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A Test for the Charge-Symmetry Hypothesis

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YANG has recently suggested[1] an experiment to test the charge-symmetry hypothesis in the coupling of mesons to nucleons. The proposed experiment is to study the following pair of reactions

$$p+p\rightarrow d+\pi^+ \quad (1a)$$
$$n+p\rightarrow d+\pi^0. \quad (1b)$$

These reactions have the advantage over those studied so far in that only one isotopic spin state of the nucleons in the initial state can contribute to the reactions if isotopic spin is conserved, i.e., if it is a good quantum number. The simple argument here is that since a nucleon in the theory of isotopic spin is defined by four variables (position, momentum, spin S, and isotopic spin I) and is a fermion, then the total wave function of a pair of nucleons must be antisymmetric. Now, since the deuteron has an even space wave function and a symmetric spin function, the isotopic spin function must be antisymmetric. The only odd isotopic spin function for two isotopic spins $\frac{1}{2}$ is that one with total $I=0$. Therefore, the final state of (1a) has $I=1$ (that of the pion, with $I_z=+1$ for π^+, 0 for π^0, and -1 for π^-) and $I_z=+1$. Thus the two protons in the initial state have total $I=1$ and $I_z=+1$. Likewise in (1b) the two nucleons have $I=1$, $I_z=0$. Since the statistical probabilities of these total spin orientations are equal, these two reactions (granting the charge-symmetry hypothesis) must have the same angular distributions, and cross sections differing exactly by a factor 2 (since only half of the state $n+p$ is the required isotopic triplet).[2]

It is the purpose of this note to point out two other reactions which give as good a test of charge-symmetry, are probably considerably easier experimentally, and in addition measure the interaction of pions and mesons in states of total isotopic spin $\frac{1}{2}$—a question in which there is considerable interest as the result of recent scattering measurements[3] which have been interpreted[4] as interactions through a resonance state of total isotopic spin $\frac{3}{2}$. The two reactions are really the two branches of the reaction

$$p+d\rightarrow d+n+\pi^+ \quad (2a)$$
$$p+d\rightarrow d+p+\pi^0. \quad (2b)$$

These reactions are the two possible breakups of a proton catalyzed by the presence of the deuteron which absorbs the extra momentum.

Here again the deuteron with $I=0$ forces the initial state to have $I=\frac{1}{2}$, $I_z=+\frac{1}{2}$, as must the final state. Therefore the odd nucleon and the pion must be in a state with $I=\frac{1}{2}$, $I_z=+\frac{1}{2}$, instead of the usual variety of states available to a nucleon and a pion. So we see that the branching ratio should be 2:1, with identical angular distributions, etc. The factor 2 comes again from the fact that the interaction leads only to a final state $I=\frac{1}{2}$, $I_z=+\frac{1}{2}$. Thus, the square of the matrix elements must be reduced to the fraction of $I=\frac{1}{2}$ contained in the ordinary $I_z=+\frac{1}{2}$ state. This is $\frac{2}{3}$ for $n+\pi^+$ and $\frac{1}{3}$ for $p+\pi^0$.

Since there are three particles in the final state one must investigate not only the angle of emission of the π^+ or p, but the energy also (or the angles of the light charged particle and the deuteron also). In a cloud chamber with magnetic field, one of the easiest things to do is to measure the energy spectrum of deuterons accompanied by positive pions in (2a) or by inelastically scattered protons as in (2b).

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A Proposed Test of the Nuclear Shell Model

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IN this note we wish to suggest certain experiments which should give direct information concerning the accuracy of the independent particle model of nuclear structure[1] in ascribing definite orbital angular momentum states to nucleons in a nucleus.

Interpretation of the angular distributions from (d, p) and (d, n) nuclear reactions[2] has shown these reactions to proceed mainly by means of a stripping process, the angular distribution of the outgoing particle in any one case being characterized by the orbital angular momentum l with which the captured particle can be accepted into the appropriate final state. The angular distributions all show a pronounced peak at small angles, this maximum lying directly forward if $l=0$, but moving progressively towards larger angles as l is increased. Also, if more than one value of l is allowed by the selection rules in a particular case, and if the initial nucleus should be indifferent as to which of these values it accepts, then it is found that the maximum in the angular distribution which is nearest the forward direction, i.e., which results from the smallest allowed l, is of much larger magnitude than the others; there is, in fact, an order of magnitude decrease in the heights of the maxima as l is increased by 2 (the allowed values of l in any one case being either all even or all odd).

According to the shell model, however, the initial nucleus will accept a particle only in a certain definite orbital angular momentum state. Although the lowest allowed l usually coincides with the value required by the shell model, there are some instances where the reverse is true, i.e., where the shell model l value is 2 units higher than the lowest value allowed by the selection rules. It is in these latter cases that we are interested. If the shell model were precise, the experimental angular distributions for such cases should show no evidence of the maximum nearest the forward direction which would be expected to be present on the grounds of selection rules alone; on the other hand, there has to be only a very small deviation from the shell model before this peak in the angular distribution is as large as the following one. The angular distributions will therefore effectively amplify by a factor $\sim$10 any admixture of the lower orbital angular momentum state in the wave function of the final nucleus, as well as any admixture of states in the initial nucleus which allow the low orbital angular momentum transfer. Such angular distributions should, therefore, provide a sensitive measure of the accuracy with which a nucleon in a nucleus can be ascribed a definite orbital angular momentum.

Some examples of reactions which satisfy the above requirements, and for which therefore it would be very desirable to have experimental angular distributions, are given in Table I. These refer to formation of the final nuclei in ground states only, since the shell model cannot be expected to give the precise ordering of excited states. In choosing such examples it is, of course, important that the lowest allowed angular momentum does not correspond to an independent particle level which is so very little separated from the predicted ground level that the two could very easily cross. Such a state of affairs occurs, for example, in the shell with neutron or proton numbers between 8 and 20 where the ordering of levels is in general $d_{5/2}$, $s_{1/2}$, $d_{3/2}$, but where the $d_{5/2}$ and $s_{1/2}$ levels are very little separated and are known in fact to