plotting W vs V and C vs V, using Briggs' infrared value<sup>6</sup>  $K = n^2$ = 16.1 to relate the C and W scales. On the basis of this procedure the points agree, and within this accuracy the experiment confirms that  $n^2$  at infrared equals K at 1000 cps. Unfortunately, the junction studied has neither a linear concentration gradient nor an abrupt transition as evidenced by the approach of the data to  $C \propto \tilde{V}^{-2}$  and in fact the curves of Fig. 2 can be equally well fitted by parabolas. This introduces an uncertainty in the Westimate of about 30 percent, which we hope to eliminate by future studies of other junctions.

We are indebted to E. Buehler, M. Sparks, and G. K. Teal, who prepared the single crystal p-n junction and to W. L. Bond, P. W. Foy, and H. R. Moore for help with the measurements.

<sup>1</sup> W. Schottky and E. Spenke, Wiss. Veröff. Siemens-Werk. 18, 1 (1939); see also S. J. Angello, Elec. Eng. 68, 865 (1949).
 <sup>2</sup> W. Shockley, Bell System Tech. J. 28, 435 (1949).
 <sup>3</sup> McAffee, Ryder, Shockley, and Sparks, Phys. Rev. 83, 650 (1951); and Goucher, Pearson, Sparks, Teal, and Shockley, Phys. Rev. 81, 637 (1951)

This junction is the same as that investigated by Goucher et al.

<sup>6</sup> The scheme as the second se

## **Electron Interferometer\***

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HIS brief report contains the basic principles of an interferometer operating with electron beams. The possibility of building such an interferometer may have occurred to many, but in the absence of any publications on this subject some elementary considerations may not be misplaced.

It is rather simple to conceive an interferometer operating with electron beams based on the equivalent light optical experiment of Young. In principle, the double slit method could be employed but simple calculations, based on light optical analogies, indicate that the dimensions and complexities of such an instrument are rather undesirable. Because of the short wavelength, the source size, the size of the slits and their separation, and the separation of the fringes becomes so small that a major experimental effort may be needed for coping with them. In view of these difficulties it is worthwhile to explore the possibilities of a wide beam interferometer instead of a narrow beam instrument. In principle a wide beam interferometer of, let us say, the Michelson or the Jamin type is possible provided an efficient beam splitter is available. Such a good beam splitting mechanism exists for electrons, although not in the customary sense of light optics. Diffraction from thin crystal lamellae offers an excellent mechanism for carrying out such an interferometer experiment. Several lamellar crystals are needed in the manner indicated in Fig. 1. Let us assume an incident parallel beam of electrons passes through a thin crystal in the manner indicated by Fig. 1(a). Part of the beam is transmitted and part of it is diffracted. At a certain distance a second crystal is placed. Part of the original beam again is transmitted and part of it is diffracted as indicated. The same applies to the beam diffracted on the first crystal. The same phenomenon is repeated again on a third crystal placed at equal distance. By placing convenient limiting apertures we can select two diffracted beams out of the multitude of all the beams indicated on Fig. 1(a) and have a total path indicated on Fig. 1(b). The resulting trajectories correspond roughly to the equivalent of the Mach-Zehnder type interferometer. Figure 1(b) indicates the optical path for zero-path difference. A field gradient across the two paths will produce a path difference which can be observed by means of the shifting of the fringes localized at infinity.

To convince ourselves that the proposed scheme is feasible, we carried out light optical analog experiments. They consisted in reproducing the optical path indicated on Fig. 1(b) by means of transmission-type grating replicas. It was found that while such a system does not offer any particular advantages as compared to the conventional interferometer, it constitutes a perfectly good light optical interferometer and it helped us to compute some of the design characteristics of the electron beam instrument. Further proof for the soundness of the idea is furnished by the electron microscope observations of interference fringes published by Mitsuishi, Nagasaki, and Uyeda.<sup>1</sup> Similar observations have been reported also by Rees<sup>2</sup> from Australia and also by Hillier.<sup>3</sup> Observations in all three places indicate that the type of interference necessary for the operation of an electron interferometer is produced on lamellar thin crystals.

Calculations have been carried out to determine the tolerances for misalignment of the elements of an electron beam interferometer. These calculations will be reported later by J. Arol Simpson. Anticipating this report it may be mentioned that the dimensional and other tolerances of the instrument are well within experimental possibilities.

The crystals required for carrying out such experiments can be either selected from natural crystals or grown for the purpose.



FIG. 1(a). Schematic representation of rays passing through three crystals.



FIG. 1(b). Rays as limited by apertures.

It has been shown in the past by numerous observers that epitaxy offers a good way for the production of thin lamellar crystals. Experiments have been carried out in this laboratory by O. G. Engel, J. A. Simpson, J. Suddeth, and T. McCraw, and thin films of different metals having thicknesses of the order of 100 to 200A have been produced with the required characteristics. While the produced films may not be single crystals in the true sense of the word, electron diffraction patterns indicate that if they are constituted by smaller crystals all the constituent crystals are so well aligned that the conditions for the interferometer experiment are fulfilled. Details about these experiments will also be contained in later publications.

In principle, the optics of such a wide beam interferometer could be simplified, to contain only one thin crystal, as shown in Fig. 2. In this case the incident beam is deflected at right angles by a first magnet A to impinge on a thin crystal. The incident beam again is partly transmitted and partly diffracted. The transmitted beam is intercepted by a conveniently located stop. The two diffracted beams are deflected by the magnets  $B_1, B_2, C_1, C_2$  in the manner shown on the figure and are reunited by the crystal to form an emerging beam proceeding in the opposite direction from the incident one. Magnet A deflects then this emerging beam in a direction coincident with the original one of the incident beam.



