measured by total reflection agree sufficiently well with those from transmission and crystal diffraction methods to verify the applicability of the usual formulas for solids or liquids, involving coherent cross section, to calculating the index of neutron refraction of a gas. Cross section values were determined for He, A, and N as listed in Tables II and III.

ACKNOWLEDGMENTS

The author is indebted to G. W. Johnson for his collaboration in many of the measurements, to E. O. Wollan of Oak Ridge National Laboratory for the use of facilities and apparatus there and for many valuable suggestions, and to other members of the Oak Ridge and Brookhaven staffs for assistance in the experiments.

PHYSICAL REVIEW VOLUME 84, NUMBER 5 DECEMBER 1, 1951

The Significance of the Absence of Primary Electrons for Theories of the Origin of the Cosmic Radiation

T. M. DONAHUE*
Johns Hopkins University, Baltimore, Maryland
(Received July 11, 1951)

Electrons of energy above 5 Bev appear to constitute less than 0.4 percent of the primary cosmic radiation incident on the earth. An analysis of acceleration mechanisms reveals no distinctions can readily be made in acceleration per se on the basis of sign of charge or mass. The absence of high energy electrons must be explained on the basis of selective absorption. Bremsstrahlung collisions in the galaxy or the solar system and radiation caused by motion in galactic or local magnetic fields are inadequate to account for the large absorption of electrons compared with heavy particles. In collisions between energetic electrons and thermal photons losses approaching the total electron energy occur. An analysis of such collisions reveals that if cosmic rays are confined to the solar system these collisions are so frequent that no electrons should be present at energies higher than 5 Bev. The photon density is too low in interstellar space to cause a similar removal of electrons there. These results favor the solar or stellar origin theories of the cosmic radiation.

I. INTRODUCTION

EXPERIMENTAL evidence is by this time preponderantly against the presence of electrons at energies greater than 5 Bev in detectable numbers among the primary cosmic-ray particles incident on the earth.[1,2] An effect so gross as to exclude completely high energy electrons from the spectrum at the earth should, it would seem, be accounted for unambiguously by any successful theory for the origin of the cosmic radiation. Such an effect is to be sought in an accelerating mechanism which is capable of discriminating against particles on the basis of their mass or, perhaps, their charge, or else in a form of energy degradation which is selective for electrons and can either compete effectively with the acceleration or remove most of the electrons from the high energy spectrum before they reach the earth.

A. Acceleration Mechanisms

To require an accelerator which almost completely discriminates against the emergence of electrons comparable in number and in energy to protons and heavier nuclei seems quite objectionable. Perhaps the most likely requirement for a generator which would discriminate is a minimum injection energy which electrons would not in large numbers be capable of attaining. This, however, merely removes the difficulties to a different range of energies. The only obvious process by which nuclei or nuclear fragments can attain moderately high energies without similar electron acceleration would require already the existence of energetic bombarding particles.

For example, it can be noted that the two most recent proposals for the origin of cosmic rays[3-5] should provide equally well for electron and heavy particle acceleration. Where particles are accelerated in the galaxy by collisions with wandering regions of high magnetic field strength[3] a minimum injection energy is required, it is true, but, as will be shown, this energy is about the same for electrons and protons and, after injection, electrons and protons should be equally well accelerated. On the other hand, there is no apparent reason that electrons should not be available equally with positive ions among the initial particles in a system of local solar or stellar origin.[4,5] The mechanics of the betatron type of accelerator envisioned by Alfvén[5] apply as well to electrons as to heavier particles.

