The Distribution of Multiplicities of Neutrons Produced by Cosmic-Ray μ-Mesons Captured in Lead*

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The nature of the μ-meson capture process in lead has been investigated by studying the number of neutrons emitted by the excited nucleus. Working under 2000 g cm^{-2} of clay and limestone and a 144 g cm^{-2} lead filter, events were studied in which a single charged particle penetrated a triple coincidence telescope and stopped in an 86 g cm^{-2} lead absorber, with one or more delayed coincident neutron counts from an array of ten $B^{10}F_3$ counters in a paraffin moderator placed below the absorber. Events in which more than one G-M tube in any coincidence tray was discharged were rejected. A large paraffin barrier was interposed between the absorber and filter to reduce the sensitivity to neutrons originating above the absorber. The neutron detecting geometry is thought to have had an efficiency sensibly independent of neutron energy up to about 10 Mev. On the basis of 327 delayed neutron coincidences, the mean multiplicity of disintegration neutrons per stopped negative μ-meson was found to be 2.16±0.15, with ±10 percent additional error due to the uncertainty in the strength of the standard neutron source used to determine neutron detecting efficiency. The mean squared multiplicity was found to be 5.2±1.9 on the basis of 3 double-neutron coincidences. On the evaporation model the expected mean multiplicity for 100-Mev excitation energy is about 6, while a calculation based on the $\mu^- + P \rightarrow N + \nu$ hypothesis, using the distribution of excitation energy calculated on the free-particle model, and Weisskopf's statistical theory of the nucleon evaporation process, leads to an expected mean multiplicity of 0.95.

I. INTRODUCTION

It has been shown by various investigators, using cloud chamber,[1–3] photographic emulsion,[4–8] and crystal counter[9] techniques, that the capture of a negative μ-meson by a nucleus does not lead, in the great majority of cases, to the emission of any charged particles. The nuclear excitation is accordingly presumed to be considerably less than the 107-Mev rest energy of the captured meson. However, it has been observed that the nucleus receives sufficient excitation to emit one or more neutrons,[10,11] and the study described represents an effort to investigate the distribution of nuclear excitations by determining the mean number $\langle m\rangle_{Av}$ of neutrons emitted in the meson capture process in lead. In addition, an attempt has been made to determine the mean squared multiplicity $\langle m^2\rangle_{Av}$ which gives information about the shape of the distribution function for excitations. The experimental results, based on data obtained underground with a counter telescope, are compared with the theoretical predictions of the values of $\langle m\rangle_{Av}$ and $\langle m^2\rangle_{Av}$ for two different assumed distributions of excitation energy. The first of these is the case of 100-Mev excitation for each interaction, and the second is the distribution given by a theory developed by Tiomno and Wheeler and by Rosenbluth[12] to account for the observed rarity of charged particles from μ-meson capture. In this theory most of the meson's rest energy is carried away by a neutrino or neutretto, leaving the nucleus with a mean excitation energy of about 10 Mev.

II. APPARATUS

The arrangement of components is indicated in Figs. 1 and 2. There are three trays of Geiger counters designated A, B', and B[13] which are connected as a cosmic-ray coincidence telescope to detect penetrating charged particles. Above the telescope is a lead filter of thickness 144 g cm^{-2}. The entire apparatus was situated in one of the tunnels of the Lemp Brewery cave in St. Louis under about 2000 g cm^{-2} of clay and limestone, in the hope that other charged particles present in the cosmic radiation would thereby be reduced in intensity to the point where events could be ascribed to the μ-meson component alone. It is estimated

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[8] E. P. George and J. Evans, Proc. Phys. Soc. (London) **A64**, 193 (1951). These investigators report underground photoplate evidence that in about 10 percent of the capture processes a one-prong star is produced.
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[12] J. Tiomno and J. A. Wheeler, Revs. Modern Phys. **21**, 153 (1949); M. N. Rosenbluth, Phys. Rev. **75**, 532 (1949). Priority for the $\mu^- + P \rightarrow N + \nu$ hypothesis seems to belong to Sakata and Inoue, Prog. Theor. Phys. **1**, 143 (1946), and B. Pontecorvo, Phys. Rev. **72**, 246 (1947).
[13] Each tray contains five tubes, of 4.97 cm inside diameter and 25.4 cm active length.

that at the depth chosen neutron production by these other charged particles could not introduce an error of more than 5 percent in $\langle m\rangle_{Av}$ and 20 percent in $\langle m^2\rangle_{Av}$.[14] Below the counter telescope is placed a lead absorber of thickness 86 g cm^{-2} (3 inches), and below it a tray of anticoincidence counters C[15] to permit detection of events in which charged particles that have passed through the telescope stop in the lead absorber. Immediately below the anticoincidence tray C is placed a 1500-pound paraffin moderator in which is imbedded an array of ten $B^{10}F_3$ slow neutron counters.[16] Its purpose is to detect disintegration neutrons originating in the absorber that have been slowed down in the paraffin. Finally a paraffin barrier (shown removed in Fig. 1) is interposed between the lead filter and the lead absorber in order to discriminate against neutrons coming from above the absorber, as, for example, from

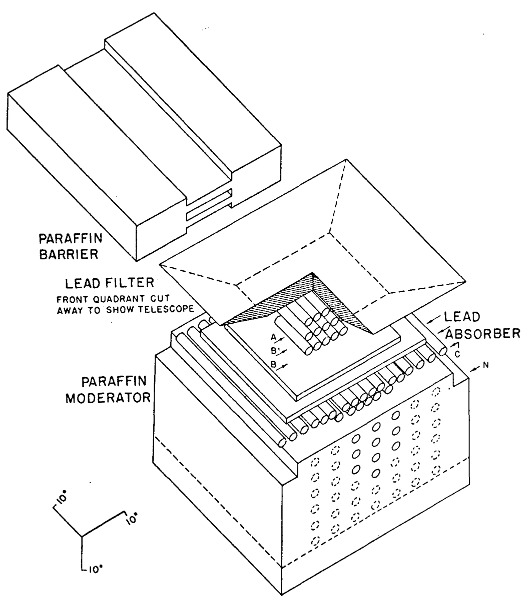

FIG. 1. The experimental arrangement (view from above). For clarity the paraffin barrier is shown removed from the telescope.

nuclear interactions occurring in the filter that produce a number of charged particles and neutrons which discharge counters in the telescope trays and give a (delayed) coincident pulse in the neutron detector (see Fig. 3). Besides decreasing the efficiency with which neutrons from above the absorber are detected, the barrier also increases, by its reflecting action, the efficiency for detecting neutrons from the absorber, and

