda)$ is the Fredholm determinant and $\Delta^{(1)}(\lambda)$ the first minor. The operational expressions for these quantities are

$$\Delta^{(0)}(\lambda)=\exp[Tr\log(1-\lambda K)], \quad (9)$$

$$\Delta^{(1)}(\lambda)=[1-\lambda K]^{-1}K\exp[Tr\log(1-\lambda K)]. \quad (10)$$

From these representations one readily infers that

$$Tr\,\Delta^{(1)}(\lambda)=-\Delta^{(0)\prime}(\lambda), \quad (11)$$

$$\Delta^{(1)}(\lambda)=K\Delta^{(0)}(\lambda)+\lambda K\Delta^{(1)}(\lambda) =K\Delta^{(0)}(\lambda)+\lambda\Delta^{(1)}(\lambda)K, \quad (12)$$

where $\Delta^{(0)\prime}(\lambda)$ is the derivative of the determinant with respect to λ. It is seen from (12) that if $\Delta^{(0)}(\lambda)=0$, then $\Delta^{(1)}(\lambda)$ is a solution of the homogeneous equation associated with (7).

The second Fredholm minor may be defined operationally as

$$\Delta^{(2)}(\lambda)=[[1-\lambda K]^{-1}KX]^2\exp[Tr\log(1-\lambda K)]. \quad (13)$$

The well-known properties of this operator

$$Tr\,\Delta^{(2)}(\lambda)=\Delta^{(0)\prime\prime}(\lambda),$$

$$\Delta^{(2)}(\lambda)=K\times\Delta^{(1)}(\lambda)+\lambda K\cdot\Delta^{(2)}(\lambda), \quad (14a)$$

$$\Delta^{(2)}(\lambda)=\Delta^{(1)}(\lambda)\times K+\lambda K.\Delta^{(2)}(\lambda), \quad (14b)$$

$$\Delta^{(2)}(\lambda)=K\times\Delta^{(1)}(\lambda)^+ +\lambda\Delta^{(2)}(\lambda)\cdot K, \quad (15a)$$

$$\Delta^{(2)}(\lambda)=\Delta^{(1)}(\lambda)\times K^2+\lambda\Delta^{(2)}(\lambda).K \quad (15b)$$

are readily deduced with the aid of the formulas given in the previous section.

IV. THE EXTERNAL ELECTROMAGNETIC FIELD

The equation we shall consider is

$$\psi=\psi_0+eK\psi \quad (16)$$

with

$$\langle\alpha x|K|\beta x'\rangle=iS_{\alpha\sigma}{}^F(xx')\gamma_{\sigma\beta}{}^\mu A_\mu(x'). \quad (17)$$

If A_μ is independent of time one readily sees that the boundary conditions incorporated into (16) with $\psi_0=0$ are the correct ones for a bound state. The Fredholm minors constructed on this kernel involve two parameters e and κ. We shall denote them bu $\Delta^{(n)}(e,\kappa)$. These are related to quantities appearing in the S matrix theory by

$$\langle x_1\alpha_1\cdots x_n\alpha_n|\Delta^{(n)}|y_1\beta_1\cdots y_n\beta_n\rangle$$

$$=\Big[\phi_0, T\psi_{\alpha(1)}(x_1)(\bar\psi(y_1)\gamma^\mu)_{\beta(1)}A^\mu(y_1)\cdots$$

$$\times\psi_{\alpha(n)}(x_n)(\bar\psi(y_n)\gamma^\nu)_{\beta(n)}A^\nu(y_n)$$

$$\times\exp\Big(-i\int_{-\infty}^{+\infty}H(x')dx'\Big)\phi_0\Big], \quad (18)$$

where ϕ_0 is the vacuum state; $H(x)$, the interaction hamiltonian and T the temporal ordering operator. This

relation was shown[5] to hold for $\Delta^{(0)}$:

$$\Delta^{(0)}(e,\kappa)=(\phi_0, S\phi_0). \tag{19}$$

With the operational representations developed in the previous sections similar relations may be shown to obtain for the higher Fredholm minors. That a quantity constructed on a Feynman kernel is related to one appearing in the solution of an integral equation with retarded boundary conditions is not too surprising. Somewhat similar relations were noticed by Jost and Pais in their treatment of the nonrelativistic problem.[6] Equation (16) with $\psi_0=0$ is equivalent to the conventional formulation of the bound state problem. If A_μ is independent of x_0, Eq. (16) has translational symmetry in time. We may therefore look for solutions of the type $\psi(\mathbf{r})\exp[-ik_0x_0]$. For sufficiently localized potentials it is then seen that $\psi(\mathbf{r})$ has a clear-cut asymptotic behavior. If $k_0^2>\kappa^2$, $\psi(\mathbf{r})\sim\exp[i(k_0^2-\kappa^2)^{\frac{1}{2}}r]$ while if $k_0^2<\kappa^2$, $\psi(\mathbf{r})\sim\exp[-(\kappa^2-k_0^2)^{\frac{1}{2}}r]$. In the first case the potential acts as a source of outgoing waves in the second we have a particle bound by the external field. The existence of these solutions will depend on whether certain relations between k_0^2, κ^2 and e can be satisfied.

In the next sections we shall work with equations that have translational invariance not only in time but also in space. The relations that will have to be satisfied will then involve the effective mass $-k_\mu k_\mu$ the mechanical mass κ^2 and the charge e.

Equation (16) is a well-defined integral equation even if the external potentials do depend on the time. The restrictions on e and κ that are necessary for the existence of a solution of the homogeneous integral equation are expressed by

$$\Delta^{(0)}(e,\kappa)=0. \tag{20}$$

With the parameters so restricted the solution is then given according to (12) by $\langle x|\Delta^{(1)}(e,\kappa)|y\rangle$, since then

$$\Delta^{(1)}(e,\kappa)=eK\Delta^{(1)}(e,\kappa). \tag{21}$$

If this quantity should vanish identically for the particular values of e amd κ we may obtain a solution (according to (14) satisfying the two integral equations,

$$\Delta^{(2)}(e,\kappa)=eK\cdot\Delta^{(2)}(e,\kappa), \tag{22a}$$

$$\Delta^{(2)}(e,\kappa)=eK.\Delta^{(2)}(e,\kappa). \tag{22b}$$

We observe that

$$\langle x_1x_2|\Delta^{(2)}(e,\kappa)|y_1y_2\rangle=-\langle x_2x_1|\Delta^{(2)}(e,\kappa)|y_1y_2\rangle.$$

This quantity may therefore be taken as the wave function for two noninteracting Dirac particles subject to an external electromagnetic field.

V. THE QUANTIZED RADIATION FIELD

Equations (21) and (22) are now taken to hold even if the external field is replaced by photon operators. To introduce the action of a particle on itself or the interaction between particles we order the photon operators in time and take the expectation value of both sides relative to a state in which no photons are present. The photon operators are taken to satisfy Maxwell's equations without sources. Such operators commute among themselves inside a temporally ordered bracket.

We shall treat the simpler equation (21) in some detail. Writing $\psi(x)$ for $\langle x|\Delta^{(1)}(e,\kappa)|y\rangle$ with y fixed we have

$$\psi=e^2K^2\psi. \tag{23}$$

The iteration procedure is now carried out in the following fashion

$$\begin{aligned}\langle\psi\rangle&=e^2\langle K^2\psi\rangle\\&=e^2\langle K^2\rangle\langle\psi\rangle+e^4[\langle K^4\psi\rangle-\langle K^2\rangle\langle K^2\psi\rangle]\\&=e^2\langle K^2\rangle\langle\psi\rangle+e^4[\langle K^4\rangle-\langle K^2\rangle\langle K^2\rangle]\langle\psi\rangle\\&\quad+e^6[\langle K^6\psi\rangle-\langle K^2\rangle\langle K^4\psi\rangle-\langle K^4\rangle\langle K^2\psi\rangle\\&\qquad+\langle K^2\rangle\langle K^2\rangle\langle K^2\psi\rangle]=\cdots.\end{aligned} \tag{24}$$

