PHYSICAL REVIEW VOLUME 85, NUMBER 1 JANUARY 1, 1952

Magnetic Properties of a Hollow Superconducting Lead Sphere *

JULIUS BABISKIN
Naval Research Laboratory, Washington, D. C.

(Received August 20, 1951)

The magnetic hysteresis of a hollow superconducting lead sphere at 4.2°K has been studied by measuring the magnetic field distributions along the equatorial plane with bismuth probes. After cooling in the absence of a magnetic field, the hollow sphere was found to be a perfect magnetic shield in the superconducting state. The magnetic field penetrated into the interior of the sphere when the magnetic field at the surface of the sphere exceeded the threshhold value. Frozen-in fields were observed upon demagnetizing the sphere from the normal state. Upon reversing the applied field, the frozen-in field could be made to vanish, so that the sphere again became a perfect magnetic shield. The magnitude of the applied field necessary to extinguish the frozen-in field was found to be the same independent of the path of approach. The internal frozen-in fields did not change over specified ranges of the applied field, so that the existence of other equilibrium superconducting states are indicated for the hollow sphere. In the intermediate and frozen-in states, 10 to 30 minutes were required for the magnetic field to reach an equilibrium value.

I. INTRODUCTION

PREVIOUS experiments,[1–4] which studied the magnetic properties of superconducting spheres, did not measure the magnetic fields inside a hollow superconducting sphere. The present paper gives the results of an experimental study of the magnetic properties of a hollow lead sphere in the superconducting, intermediate and frozen-in states. This experimental study was performed at the boiling point (~4.2°K) of liquid helium under normal atmospheric pressure by varying the applied field through the magnetic hysteresis cycle. The magneto-resistance of calibrated bismuth probes was used to measure the equatorial magnetic field distributions inside and outside the hollow sphere. Since the magneto-resistance of bismuth exhibits greater sensitivity at higher magnetic fields, it is advantageous to use a superconductor having relatively high critical magnetic fields (H_c). Therefore, lead ($H_c \cong 540$ gauss at 4.2°K) was chosen as the superconductor.

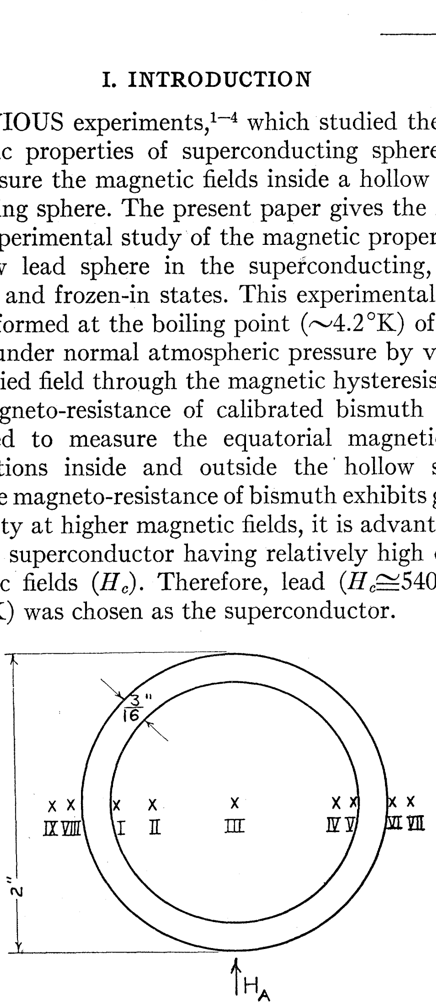

FIG. 1. Cross-sectional view showing positions of the bismuth probes on the equatorial plane of the hollow lead sphere.

II. EXPERIMENTAL METHODS

The bismuth probes were mounted in the fixed positions shown in Fig. 1 and were calibrated in a uniform magnetic field at 4.2°K. Although the magneto-resistance calibration curves for any single bismuth probe were not observed to be reproducible upon cycling from room temperature to 4.2°K. Their shapes were reproducible to form a family of curves. Thus a calibration could be reconstructed to 1 percent from the measurement of a single point on the curve. The probe mounts were then placed on the equatorial plane of the hollow sphere with respect to the applied field (H_A). The magnetic field distributions for the hollow sphere in applied fields below H_c were determined from the graphical extrapolation of normal state measurements using the family of calibration curves. The measured magnetic fields (H_M) as determined from the calibration curves are the absolute values of the average magnetic field over the dimensions of the bismuth probes (0.011 in. diam, 0.16 in. long). When the magnetic field was homogeneous over the dimensions of the probe, the error in H_M was of the order of 1 percent, except for $H_M < 20$ gauss where the effects of thermal electromotive forces become appreciable. For inhomogeneous fields, the error in H_M was greater depending upon the structure of the magnetic field.