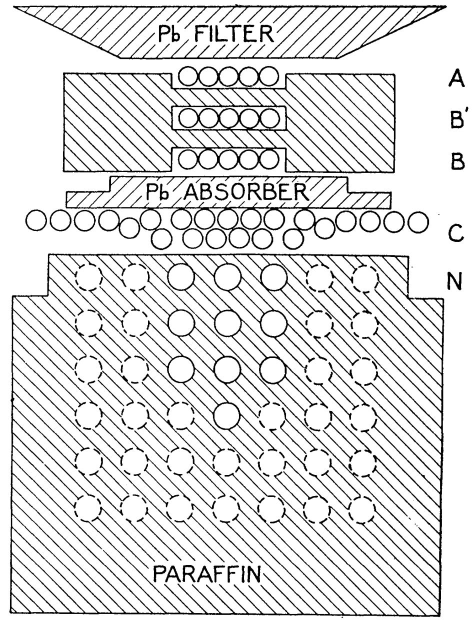

FIG. 2. The experimental arrangement (vertical section).

reduces the contribution to the telescope anticoincidence rate from knock-on electrons and side showers. The lead filter, paraffin barrier, and lead absorber were all mounted on wheeled dollys arranged to run on tracks, so that they could be readily placed in the telescope array or rolled back onto a siding completely out of the telescope cone. This arrangement facilitated control experiments to ascertain the effects of the various layers. For clarity the paraffin barrier in Fig. 1 is shown on the siding rather than in the telescope.

Figure 4 is a simplified block diagram of the circuits used, with amplifier stages, scalers and other details omitted. The signals from trays A, B', and B are

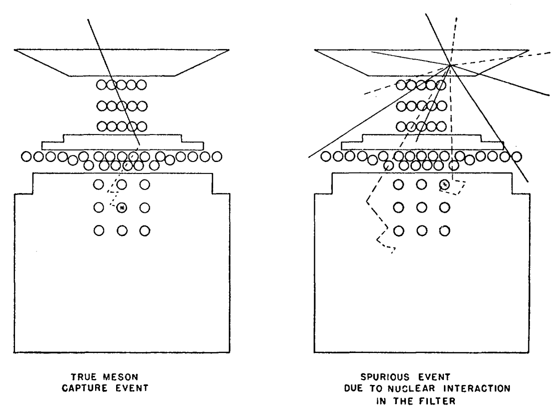

FIG. 3. True meson capture event and spurious event due to interaction in filter. The drawing on the left shows the stopping of a μ-meson with emission of a neutron that enters the paraffin thermalizer. The drawing on the right shows a nuclear interaction in the filter that could be recorded as an **A:N** event. Insertion of the paraffin barrier between the filter and absorber and restriction to "single" telescope coincidences greatly reduce the chance of recording such an event.

[14] The recent work of C. Cocconi and V. Cocconi-Tongiorgi [Phys. Rev. **82**, 335 (1951)] on the multiple production of neutrons in Pb at various depths under water shows that this estimate is most conservative. They report that 1000 g cm^{-2} is sufficiently deep to bring the neutron shower rate into equilibrium with the penetrating component.

[15] This tray contains 23 tubes, each of 4.97 cm inside diameter and 84.6 cm active length.

[16] The enriched BF_3 was furnished by the Isotopes Division, Atomic Energy Commission, Oak Ridge, Tenn. The counters were made by the N. Wood Counter Laboratory, Chicago, Illinois.

connected to a threefold coincidence circuit with a resolving time of ± 1 μsec. These coincidence events will be designated as events **C**. The four adjacent blocks represent circuits which cause anticoincidence events, denoted **A**, to be recorded whenever no C counter is discharged during an interval from 2 μsec before to 5 μsec after a triple coincidence event **C**. The next circuits, reading from the top of the diagram, record delayed coincidences **C**:**N** between an event **C** and one or more counts **N** in the neutron detecting system, as well as delayed coincidences **A**:**N**. For both these events the neutron count must occur during the interval from 6 to 166 μsec after the **C** event. This long resolving time was chosen to match approximately the

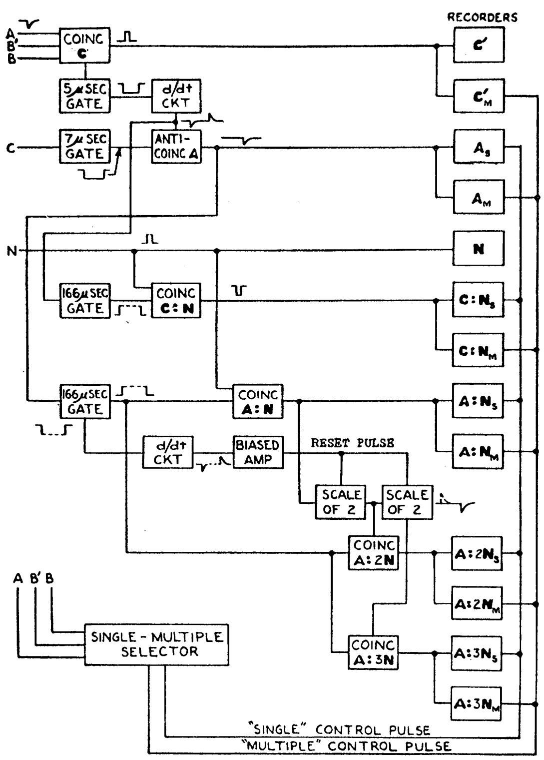

FIG. 4. Simplified functional diagram of the circuits.

mean life of a neutron in the detecting system. Under the conditions of low background that prevailed, the correction for accidental coincidences was always small in spite of the rather long resolving time. By requiring the **N** pulse to be delayed at least 6 μsec from the **C** event, the possibility of spurious events due to shower-produced prompt pulses from the N counters is excluded. As it takes about 5 μsec for a fast neutron to reach thermal speed, no loss in efficiency is entailed. Finally, by means of a scale-of-four reset after each **A**:**N** coincidence gate, anticoincidence events **A** accompanied by two or more and three or more delayed counts in the neutron detector channel are detected, and recorded as **A**:2**N** and **A**:3**N** events. The Single-Multiple selector circuits are provided for the purpose of distinguishing between "single" events in which a single charged particle penetrates the telescope, discharging only one counter in each tray, and "multiple" events in which more than one counter is discharged in any one of the trays and at least one is discharged in each of the two others. It was hoped that by considering rates of "single" events alone a further discrimination against interactions produced by particles other than μ-mesons might be effected. Hence pairs of recorders and associated circuits were provided for each event, to totalize separately cases in which a "single" telescope event is observed and cases in which the telescope event is "multiple." Thus, for example, **A**:**N** events are subdivided into $\mathbf{A}:\mathbf{N}_s$ and $\mathbf{A}:\mathbf{N}_m$ events. Each recorder is also connected in parallel with a pen element of a 20-pen Esterline-Angus "Operation Recorder." The chart record from this instrument gave a valuable check on normalcy of operation throughout each run, the extra pens provided being all used to check continuously some aspect of the operation of the equipment.