The bracket $\langle\ \rangle$ denotes here vacuum expectation values as well as ordering of photon operators with respect to time. Retaining the first two terms of this expansion we have

$$\begin{aligned}\langle\psi(x)\rangle=&-ie^2\int S^F(x,\tau)\gamma^\mu S^F(\tau y)\gamma^\mu\\&\times D^F(\tau y)\langle\psi(y)\rangle d\tau dy\\&-e^4\int S^F(x\tau_1)\gamma^\mu S^F(\tau_1\tau_2)\gamma^\nu S^F(\tau_2\tau_3)\gamma^\mu\\&\times S^F(\tau_3 y)\gamma^\nu D^F(\tau_1\tau_3)D^F(\tau_2 y)\langle\psi(y)\rangle d\tau dy.\end{aligned} \tag{25}$$

The form of the interaction becomes clearer when we go over to the differential equation,

$$\begin{aligned}\left(\gamma\frac{\partial}{\partial x}+\kappa\right)\langle\psi(x)\rangle=&-e^2\int V^{(2)}(xy)\langle\psi(y)\rangle dy\\&-e^4\int V^{(4)}(xy)\langle\psi(y)\rangle dy,\end{aligned}$$

where

$$\begin{aligned}V^{(2)}(xy)&=i\gamma^\mu S^F(xy)\gamma^\mu D^F(xy)\\V^{(4)}(xy)&=\int\gamma^\mu S^F(x\tau_1)\gamma^\nu S^F(\tau_1\tau_2)\gamma^\mu S^F(\tau_2 y)\gamma^\nu\\&\qquad\times D^F(x\tau_2)D^F(\tau_1 y)d\tau.\end{aligned} \tag{26}$$

We have thus isolated the "self-energy potential" acting on the electron due to its interaction with the bare radiation field. Evidently $V^{(2)}$ and $V^{(4)}$ are due to interaction patterns given in Figs. 1(a) and 1(b), respectively. It will be noticed that interactions of the type illustrated in Figs. 1(c) and 1(d) will not appear

[5] M. Neuman, Phys. Rev. **83**, 1258 (1951).
[6] R. Jost and A. Pais, Phys. Rev. **82**, 840 (1951).

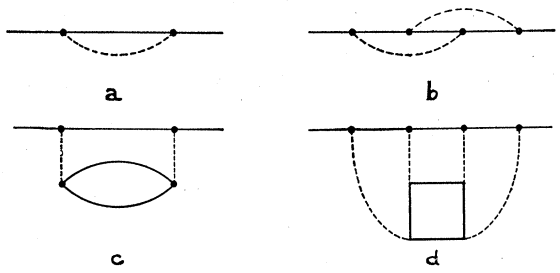

FIG. 1. Interaction patterns for the one-particle problem.

in this formulation. The reason of their absence is the fact that we replaced the external field by a bare photon field satisfying Maxwell's equations without sources. A radiation field of this nature is linear. The virtual photons do not polarize the vacuum and do not scatter each other.

We shall now discuss the self-energy of the electron in the first pattern approximation. Equation (25) is readily seen to have translational symmetry under displacements in space as well as in time. In accordance with the remarks of the previous section we therefore look for relations between the invariant quantity $p_\mu p_\mu$ and the parameters e and κ. More precisely, we try to determine the effective mass of the electron when the particle is subject to a potential arising from the emission and subsequent reabsorption of a single photon. The fourier transform of (25) (retaining only the first term on the right side of this equation) is given by

$$\psi(p)=-4i\alpha(2\pi)^{-3}[p^2+\kappa^2-i\epsilon]^{-1}\phi(p)\psi(p) \quad (27)$$

where

$$\phi(p)=\int_0^1 dx_1\int_0^1 dx_2\delta(x_1+x_2-1)(i\gamma p-\kappa)(i\gamma p x_2+2\kappa)$$
$$\times\int d^4k[k^2+p^2x_1x_2+\kappa^2x_1-i\epsilon]^{-2}. \quad (28)$$