The hollow sphere (2 in. o.d., $\frac{3}{16}$ in. wall thickness) was made by welding together two hemispherical lead shells (99.996 percent pure, National Lead Company). The current and potential leads from the internal bismuth probes were brought out of the hollow sphere through a 0.05-in. hole at the pole of the sphere. The effects of the hole have been shown to be minimized there.[3]

The applied magnetic field (H_A) was furnished by a liquid nitrogen-cooled solenoid (16 in. long, 4 in. i.d.), which surrounded the liquid helium flask. H_A was uniform to within 0.2 percent over the volume of the sphere. The over-all error in the measurement of H_A was less than 0.5 percent. Since the solenoid obstructed

---

* This work is part of a thesis under the direction of Professor M. H. Johnson and submitted in partial fulfillment of the requirements for the M.S. degree at the University of Maryland.

[1] W. J. de Haas and A. Guinau, Physica 3, 182 and 534 (1936).

[2] K. Mendelssohn and J. Babbitt, Proc. Roy. Soc. (London) **151**, 316 (1935).

[3] A. Shalnikov, J. Phys. U.S.S.R. **6**, 53 (1942).

[4] de Haas, Engelkes, and Guinau, Physica **4**, 494 (1937).

direct observation, a float[5] was used to indicate the liquid helium level.

III. EXPERIMENTAL RESULTS AND DISCUSSION

A. Complete Hysteresis Cycles

The magnetization curves for the interior of the hollow sphere are shown in Fig. 2. In the superconducting state along OA, the hollow sphere was a perfect magnetic shield. Complete reversibility was maintained along OA. The first detectable penetration of the magnetic field into the hollow section of the sphere occurred at A. In the intermediate state along AB, the maxima of the internal magnetic field distributions were at the center of the sphere. The sphere went completely into the normal state at B.

Figure 3 shows a magnetization curve for the exterior of the hollow sphere. The observed slope of OA was 1.45 as compared to the theoretical value of 1.5. This difference can be accounted for on the basis of experimental limitations. H_M became equal to H_c at A and remained equal to H_c along AB.

The magnetization curves in Figs. 2 and 3 were observed to be time-dependent[4, 6] both in the intermediate state and along BD, which represents the frozen-in state. All plotted points are equilibrium values of H_M at constant H_A. Along AB, from 30 to 40 minutes were required to reach equilibrium for each increment of H_A. Along BD, the time dependence varied from 30 minutes near B to 10 minutes at $H_A=0$. No time dependence was observed along BD from $H_A=0$ to D. For every time dependent increase or decrease of H_M inside the sphere, there was a simultaneous time-dependent decrease or increase respectively of H_M outside the sphere.

The equilibrium values of H_M outside the sphere along AB in Fig. 3 were equal to H_c. When H_A was given an increment from an equilibrium value along AB, H_M would slightly exceed H_c at first and would then decrease with time until H_M again equaled H_c. Simultaneously, there was a time-dependent increase of H_M inside the sphere, in which the total change was a much larger percentage of the equilibrium value of H_M. This may be interpreted as the resistance of the sphere to the penetration of H_A into the interior, when it is in the intermediate state.