III. EXPERIMENTAL PROCEDURE

The relations between the quantities to be measured and determined will first be stated, after which the procedures for evaluating each quantity will be considered in turn.

The mean multiplicity and mean squared multiplicity are defined by Eqs. (1) and (2), respectively:

$$\langle m\rangle_{\mathrm{Av}}=\sum_{m=0}^{\infty} m a_m, \quad (1)$$

$$\langle m^2\rangle_{\mathrm{Av}}=\sum_{m=0}^{\infty} m^2 a_m, \quad (2)$$

where a_m is the probability that a given meson capture process will lead to the emission of exactly m neutrons. If we now introduce the following quantities: M_- = negative meson stopping rate, S = delayed neutron coincidence rate (one or more), D = delayed double neutron coincidence rate (two or more), and ϵ_c neutron detection efficiency (coherent), we can write the following exact expression:

$$S=M_-\sum_{m=1}^{\infty} a_m \sum_{r=1}^{m} \frac{m!}{r!(m-r)!}\epsilon_c{}^r(1-\epsilon_c)^{m-r}. \quad (3)$$

For small values of ϵ_c Eq. (3) can be represented with sufficient accuracy by the following simplified expression:

$$S=M_-\langle m\rangle_{\mathrm{Av}}\epsilon_c. \quad (4)$$

The corresponding approximate expression for the delayed double neutron coincidence rate is

$$D=\tfrac{1}{2}M_-\langle m(m-1)\rangle_{\mathrm{Av}}\epsilon_c{}^2. \quad (5)$$

Equations (4) and (5) can be solved for $\langle m\rangle_{\mathrm{Av}}$ and $\langle m^2\rangle_{\mathrm{Av}}$ to give the desired expressions for these quantities in terms of observable rates and neutron detector efficiency, thus:

$$\langle m\rangle_{\text{Av}}=S/(M_{-}\epsilon_c), \quad (6)$$

$$\langle m^2\rangle_{\text{Av}}=\langle m\rangle_{\text{Av}}+2D/(M_{-}\epsilon_c^2). \quad (7)$$

The difference between the rates of "single"[17] anticoincidences with and without the Pb absorber was taken to be the meson stopping rate, and 0.44 of this difference was taken to be the negative meson stopping rate. (In spite of the rather high anticoincidence background, which amounts to 50 percent of the effect produced by the absorber and 1.2 percent of the coincidence rate, a telescope geometry similar to ours has been shown by W. L. Kraushaar (Phys. Rev. **76**, 1045, Experiments VIII and II (1949)) to give an apparent meson stopping rate which agrees within $2\frac{1}{2}$ percent with the "best value" obtained using a more elaborate geometry in which the anticoincidence background was held down to 0.5 percent of the coincidence rate.) This factor is that obtained by Wilson[18] for μ-mesons of momentum 4×10^9 ev/c at sea-level; it is these mesons which stop in the absorber under 2000 g cm^{-2} of earth. S and D were taken to be the rates $\mathbf{A}:\mathbf{N}_s$ and $\mathbf{A}:2\mathbf{N}_s$, respectively. The neutron detector efficiency ϵ_c varies somewhat depending upon the particular region of the absorber in which the capture process takes place, so that (4) should actually be written

$$S=M_{-}\langle m\rangle_{\text{Av}}\int d\tau f(x,y,z)\epsilon_c(x,y,z)=M_{-}\langle m\rangle_{\text{Av}}\langle\epsilon_c\rangle_{\text{Av}} \quad (8)$$

where $f(x,y,z)d\tau$ is the probability that a meson capture takes place in $d\tau$ at (x,y,z). A rough approximation to the above process of integration was realized by dividing the portion of the absorber in the telescope cone into imaginary blocks $4\frac{1}{2}$ inches square, after which a standard Ra-α-Be source was placed successively at the centers of the top surface, mid-plane, and bottom surface of each block by means of a jig. The Pb absorber was, of course, in place during these measurements. The measured efficiencies with the source at these various positions, corrected as explained below for the coherent counting loss, were taken as the $\epsilon_c(x,y,z)$ in Eq. (8). The values of the weighting factors $f(x,y,z)$ depend upon the angular distribution of the cosmic-ray particles which stop in the absorber, the thickness of absorbing material above the telescope for rays of different directions, and the variation of effective area of the counter telescope with the direction of the incident radiation. Instead of attempting to compute values of $f(x,y,z)$ by evaluating all of these effects, it was decided to use an empirical procedure. It was assumed that the same $f(x,y,z)$ would describe the probability that a given *penetrating* charged particle would pass through the mid-plane of a particular $d\tau$ at (x,y,z), and an experimental determination of these probabilities was made. This was done by removing the absorber and putting small 3 inches$\times3\frac{1}{4}$ inches Geiger counter trays B'' and B''' (two to speed measurements), successively, at points corresponding to the centers of the mid-plane of each block. The coincidence rates $AB'B''B$ and $AB'B'''B$, obtained with an auxiliary dual coincidence unit,[19] properly normalized, were taken as the weighting factors $f(x,y,z)$. Most of the events (90 percent) were found to take place in the center block and those blocks adjacent to it, so that the value found for $\langle\epsilon_c\rangle_{\text{Av}}$ was very close to the true values of ϵ_c for most of the individual events. It is true that this efficiency value is an average over the broad spectrum of the Ra-α-Be source, while the spectrum of the neutrons produced in the cosmic-ray interactions is not known. However the neutron detector geometry used (see Figs. 1 and 2) is believed to have an efficiency reasonably independent of incident neutron energy up to about 10 Mev, and the theoretically predicted spectrum of neutrons evaporated from an excited nucleus does not extend to higher energies. Moreover, the measured efficiency and the delayed neutron coincidence rate were found to change by about the same factor when two different neutron detector geometries were tried, so that it seems reasonable to regard the value of efficiency for Ra-α-Be neutrons as applicable to the neutrons being studied.