FIG. 2. Interferometer arrangement with one crystal and magnetic field

A number of other possible combinations with varying number of crystals and fields could be listed. It is conceivable that Bragg reflection on thicker crystals could also be used. The geometries which reduce the number of crystals merely shift the required tolerances from the crystal alignments to the magnetic field alignments. No estimates have been made up to now as to the degree of accuracy required in this latter operation.

The usefulness of an electron interferometer can be manifold. As mentioned above, fringe displacement is produced by a field gradient and therefore such an instrument constitutes an extremely sensitive device for measuring gradients of magnetic or electrostatic fields. By producing large differences of optical paths it is intended to use the instrument for the determination of the limit of coherence of an electron beam. In analogy to interference spectroscopy of light optics it could be used for the determination of the band structure of electron emitters. Another possible use may be indicated by applying it to the study of internal potentials in solids.

Construction of an electron interferometer is underway and results will be reported later.

\* This work was done as part of a cooperative program of research and development in basic instrumentation sponsored jointly by the National Bureau of Standards, ONR, Office of Air Research, and the AEC. <sup>1</sup> Mitsuishi, Nagasaki, and Uyeda, Proc. Japan Acad. 27, 86 (1951). <sup>2</sup> Private communication from Dr. Rees. <sup>3</sup> Reported at the National Bureau of Standards Electron Physics Surprogram.

Symposium.

## Stopping Power of Heavier Substances

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N recent years a considerable number of experiments have been performed on the stopping of swift particles of low charge. The results have been discussed on the basis of the treatments by Bethe and by Bloch. The purpose of the present note is to point out a simple way in which to compare the experimental data for substances of fairly high atomic numbers.

When the dynamics of atoms is described on classical lines on the basis of the Thomas-Fermi model a striking similarity appears in the behavior of different atoms for the following reason. In the dynamics of a Fermi gas of density  $\rho$  there enters directly a frequency, namely the classical resonance frequency  $\omega_0 = (4\pi\rho e^2/m)^{1/2}$ , involving the interaction of the electrons. In the statistical model of the atom, where the common unit of length is proportional to  $Z^{-1/3}$ , the scale serving as a norm for the frequencies  $\omega_0$  is proportional to Z. Similarly, since the velocities involved behave as  $\rho^{1/3}$  or  $Z^{2/3}$ , the scale measuring frequencies of revolution  $\omega_r$  in the atom is also proportional to Z. This gives the characteristics of the dynamics of heavy atoms on the classical approach, in which the motion is shared by many electrons.

Consider now the process of slowing-down of a swift atomic particle of low charge, z, so that the energy transfer is proportional to  $z^2$ . The effects of the particle are then completely specified by its velocity v, corresponding to a frequency  $\omega_{\rm max} = 2mv^2/\hbar$ . The stopping will be determined by the relative value of the frequencies in the atom measured in terms of  $\omega_{max}$ . One then finds for dimensional reasons that the specific energy loss as a function of v and Z, may be written  $(4\pi z^2 e^4/mv^2) \cdot N \cdot Z$  times a dimensionless function of  $(\omega_0 \text{ or } \omega_r)/\omega_{\max} \propto Z/v^2$ , and one may write

$$\frac{1}{N}\frac{dE}{dR} \cdot \frac{mv^2}{4\pi z^2 e^4 Z} = L = L(Z/v^2),$$
(1)

where L is a so far unspecified function, determined by the distribution of the proper frequencies in the atom. When relativistic effects are of importance, the familiar quantity  $\log(1-v^2/c^2)+v^2/c^2$ is to be added on the left hand side. But corrections for K-shells are not to be introduced in a statistical description of atoms of the kind considered here.

By using a simplified classical hydrodynamical treatment of the motion of a Fermi gas, Bloch has obtained a formula of the type (1) with  $L = \log(2mv^2/Z \cdot I_0)$ ,  $I_0$  being a constant. This expression should have approximate validity for very high velocities v.

The considerations leading to (1) may be given a more quantitative form. It is here useful to note that for a free Fermi gas of electrons the frequency giving the adiabatic limit for a particle of high velocity is precisely the classical resonance frequency  $\omega_0$ . Averaging over the density distribution in the atom and including the above frequencies of revolution, one obtains the formula of Bloch for a very swift particle, while for lower velocities the stopping power of the medium is found to be nearly proportional to  $Z^{1/2}/v$ . The constants obtained are in both cases of the right order of magnitude. Moreover, it can be shown that similar results are obtained directly from the general stopping formula of Bethe when one introduces the frequencies and oscillator strengths of the individual atomic transition processes.

According to Eq. (1) the data on stopping may be approximately described by a single function of  $Z/v^2$  common to all elements of not too low atomic number. In Fig. 1 are shown some experimental values of L determined from the left hand side of (1); the abscissa used is  $x = (v/v_0) \cdot Z^{-1/2}$ . We have included a number of absolute measurements on the stopping of protons of energy between 1-200 kev and 340 Mev,1-3 for metals ranging from uranium to the extreme case of beryllium. The points on the figure show a rather well-defined curve. The placing of other elements may be obtained from the numerous relative measure-