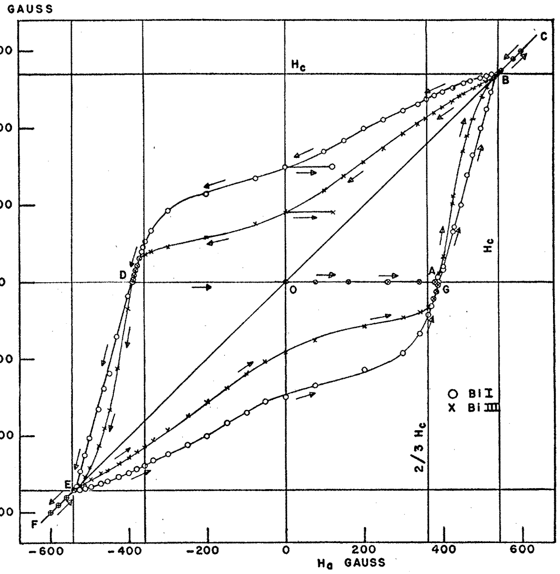

FIG. 2. Magnetization curves for the interior of the hollow sphere. Bi I at the inside edge; Bi III at the center. The magnetization curves for the other internal bismuth probes of Fig. 1 fall within the envelope of the above curves.

Upon demagnetizing the sphere from the normal state, the initial magnetization OAB was not reversible, as would be the case for an ideal solid sphere. The hollow sphere exhibits magnetic hysteresis in going into the frozen-in state. Frozen-in fields are inherently linked to multiply-connected geometries by London theory.[7] Although the hollow sphere is topologically a singly-connected body, frozen-in fields were observed for the hollow sphere. This suggests the enclosure of non-superconducting regions within the superconducting material, so that the hollow sphere in the frozen-in state may be considered to be a multiply-connected body with respect to the superconducting material.

The minima of the internal magnetic field distributions in the frozen-in state were at the center of the hollow sphere. In Fig. 2, a relatively large frozen-in field having the same direction of the original H_A was observed at $H_A=0$. Upon reversing H_A, the internal frozen-in field was reduced and finally disappeared at

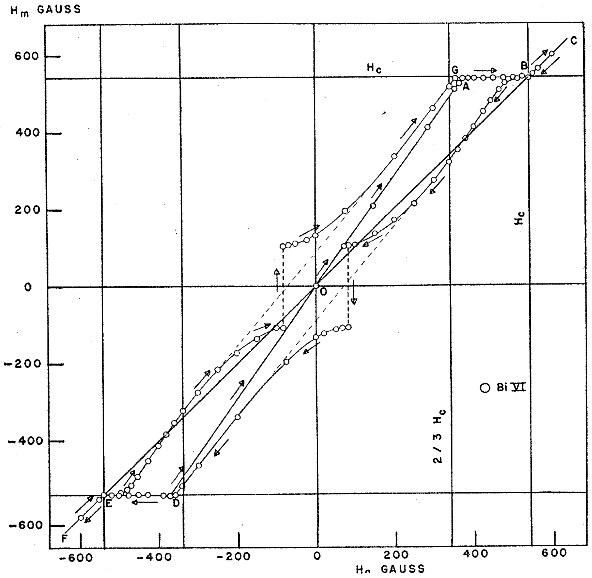

FIG. 3. A magnetization curve for the exterior of the hollow sphere. Bi VI at the outside edge. The magnetization curves for the other external bismuth probes of Fig. 1 were similar to the above curves.

---

[5] J. Babiskin, Rev. Sci. Instr. **21**, 941 (1950).

[6] W. Tuyn, Leiden Communications No. **198** (1929).

[7] F. London, *Superfluids* (John Wiley & Sons, Inc., New York, 1950), Vol 1, p. 47.

D. The curves for all five internal probes converged at *D*, where a uniform distribution of $H_M=0$ was observed. Continuing the cycle from *D*, the intermediate state commences again along *DE* and the complete symmetric hysteresis cycle *CBDEFEGBC* can be repeated.

In Fig. 3 along *BD*, a high degree of inhomogeneity of H_M existed over the dimensions of the external bismuth probes. Since the probes only measure the absolute value of the magnetic field, the reversal of H_M along *BD* was determined by observing a resistance minimum followed by a continuous increase of resistance until the normal state was reached. The discontinuity in *BD* has been drawn in at the resistance minimum. The dashed line, which intersects the discontinuity in *BD*, is an approximation of the average H_M in this region.

Outside the sphere along *BD*, a frozen-in field in the opposite direction to the original H_A was observed at $H_A=0$. The symmetric hysteresis cycle in Fig. 3 can also be repeated.