The neutron detector efficiency as measured directly with a standard source differs from that with which cosmic-ray neutrons are detected because of the coherency requirement imposed by the delayed coincidence circuit. The neutrons are slowed down to thermal speeds in about 5 μsec, and thereafter they have an exponentially decreasing probability of being detected, with a characteristic time, or "mean life," τ, associated with their diffusion and capture. The finite gate length, T, in the delayed coincidence neutron measurements entails a reduction in the efficiency by the factor

$$\epsilon_c/\epsilon_i=1-\exp(-T/\tau). \quad (9)$$

The mean life τ was determined in the following manner. The trailing edge of the 166 μsec gate of the delayed neutron coincidence circuit was made to trigger a second gate associated with a similar coincidence circuit. For coherent neutrons, the ratio of counts in the two gates is

$$[1-\exp(-T_1/\tau)]/[\exp(-T_1/\tau)-\exp(-T_2/\tau)] \quad (10)$$

where T_1 is the stopping time of the first gate, measured from its starting time, and T_2 is the stopping time of the second gate measured from the starting time of the first. Two different sources of coherent neutrons were

[17] It is recognized that a small fraction of the mesons that stop in the absorber are accompanied by a knock-on electron on entering the telescope. Exclusion of these mesons does not introduce any error in $\langle m\rangle_{\text{Av}}$ and $\langle m^2\rangle_{\text{Av}}$; it only reduces slightly the rate at which data is obtained.

[18] J. G. Wilson, Nuovo cimento **6**, Supplement **3**, 523 (1949).

[19] A. H. Benade and R. D. Sard, Phys. Rev. **76**, 488 (1949).

used. The first was provided by the cosmic radiation: a count in the B' tray placed on the absorber was used to trigger the delayed neutron coincidence gates. The usual **C**:**N** events were of course included here. The high rate of delayed coincidences made it possible to obtain enough counts in about four days' time to give τ to within 5 percent standard error: $\tau=169\pm9$ μsec. The second source was provided by the $Be^9(\alpha,n)C^{12*}$ reaction, in which the excited carbon nucleus emits a γ-ray immediately. The γ-ray was used to trigger the delayed neutron coincidence gates. An old Po-Be source which had decayed to only $\sim$20 neutrons/sec was placed on top of the paraffin moderator on the telescope axis, and the B' tray was centered immediately above the source. The B' tray served to detect γ-rays from the source. The neutron coincidence rate was well above the casual and cosmic-ray rate, the efficiency of B' for the γ-rays being apparently about 0.8 percent. Two days' data sufficed to determine τ as $154\,{}^{+9}_{-8}$ μsec. The coherent neutrons from the Po-Be source have a different spectrum of initial energies from that of the neutrons produced in the cosmic-ray events, but the time required and distance traveled in the slowing down process should differ but little. Indeed, the values of τ obtained from the two methods agree well within the statistical uncertainties, and use of the weighted mean, $162\,{}^{+7}_{-6}$ μsec, seems justified. The result, which should depend to some extent on the particular moderator-counter geometry used, is in good agreement with the value of 155 ± 5 μsec found by Cocconi, Tongiorgi, and Widgoff[20] with a similar geometry. Using $T=166\pm5$ μsec for the average gate length over the period in which the multiplicity measurements were made, we find

$$\epsilon_c/\epsilon_i=0.64\pm0.02.$$

This ratio is essentially independent of the position in the absorber at which the neutron originates, so that we can take it outside the integral in (8),

$$\langle\epsilon_c\rangle_{\text{Av}}=(\epsilon_c/\epsilon_i)\int d\tau f(x,y,z)\epsilon_i(x,y,z)=(\epsilon_c/\epsilon_i)\langle\epsilon_i\rangle_{\text{Av}}. \quad (11)$$

The measurements of $\epsilon_i(x,y,z)$ were made with a 70 μc Ra-α-Be source emitting $(1.69\pm0.17)\times10^3$ neutrons/sec. This figure is based on several recent comparisons of the source with the laboratory's 20-mC Ra-α-Be standard and a recent calibration of the latter by the National Bureau of Standards. The Bureau of Standards, calibration agrees very closely with one made of the same source two years previously by D. J. Hughes at the Argonne National Laboratory. In both cases, the ultimate standard used was the Argonne Laboratory's "Source No. 38," and the uncertainty quoted above represents an estimated 10 percent uncertainty in the flux from this standard. As this uncertainty corresponds to a systematic error which could eventually be corrected for, it will not be mixed with the statistical uncertainties in the computations below. The indicated errors will be simply the statistical standard errors estimated from the number of counts. Numerical integration, using the measured values of $\epsilon_i(x,y,z)$ and $f(x,y,z)$, yields

$$\langle\epsilon_i\rangle_{\text{Av}}=2.83 \text{ percent}$$

and, thence,

$$\langle\epsilon_c\rangle_{\text{Av}}=(1.81\pm0.06) \text{ percent}.$$

These figures refer to the arrangement of neutron counters shown in Figs. 1 and 2, the ten counters being connected in parallel and the 32 unused holes in the paraffin moderator being filled with close-fitting paraffin plugs ("333100" geometry). Some runs were made in Lemp Cave as well as near sea-level (Crow Hall subbasement, under 0.5 meter reinforced concrete) with an alternative geometry chosen to give high efficiency with some sacrifice of flatness. Seven of the counters were placed in the row of cavities nearest the absorber, with the remaining three counters in the center of the next row ("730000" geometry). For this geometry the measurements gave

$$\langle\epsilon_i\rangle_{\text{Av}}=3.87 \text{ percent},$$

whence

$$\langle\epsilon_c\rangle_{\text{Av}}=(2.48\pm0.08) \text{ percent}.$$

The "333100" geometry is thought to be essentially "flat" (efficiency independent of energy) over the spectrum of the evaporation neutrons, the situation resembling that in which a plane source of neutrons is immersed in an infinite mass of paraffin and the thermal neutron density is sampled at small equal intervals along a line perpendicular to the plane. It was verified that very few delayed coincidence neutrons are detected by counters placed further below the absorber.