Writing

$$[k^2+p^2x_1x_2+\kappa^2x_1-i\epsilon]^{-2}$$
$$=\int_{\kappa^2}^\infty dM 2x_1[k^2+p^2x_1x_2+Mx_1-i\epsilon]^{-3} \quad (29)$$

we immediately obtain

$$[p^2+\kappa^2-i\epsilon]^{-1}\kappa 2\gamma p\psi(p)=[\pi^{-1}\alpha\chi^{(1)}]^{-1}$$
$$\times[1+\pi^{-1}\alpha[\chi^{(0)}-(p^2+\kappa^2)^{-1}p^2\chi^{(1)}]]\psi(p) \quad (30)$$

with

$$\chi^{(0)}=\int_{\kappa^2}^\infty dM(p^2)^{-1}\log[(M-i\epsilon)^{-1}(p^2+M-i\epsilon)]$$
$$\underset{\lambda\to\infty}{\approx} \log\lambda+\text{finite terms}, \quad (31a)$$

$$\chi^{(1)}=\chi^{(0)}-(1/2)\int_{\kappa^2}^\infty dM(p^2)^{-1}[1-(p^2)^{-1}M$$
$$\times\log[(M-2\epsilon)^{-1}(p^2+M-i\epsilon)]]$$
$$\underset{\lambda\to\infty}{\approx}(3/4)\log\lambda+\text{finite terms}. \quad (31b)$$

In the conventional treatment of the self-energy one replaces $i\gamma p$ in (30) by $-\kappa$ and obtains a linear equation in p^2 with the root

$$\kappa^{-2}p^2=-[1+\pi^{-1}\alpha(\chi^{(0)}-\chi^{(1)})]^{-1}[1+\pi^{-1}\alpha(\chi^{(0)}+\chi^{(1)})]$$
$$\underset{\lambda\to\infty}{\approx}-[1+(4\pi)^{-1}\alpha\log\lambda]^{-1}$$
$$\times[1+(4\pi)^{-1}7\alpha\log\lambda]$$
$$\underset{\lambda\to\infty}{\approx}-[1+(2\pi)^{-1}3\alpha\log\lambda]. \quad (32)$$

The right member of (32) agrees with the expression conventionally given for the self-energy of the electron in the α-approximation. In the pattern approximation, however, neither the replacement nor the expansion are legitimate. It is therefore necessary to iterate (24) which leads to an equation for p^2 of the form,

$$-[p^2+\kappa^2]^{-2}p^2\kappa^2=[\pi^{-1}\alpha\chi^{(1)}]^{-2}$$
$$\times[1+\pi^{-1}\alpha[\chi^{(0)}-(p^2+\kappa^2)^{-1}p^2\chi^{(1)}]]^2. \quad (33)$$

According to the evaluations (31a) and (31b) of $\chi^{(0)}$ and $\chi^{(1)}$ as $\lambda\to\infty$ the left side of 33 approaches a finite value. The roots of (33) become

$$\kappa^{-2}p^2=-(17/2)\pm 4[(17/8)^2-1]^{\frac{1}{2}}. \quad (34)$$

It is clear that the number of roots will increase with the number of photons involved in the interaction. In this treatment of the one-body problem some of the degrees of freedom of the radiation field are apparently transferred to the particle.

No physical significance is of course to be attached to the numbers appearing in Eq. (34). They merely reflect the difficulties that are encountered in an attempt to treat a one-body problem along the same lines as a two-particle system. An electron with a definite mechanical mass could apparently under the influence of its interaction with the vacuum fluctuations of the electromagnetic field assume any one of a large number of experimentally observable masses. This difficulty is by no means peculiar to the quantum theory of fields. Its classical counterpart is the appearance of time derivatives of high order in the equation of motion of an electron interacting with its own radiation field. It has been suggested by Snyder and Snow[7] that ambiguous quantities like $\chi^{(0)}$ and $\chi^{(1)}$ should be subjected to suitable limiting processes and evaluated to yield zero. An examination of (33) shows indeed that the difficulty of multiple roots could be removed in this manner. However, the order of the divergence of an

[7] H. S. Snyder, Phys. Rev. **78**, 95 (1950); Phys. Rev. **79**, 520 (1950); G. Snow and H. S. Snyder, Phys. Rev. **80**, 987 (1950).

expression, even though it can be changed by certain rearrangements, and limiting processes, is a well-defined mathematical concept and the reasons for the appearance of divergent quantities in the particular physical framework on which our theories are based is rather clear. It therefore seems to the author that such forceful methods, even though they formally might solve the problem in this particular case, could hardly be considered as removing the physical difficulty.