B. Restoration of Superconductivity

Upon increasing H_A from $H_A=0$ gauss in Fig. 2, the initial horizontal sections of the branch curves emanating from *BD* show that the internal frozen-in field was unchanged up to $H_A\cong 120$ gauss. This indicates that superconducting currents are induced on the exterior surface of the hollow sphere in the frozen-in state. For $H_A>120$ gauss, the internal frozen-in field was changed and the curves merged into their respective curves in the intermediate state.

Observations were made on branch curves emanating from points along *BD* other than those at $H_A=0$ in Fig. 2. All these curves exhibited initial horizontal sections which became progressively longer as the starting point approached *D*. The length of these initial horizontal sections indicates the extent of magnetic shielding in the frozen-in state. Finally, the initial horizontal section of the branch curve emanating from *D* is *DOA*, so that the observed internal uniform distribution of $H_M=0$ gauss at *D* is shielded upon the removal of H_A. Thus a uniform distribution of $H_M=0$ gauss exists again at point *O*, which was the initial condition of the experiment. The hollow sphere is again a perfect magnetic shield as in the superconducting state. Therefore, the initial conditions of the superconducting state has been restored inside the sphere and the cycle *OABCBDO* can be repeated.

Upon removing H_A from point *D* in Fig. 3, the curves for all four external probes converged at point *O* resulting in a uniform distribution of $H_M=0$ again outside the sphere at that point. Therefore the initial conditions for the superconducting state have been restored outside the sphere and the cycle *OABCBDO* in Fig. 3 can also be repeated.

The point *D* occurred at $H_A\cong 390$ gauss in Fig. 2 and at $H_A\cong 374$ gauss in Fig. 3. Therefore, the restoration of the superconducting state at point *O* was not simultaneous for the interior and the exterior of the sphere. Some preliminary experiments were performed with a hollow sphere in which the hemispherical shells were poorly welded. In this case, the initial conditions of the superconducting state were restored simultaneously inside and outside the sphere. Upon rewelding the poorly welded hollow sphere, the magnetization curves were in good agreement with those shown in Figs. 2 and 3.

In Fig. 2, a number of minor hysteresis cycles were observed by generating branch curves from the intermediate state along *AB* by decreasing H_A. These branch curves also exhibited initial horizontal sections, which became progressively shorter as their starting points approached the midpoint of *AB*. It was found that all the observed branch curves emanating from *AB* for all the internal bismuth probes converged at *D*, so that all curves in the upper half of Fig. 2 cross the $H_M=0$ axis at that point. Therefore, all frozen-in fields vanish at *D* regardless of the path of approach. By symmetry, all curves in the lower half of Fig. 2 cross the $H_M=0$ gauss axis at *G*. Thus, *B*, *D*, *E*, and *G* are four focal points into which all the branch curves converged.

Upon approaching the focal points *D* and *G* from the frozen-in state, one can branch off either into the initial superconducting state or the intermediate state. It is noted that the initial superconducting state *DOA* is a bounding horizontal section between the upper and lower halves of Fig. 2 for the initial horizontal sections of the families of branch curves which emanate from the intermediate and frozen-in state. Therefore, the existence of equilibrium superconducting states other than *DOA* are indicated for the hollow sphere, and *DOA* can be considered as the limiting case for these states.

C. Magnetic Shielding of Internal Applied Fields

An experiment was performed to determine the effects of a magnetic field applied by a small field coil in the hollow section of the sphere. The magnetic field distribution of the small field coil was first determined in the absence of the hollow sphere. The small field coil was then placed at the center of the hollow sphere and measurements were made on the magnetic field along the equatorial plane.

No measurable magnetic fields due to the small field coil were detected just outside the hollow sphere in the superconducting state, although a maximum magnetic field of about 30 gauss was detected in the same position in the absence of the hollow sphere. For a given current in the small field coil, the magnitude of the internal magnetic field distributions was greater in the presence than in the absence of the hollow sphere. Thus, to the accuracy of the measurements, the exterior of the sphere is completely shielded from magnetic fields applied in the interior of the sphere.

The author wishes to express his appreciation to Dr. R. L. Dolecek and Dr. J. de Launay for having suggested this problem, and to M. C. Steele, Dr. R. T. Webber, and M. F. M. Osborne for profitable suggestions and discussions.