In applying (5), a correction must be made for the fact that the circuits have nonzero resolving time, so that a second **N** pulse can only register an **A**:2**N** event if it occurs at least 7.8 μsec after the first **N** pulse. Calling the resolving time d and expanding in powers of d/τ, we find the correction factor to (5) to be

$$1-\frac{d}{\tau}\frac{1+\exp(-T/\tau)}{1-\exp(-T/\tau)}+\cdots,$$

or 0.90. In (7), ϵ_c^2 will therefore be multiplied by 0.90.

In the cave, several runs were taken without the paraffin barrier in place, in order to determine whether any of the observed neutron events originated in the lead filter. By comparing rates with and without the barrier, and determining by means of the source the factor of change in neutron detector efficiency for neutrons produced in the filter and in the absorber, it was possible to establish that with the barrier in place

[20] Cocconi, Tongiorgi, and Widgoff, Phys. Rev. 79, 768 (1950).

only a negligible fraction of the $\mathbf{A}:\mathbf{N}_s$ events observed had their origin in the filter.

Runs were also taken with the absorber removed to determine the background $\mathbf{A}_s$ and $\mathbf{A}:\mathbf{N}_s$ rates. The delayed coincidence rate $\mathbf{A}:\mathbf{N}_s$ dropped to essentially zero (2 counts in 118 hours), while the anticoincidence rate $\mathbf{A}_s$ decreased to (0.519 ± 0.009) min^{-1}, indicating an anticoincidence efficiency of at least 98.8 percent.

Since numerous earlier measurements made with cadmium sheaths over the neutron counters in a similar counter array had shown the delayed coincidence rate to drop to zero each time,[10] only one such run was made with this apparatus, verifying that no delayed coincidence counts occurred with cadmium sheaths in place.

IV. TEST PROCEDURE

In general, data were taken during runs of about 20 hours, between which rather thorough equipment checks requiring three or four hours were made. Since the trustworthiness of the data is directly dependent on the degree of thoroughness of the equipment check, some description of this procedure will be given here. These checks apply only to the Lemp Cave data. Data taken at sea level were not checked so systematically, but the results given in Sec. V are based on a series of runs during which the rates were very steady, and the chart records for these runs were checked as described below, so that these data are believed to be reliable.

Each chart was inspected, ordinarily before the start of the next run, and the following matters observed and noted:

1. Steadiness of rates throughout run. 2. Proper association of events (e.g., presence of an **A** pip was verified each time an **A**:**N** pip was observed). 3. Agreement of number of pips for delayed coincidence events with the message register totals. 4. Possible correlation of events with line voltage fluctuations. 5. Possible correlation of delayed coincidence events with the occasional bursts of **N** counts due to insulator leakage. 6. Recording of single events in both single and multiple channels. This effect occurred occasionally because of overlapping of the long gates employed in the thyratron message register circuits, but it was possible to correct for it. In the final data analysis, runs or portions of runs were rejected whenever this inspection showed abnormalities in the run. At the end of each run all rates were computed for the run and plotted on a "rate graph," so that it was possible to tell at a glance whenever one of the rates deviated from the average of the preceding runs.

After this a set of six artificial pulses whose relative times could be adjusted in any manner desired were connected at the counter inputs in such a way as to simulate the occurrence of pulses in the A, B', B, and N channels (3 pulses in the latter). By delaying these suitably with respect to one another it was possible to verify that all circuits were operative and that their resolving times were approximately correct. This was done first using one signal per telescope tray, and then repeated with 4 simulated counter signals (multiple event). Thus every circuit in the apparatus was operationally checked, from preamplifiers to message registers. This check was made without fail every day, since some of the events were so rare (e.g., no count was recorded on $\mathbf{A}:3\mathbf{N}_s$ during the entire Lemp Cave series of measurements) that it was impossible to detect circuit failure from the cosmic-ray data.

Other checks performed daily without fail were a measurement of neutron detector efficiency with the Ra-α-Be source placed in a standard position at the top of the lead absorber, and a measurement with a Los Alamos-type "precision pulser" of the threshold of the neutron channel amplifier and discriminator.[21] Measurement of these quantities, along with the value of the high voltage applied to the BF_3 counters (2800 volts) and Geiger counters, followed by daily resetting of the controls to correct for any drift of characteristics, resulted in extreme constancy of the efficiency for neutron detection.

Other checks made with a frequency dictated by discretion were measurements of bias and plate voltages in the electronic circuits, measurement of plateaus and pulse heights of Geiger counters, and investigation of any miscellaneous troubles which might be suspected.

Finally calibration of all coincidence circuit resolving times was made at approximate two week intervals using the multiple pulse generators employed in the daily check. The time relation of the six pulses was determined by the settings of six multi-revolution potentiometers, after which an oscilloscope and calibrator were used to translate the settings of the potentiometers into relative time values. As a final step the calibrator was checked against an Army Signal Corps SCR-211 frequency meter.

V. EXPERIMENTAL RESULTS

The rates measured with the normal geometry are given in Table I. Those obtained with the "flat" N detector in the cave (second column) are the basic data of this experiment. The third and fourth columns give the rates obtained using the more efficient neutron detecting geometry, both in the cave and in the subbasement of Crow Hall. The rates are corrected for casual coincidences, computed from the measured individual counting rates and the gate-lengths. This correction is always small, its greatest value being 7 percent, with no correction greater than 0.7 percent in the quantities used to calculate $\langle m \rangle_{\text{Av}}$ and $\langle m^2 \rangle_{\text{Av}}$. The

[21] Atomic Instrument Company Type 204-B (modified) with preamplifier Type 205. The threshold signal level was maintained at 1.2 millivolts. The rise time-decay time setting of the amplifier was 0.2 μsec–0.4 μsec, and examination of the output with a sweep triggered by the counter pulse has shown that the afterpulse due to attached electrons does not get through when this short decay time is used (Cocconi Tongiorgi, Hayakawa, and Widgoff, Rev. Sci. Instr., in press).