The renormalization scheme is an attempt to extract from expressions relating to real processes the terms reflecting the change produced in the inertial properties of the electron due to its interaction with the electromagnetic field. In contrast to the suggestions advanced in reference 7, the order of divergence and the form of various ambiguous quantities is respected and their cancellation after renormalization is noted in each power of the coupling constant. From the point of view outlined in this note that the physical arguments advanced in favor of renormalization may be objected to on the grounds that they disregard the question of transfer of degrees of freedom from the radiation to the matter field and that this aspect should also be taken into account, if the conventional treatment of divergent expressions is being adhered to. One would therefore be inclined to look upon renormalization as a tentative formal prescription, as has been emphasized by many workers in this field, applicable to a class of problems which can be treated in terms of an expansion in powers of the coupling constant, rather than the taking into account of a real physical change that would be produced in the effective mass of the electron if the bare particle were subject to the action of the vacuum fluctuations of the electromagnetic field. Whatever its logical merits, once this rather simple prescription is accepted and a certain computational technique consistently adhered to, finite and unambiguous answers emerge in every power of the coupling constant. It should be quite clear that the class of rearrangements to which the divergent expression are subjected in this scheme are from a purely mathematical point of view no more legitimate than other classes that could easily accomplish the opposite purpose: turn the theory into a completely meaningless array of symbols. Whether one could admit all possible rearrangements of ambiguous expressions and still extract from the theory anything meaningful using sounder physical criteria, as has been attempted in the work cited in reference 7 is an open question. These attempts, however, do emphasize the fact that divergent expressions are indeed ambiguous quantities, and therefore a theory involving them can only be regarded as a tentative prescription, that procedures alternate to the ones that accomplish the isolation of singularities are also possible and are mathematically no less legitimate, and that attempts to resolve the ambiguities on a sounder theoretical basis encounter great difficulties.

We shall now give a purely formal derivation of the Bethe-Salpeter[1] equation in the approximation in which a single photon gives rise to the potentials. The interaction pattern is easily visualized and the symbolic expression corresponding to it is obtained by combining Eqs. (22a) and (22b) and is of the form

$$\Delta^{(2)}=e^2\langle K\cdot K.\rangle\Delta^{(2)}. \tag{35}$$

Denoting the matrix elements of Δ^2 with y_1, and y_2 fixed by

$$\langle x_1x_2|\,\Delta^{(2)}\,|y_1y_2\rangle=\psi_{\alpha(1)\alpha(2)}(x_1x_2)$$

Eq. (35) assumes the explicit form

$$\psi_{\alpha(1)\alpha(2)}(x_1x_2)$$
$$=-ie^2\int[S^F(x_1\tau_1)\gamma^\mu]_{\alpha(1)\beta(1)}[S^F(x_2\tau_2)\gamma^\mu]_{\alpha(2)\beta(2)}$$
$$\times D^F(\tau_1\tau_2)\psi_{\beta(1)\beta(2)}(\tau_1\tau_2)d\tau. \tag{36}$$

The corresponding differential equation is

$$\left(\gamma^{(1)}\frac{\partial}{\partial x_1}+\kappa\right)_1\left(\gamma^{(2)}\frac{\partial}{\partial x_2}+\kappa\right)_2\psi(x_1x_2)$$
$$=-ie^2\gamma^{(1)}\gamma^{(2)}D^F(x_1x_2)\psi(x_1x_2). \tag{37}$$

In (37) $\gamma^{(1)}$ acts and the first, $\gamma^{(2)}$ on the second set of spinor indices of $\psi(x_1x_2)$. These operators therefore commute. Other iterations of Eqs. (22) giving rise to potentials due to a single photon in the field

$$\Delta^{(2)}=e^2\langle K\cdot K\cdot\rangle\Delta^{(2)}, \tag{38a}$$
$$\Delta^{(2)}=e^2\langle K.K.\rangle\Delta^{(2)} \tag{38b}$$

are, of course, also possible. The meaning, consistency, and extensions of these equations to include more complex interaction patterns are currently being investigated.

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