TABLE I. Measured rates[a] in principal geometry.[b]

Type of event	Locale		
	Lemp Cave (flat N detector, 333100)	Lemp Cave (efficient N detector, 730000)	Crow Hall (efficient N detector, 730000)
$C^{\text{single}}_{\text{multiple}}$ (min^{-1})	44.01 6.91	44.53 6.98	137.0 15.42
$C:N^{\text{single}}_{\text{multiple}}$ (hr^{-1})	2.28±0.10 0.91±0.06	3.15±0.21 1.31±0.13	14.26±0.28 4.73±0.16
$A^{\text{single}}_{\text{multiple}}$ (min^{-1})	1.50 0.23	1.53 0.23	6.37 0.75
$A:N^{\text{single}}_{\text{multiple}}$ (hr^{-1})	1.03±0.06 0.15±0.02	1.35±0.13 0.18±0.05	7.77±0.20 1.05±0.07
$A:2N^{\text{single}}_{\text{multiple}}$ (hr^{-1})	0.012±0.007 0.012±0.006	0 0.01±0.01	0.41±0.05 0.15±0.03
$A:3N^{\text{single}}_{\text{multiple}}$ (hr^{-1})	0 0.006±0.004	0 0.01±0.01	0.03 ±0.01 0.005±0.005

[a] The neutron coincidence rates are corrected for accidental coincidences.
[b] Pb filter above telescope, paraffin barrier in telescope, Pb absorber in place.

estimated statistical standard deviations are the square roots of the uncorrected numbers of counts divided by the time.

Table II gives the rates used in calculating $\langle m\rangle_{\text{Av}}$ and $\langle m^2\rangle_{\text{Av}}$ according to (6) and (7). The first, third, and fifth rows correspond to rates listed in the second column of Table I. The other two rows give rates for measurements made with the Pb absorber removed. The first row of Table III gives the result of substituting these figures in (6) and (7), the errors indicated being purely statistical. As explained in Sec. III, the fraction of stoppings corresponding to negative mesons has been taken as 0.44, the coherent neutron detecting efficiency as (1.81±0.06) percent. And in using (7), ϵ_c^2 has been replaced by $0.90\epsilon_c^2$. The value of $\langle m^2\rangle_{\text{Av}}-\langle m\rangle_{\text{Av}}$ obtained from (7), 3.15±1.84, should be treated with reserve, as it is based on only three counts.

The multiplicity can also be calculated from the data (Table I, third column) obtained with the more efficient neutron detector (730000), with $\epsilon_c=(2.48\pm0.08)$ percent. The result is given in the second row of Table III, errors being stated as in the first row. It is seen to agree very well with that obtained with the geometry thought to be flatter. That is, comparing the two different neutron detecting geometries, the ratio of the rates obtained for Ra-α-Be neutrons is the same, within the statistics, as the ratio of the rates obtained for disintegration neutrons from μ-meson capture in Pb. This can be taken as evidence that the two neutron spectra are similar. During the brief run with the more efficient geometry there were no $\mathbf{A}:2\mathbf{N}_s$ events, so that all that can be concluded about $\langle m^2\rangle_{\text{Av}}$ on the basis of this run is that it is at least $(2.04\pm0.30)^2$, since in general $\langle m^2\rangle_{\text{Av}}>\langle m\rangle_{\text{Av}}^2$.

Calculation of $\langle m\rangle_{\text{Av}}$ and $\langle m^2\rangle_{\text{Av}}$ from the data obtained with only about 240 g cm^{-2} concrete between the Pb filter and the sky (Table I, fourth column, lists some of this data) throws some light on the effect of neutron production in processes other than μ-meson capture on determinations of $\langle m\rangle_{\text{Av}}$ and $\langle m^2\rangle_{\text{Av}}$. The results are given in the third row of Table III. It is seen that $\langle m\rangle_{\text{Av}}$ comes out about 20 percent high. The change in $\langle m^2\rangle_{\text{Av}}$ with depth is considerably greater, indicating that in the neutron multiplicity distribution from μ-meson capture there is less of a high multiplicity tail than in the other nuclear interactions that produce $\mathbf{A}:\mathbf{N}_s$ events near sea level.

That the separation of "single" telescope events from "multiple" ones has resulted in an increase of the relative contribution of μ-meson capture to the $\mathbf{A}:\mathbf{N}$ rate is indicated by two pieces of evidence. Control measurements in the cave made with the paraffin barrier removed show that neutron production in the filter makes no contribution to the $\mathbf{A}:\mathbf{N}_s$ rate when the barrier is in place, while some 40 percent of the $\mathbf{A}:\mathbf{N}_m$ rate must be ascribed to interactions in the filter when the barrier is in place. Secondly, in the Crow Hall results, a significant difference is observed in the quantity $\mathbf{A}:2\mathbf{N}/\mathbf{A}:\mathbf{N}$ for "singles" and "multiples." This quantity, which is approximately proportional to the ratio of $\langle m^2\rangle_{\text{Av}}$ to $\langle m\rangle_{\text{Av}}$ has the value (5.3±0.7) percent for "single" telescope events and (14.3±3.0) percent for the "multiples." Thus, near sea-level the "single" and "multiple" events concern appreciably different distributions of multiplicities.

TABLE II. Rates involved in main multiplicity determination. (Lemp Cave, flat neutron detector, single telescope events.)

Type of event	Counts	Duration (hrs)	Rate, corrected for casuals
$\mathbf{A}_s$ absorber in	29,378	326.15	1.501±0.009 min^{-1}
$\mathbf{A}_s$ absorber out	3683	118.32	0.519±0.009 min^{-1}
$\mathbf{A}:\mathbf{N}_s$ absorber in	337	326.15	1.03 ±0.06 hr^{-1}
$\mathbf{A}:\mathbf{N}_s$ absorber out	2	118.32	0.016±0.011 hr^{-1}
$\mathbf{A}:2\mathbf{N}_s$ absorber in	3	244.79	0.012±0.007 hr^{-1}

The experimental result, $\langle m\rangle_{\text{Av}}=2.16\pm0.15$, plus possible systematic efficiency error, is not in disagreement with the value 1.4±0.2 reported by Conforto and Sard,[22] since the anticoincidence rate used in their calculation of $\langle m\rangle_{\text{Av}}$ was thought to be somewhat higher than the true meson stopping rate. In their experiment, carried out at sea level, a magnetized iron lens designed to focus mesons was used to eliminate the protonic and electronic contributions to $\mathbf{A}:\mathbf{N}$.

[22] A. Conforto and R. D. Sard, Phys. Rev. 82, 335 (1951).

A similar investigation has recently been reported by Groetzinger, Berger, and McClure[23] using a magnetized iron lens to focus mesons and a neutron detecting arrangement somewhat similar to the high efficiency 730000 geometry described above. However, the low counting rate obtained with this arrangement resulted in a lower statistical accuracy, the result being stated as $\langle m\rangle_{Av}=1.96\pm0.72$ (probable error).

Recently, Cocconi and Cocconi-Tongiorgi[14,24] have carried out measurements of the neutron counting rates of various multiplicities (**N**, 2**N**, 3**N**, $\cdots$) when a Pb block surrounded by a paraffin-BF_3 counter system is held at various depths under water. They did not measure neutrons correlated with charged particles. On the basis of assumptions about the relative frequencies of occurrence and the neutron multiplicity distributions for the various processes giving rise to neutrons, they arrive at $\langle m\rangle_{Av}=2\pm0.5$. The agreement of this published estimate[14] with our more directly measured value can be considered as supporting evidence for the assumptions made by the Cocconi's.

It is noted in passing that comparison of the rates in the second and fourth rows of Table I confirms the previous evidence,[25] obtained at a shallower site, for neutron production in lead by fast μ-mesons. This question will be discussed in detail elsewhere.

VI. THE THEORETICALLY EXPECTED MULTIPLICITIES

Two possible meson-nucleon interactions have been considered. The first is that of the Yukawa particle, which gives all its rest-energy to the proton with which it interacts. The capturing nucleus then receives an excitation energy of about 100 Mev. Fujimoto and Yamaguchi[26] have calculated the disintegration of a capturing silver nucleus in this case. Taking into account the competition between neutron emission and charged particle emission at each step of the reaction, they find that on the average the reaction should lead to the emission of altogether 1.0 protons and 4.7 neutrons. Because of the higher Coulomb barrier in the Tl nucleus, it seems reasonable to expect an average of about 6 neutrons emitted in the reaction studied here. This result is entered in the fourth row of Table III.

The other interaction considered is that in which the encounter of a μ-meson with a nuclear proton results in a neutron and a light "thief" particle. Tiomno and Wheeler as well as Rosenbluth[12] have calculated the distribution function for the excitation energy, treating the nucleus as a confined Fermi gas. The function rises to a peak at about 15-Mev excitation energy and then falls rapidly to zero at about 20 Mev. It is necessary to translate this distribution function for excitation energy into one for number of neutrons emitted. The first step is to assume that proton emission is excluded, and that neutron emission occurs and re-occurs until the residual excitation of the nucleus falls below the binding energy of the last neutron, after which only γ-emission takes place. This assumption seems to be consistent with present knowledge regarding the level widths for possible competing emission processes and, moreover, leads to an upper limit for the predicted neutron multiplicity for a given distribution of initial excitations. In this connection, comparison of the distribution of excitation energy calculated on the "free-particle" model with that calculated by Tiomno and Wheeler[12] on the "dipole" model suggests that the former tends to overestimate the excitation. Thus the following calculation probably errs on the side of generosity in its predictions for $\langle m\rangle_{Av}$ and $\langle m^2\rangle_{Av}$.

The neutron multiplicity distribution is derived from the distribution function of initial excitation energy of the compound nucleus, $f(A;Q_0)$, by means of the function $g(Z,A;Q,\epsilon)$, giving the probability that a nucleus (Z,A) with excitation energy Q emits a neutron of kinetic energy in unit energy interval at ϵ. In effect, the probability of residual excitation in dQ_1 at Q_1 after emission of one neutron is

$$f(A-1;Q_1)dQ_1 = dQ_1\int_{Q_0>BE(Z,A)+Q_1} dQ_0 f(A;Q_0) \times g(Z,A-1;Q_0,Q_0-BE(Z,A)-Q_1), \quad (12)$$

where $Z=81$ for capture in Pb and $BE(Z,A)$ is the binding energy of a neutron in the nucleus (Z,A). The function $g(Z,A;Q,\epsilon)$ is normalized to 1 when integrated with respect to ϵ from $\epsilon=0$ to $\epsilon=Q-BE$. The probability of residual excitation in dQ_2 at Q_2 after

TABLE III. Mean and mean square neutron multiplicity from μ-meson capture in lead.[a]

	$\langle m\rangle_{Av}$	$\langle m^2\rangle_{Av}$
Experimental (Lemp Cave, flat neutron detector, single telescope events)	2.16 ± 0.15	5.3 ± 1.9
Experimental (Lemp Cave, efficient neutron detector, single telescope events)	2.04 ± 0.21	$>4.2\pm0.9$
Apparent experimental (Crow Hall, efficient neutron detector, single telescope events)	2.57 ± 0.11	14.8 ± 1.5
Theoretical, for excitation energy of 100 Mev[b]	6	>36
Theoretical, for $\mu^-+P\rightarrow N+\nu$, assuming free-particle model for excitation and statistical theory for evaporation	0.95	1.3

[a] The errors indicated for the experimental values are estimated statistical standard errors. They do not include systematic errors such as the estimated ±10 percent uncertainty in the strength of the neutron source used in the efficiency determination.
[b] See reference 26.

[23] Groetzinger, Berger, and McClure, Phys. Rev. **81**, 969 (1951).
[24] G. Cocconi, private communication to R. D. Sard.
[25] R. D. Sard, Phys. Rev. **80**, 134 (1950); Sard, Crouch, Jones, Conforto, and Stearns, Nuovo cimento **8**, 326 (1951).
[26] Y. Fujimoto and Y. Yamaguchi, Prog. Theor. Phys. **4**, 468 (1949); **5**, 787 (1950).

TABLE IV. Neutron binding energies and mass difference energies.

Capturing isotope	Abundance (%)	Tl-Pb mass difference energy[a] (Mev)	B. E. of first neutron[b] (Mev)	B. E. of second neutron[b] (Mev)	B. E. of third neutron[b] (Mev)
Pb^{206}	23.6	1.7	6.23	7.48	6.54
Pb^{207}	22.6	1.5	6.97	6.23	7.48
Pb^{208}	52.3	5.0	3.86	6.97	6.23

[a] See reference 30. [b] See reference 29.

emission of two neutrons is, similarly,

$$f(A-2;Q_2)dQ_2 = dQ_2 \int_{Q_1>BE(Z,A-1)+Q_2} dQ_1 f(A-1;Q_1) \times g(Z,A-1;Q_1,Q_1-BE(Z,A-1)-Q_2), \quad (13)$$

and so forth. The probability, a_m, of emitting m neutrons altogether is given by (14) and (15):

$$a_0=\int_{Q_0=0}^{BE(Z,A)} dQ_0 f(A;Q_0), \quad (14)$$

$$a_m=\left(1-\sum_{k=0}^{m-1} a_k\right)\int_{Q_m=0}^{BE(Z,A-m)} dQ_m f(A-m;Q_m). \quad (15)$$

We use for the function g the expression given by Weisskopf[27] in his statistical theory.

$$g(Z,A;Q,\epsilon)=C\epsilon \exp[-(a/\{Q-BE(Z,A)\})^{\frac{1}{2}}\epsilon], \quad (16)$$

the normalization determining C. According to Blatt and Weisskopf,[28] the value of a giving the best agreement with experiment for heavy nuclei is 10 Mev^{-1}. Table IV lists the values we have adopted for the neutron binding energy[29] in the various Tl isotopes, and for the mass difference energy[30] which is needed to build a Tl nucleus from its Pb isobar. This latter energy must be subtracted from Tiomno and Wheeler's Q values to get the true available initial excitation energy Q_0. Table V lists the computed values for the a_m, the integrations indicated in (12), (13), (14), and (15) having been done graphically with a planimeter. The values found for the coefficients a_m seem to be consistent with Wheeler's assertion that on the basis of the charge-exchange reaction emission of 2 neutrons would occur occasionally, whereas 3 would be most rare.[31] Table V also gives the values of $\langle m\rangle_{Av}$ and $\langle m^2\rangle_{Av}$ computed from these a_m. For comparison with the experimental results, these computed values of $\langle m\rangle_{Av}$ and $\langle m^2\rangle_{Av}$ are also entered in the last row of Table III.

VII. CONCLUSIONS

The mean number of disintegration neutrons produced when a Pb nucleus captures a negative μ-meson is found to be 2.16 ± 0.15, with ±10 percent additional systematic error due to the uncertainty in the strength of the standard neutron source used to determine the neutron detecting efficiency.

Despite the poor statistics available for the determination of the mean squared neutron multiplicity (3 events), it is clear that $\langle m^2\rangle_{Av}$ does not greatly exceed $\langle m\rangle^2_{Av}$. This contrasts with the results obtained near sea-level, where $\langle m\rangle_{Av}=2.6\pm0.1$, and $\langle m^2\rangle_{Av}=14.8\pm1.5$, indicating that the neutron multiplicity distribution from μ-meson capture has a considerably less pronounced high multiplicity tail than does the distribution from the other processes occurring at the shallower depth.

TABLE V. Theoretical results, using free-particle model and statistical theory.

Capturing isotope	a_m = probability per capture, of emitting m neutrons				Mean number of neutrons m_{Av}	Mean square number of neutrons m^2_{Av}
	$m=0$	$m=1$	$m=2$	$m=3$		
Pb^{206}	0.199	0.643	0.158	0	0.96	1.28
Pb^{207}	0.128	0.627	0.245	0	1.12	1.61
Pb^{208}	0.245	0.628	0.127	0	0.88	1.14
Average, weighted according to normal abundance	0.207	0.631	0.161	0	0.95	1.28

The result $\langle m\rangle_{Av}=2.16\pm0.15$, clearly rules out the Yukawa-type interaction, and is therefore consistent with the findings of the various investigators who have searched for charged particles associated with negative μ-meson capture. On the other hand, while the general picture of the charge-exchange reaction with its light "thief" particle carrying away most of the rest energy is substantiated, the observed value of $\langle m\rangle_{Av}$ is markedly higher than the value 0.95 predicted on the basis of the free-particle model (for the calculation of the excitation) and the statistical evaporation theory (for the calculation of the neutron emission). A more refined theoretical calculation is needed to decide whether the discrepancy is fundamental, revealing inadequacy of the assumed $\mu^-+P\rightarrow N+\nu$ mechanism, or whether it simply results from the approximations made in the calculation.

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[31] Proceedings of the Echo Lake Cosmic-Ray Symposium, 1949 (ONR). Discussion of paper 7, p. 29.

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Eigenvalue Problem in Quantum Electrodynamics*

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The eigenvalue problem in quantum electrodynamics is discussed from the point of view of the Fredholm theory of integral equations. Starting with positron theory—the theory of a quantized Dirac field interacting with an external field only—the external potentials are replaced by bare photon fields. To insure causality the photon operators are ordered in time. Certain integral equations for the Fredholm minors constructed on the Feynman kernel are taken to be the equations for the wave functions of *n* particle systems. An expansion in interaction patterns rather than the coupling constant is indicated. The one particle problem is treated in the first pattern approximation. Procedures proposed by Snyder and Snow and the mass renormalization scheme are discussed in this connection. Finally a purely formal derivation of the Bethe-Salpeter equation for the two-body problem in the lowest pattern approximation is given.

I. INTRODUCTION

SEVERAL new attempts[1,2] have been made recently to deduce from a field theory the equations for a system of two Dirac particles in interaction. The problem has been approached from the point of view of the S matrix[1] and by a formal extension of the theory of Green's functions.[2] The difficulties of a rigorous deduction of these equations have been emphasized by Nambu.[3] Work in this direction has therefore been either purely formal or largely heuristic.

Using arguments of the same character, it is intended to provide in this note additional motivation for the acceptance of certain equations as approximate descriptions of bound systems. The mathematical imagery employed is that of the Fredholm theory of integral equations. Intuition is relied on to extract from it a workable set of equations for one, two, and many body problems.

In brief outline the procedure is the following. We first investigate an electron-positron field subject to an external electromagnetic field only. The integral equation considered is that of a scattering problem with an inhomogeneous term corresponding to the wave function of a free particle in the absence of the external field. The associated homogeneous integral equation may have nontrivial solutions, if certain relations between the energy, the mass and the charge are satisfied. These represent the bound states of the system. The restrictions on the values of the parameters are expressed by the requirement that the Fredholm determinant constructed on the kernel vanish. The wave function of the system is then given by the first Fredholm minor. If for a particular relation between the energy, the mass and the charge the determinant as well as the first minor are identically zero the solution of the equation is given by the second minor, if this quantity does not vanish. The second minor is antisymmetric in its two indices and satisfies the homogeneous equation independently in both. It is therefore the wave function for a two-particle noninteracting system. The external field is now replaced by a quantized electromagnetic field satisfying Maxwell's equations without sources. If a radiation field of such character is to give rise to a causal interaction between charged particles the photon operators must be chronologically ordered[4] and an expectation value relative to a state without photons taken. An expansion in various patterns of interaction, rather then in the coupling constant is then carried out and the effective potentials that arise out of various modes of interaction are isolated. The Bethe-Salpeter equation is obtained in the lowest pattern approximation.

II. ALGEBRAIC PRELIMINARIES

The Fredholm theory in its conventional form is somewhat clumsy for our purpose. Our first task will

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