Therefore, it would appear that the explanation for the missing electronic component is most likely to be found by an analysis of the various ways in which

* Now at the University of Pittsburgh, Pittsburgh, Pennsylvania.
[1] R. Hulsizer, Phys. Rev. **76**, 164 (1949).
[2] Critchfield, Ney, and Oleska, Phys. Rev. **79**, 402 (1950).
[3] E. Fermi, Phys. Rev. **75**, 1169 (1949).
[4] R. D. Richtmeyer and E. Teller, Phys. Rev. **75**, 1729 (1949).
[5] H. Alfvén, Phys. Rev. **75**, 1732 (1949); **77**, 375 (1950).

particles can lose energy in space before they reach the earth. The purpose of the present paper is to show that the only energy losses frequent and serious enough to remove electrons above a few Bev from the spectrum incident on the earth are those caused by collisions between high energy electrons and thermal photons, and then only when cosmic-ray particles are forced to remain for a long time in bound orbits close to the sun or, at least, to the stars from which, then, they must originate. The absence of electrons thus argues strongly in favor of the theories of local origin for the cosmic radiation.

B. Types of Absorption Processes

The principal means by which particles in interstellar space or in local regions near stars can lose energy are the following: 1. Collisions with matter; 2. Radiation produced by acceleration in extended magnetic fields; 3. Compton collisions with photons.

Each of these will be considered as a possible cause of absorption selective for electrons in various parts of the universe.

II. BREMSSTRAHLUNG OF ELECTRONS IN INTERSTELLAR MATTER

If it is assumed that interstellar matter consists chiefly of protons with a density of 10^{-24} g/cm^3, then a rather generous value for the radiation length of 100 g/cm^2 will give a mean free path of electrons for radiative collisions of 10^{26} cm. This is of the same order of magnitude as the mean free path for proton-proton collisions taken by Fermi[3] to be

$$\Lambda=(\rho\sigma)^{-1}\cong 1/2\times 10^{-26}=5\times 10^{25}\text{ cm.} \tag{1}$$

Thus this distance, 10^8 light years, is no shorter than that which must be considered the absorption free path of the other particles in the cosmic radiation.

III. RADIATION OF ELECTRONS IN GALACTIC AND LOCAL MAGNETIC FIELDS

The power radiated by electrons at cosmic ray energies in a magnetic field is most conveniently written

$$P=\tfrac{2}{3}c\left(\frac{e^2}{mc^2}\right)^2 H^2\left(\frac{U}{mc^2}\right)^2\text{ erg/sec}$$

$$\cong 10^{-3}H^2(U/mc^2)^2\text{ ev/sec} \tag{2}$$

where U, the electron energy, is assumed to be much larger than mc^2, and the magnetic field, $\mathbf{H}$, and the electron velocity, $\mathbf{v}$, are taken to be perpendicular to each other. Thus, for a galactic magnetic field assumed as strong as 10^{-10} gauss an electron of 10^{14} ev energy radiates only at the rate of 4×10^{-7} ev sec^{-1}, or of 10^{12} ev at the rate of 4×10^{-11} ev sec^{-1}. Even if the average magnetic field through which the electrons travel in the galaxy were as high as 10^{-6} gauss the rate at which energy would be lost by a 10^{12} ev electron would be only 4×10^{-3} ev sec^{-1}, and almost 10^7 years would be required to reduce it to 10^{11} ev. Fermi[3] requires that the rate of energy gain be given by

$$dU/dt=2.5\times 10^{-14}U \tag{3}$$

in order that it compete successfully with recognized absorption losses to yield the observed primary energy spectrum. Equation (2) may be written

$$-dU/dt\cong 4\times 10^{-15}H^2U^2$$

so that any combination of H and U such that

$$H^2U<5$$

would permit electrons to attain that value of U in the cosmic radiation. Thus if H is 10^{-10} gauss, $U<5\times 10^{20}$ ev could be attained. A magnetic field of about 5×10^{-5} gauss would be necessary to exclude the acceleration of electrons to energies greater than 2.5×10^9 ev.

During Fermi's acceleration process electrons would travel in fields presumably as strong as 10^{-5} gauss at the time of their collisions with high field strength clouds. The fractional energy gained in the collision is

$$\Delta U/U\cong (V/c)^2\cong 10^{-8} \tag{4}$$

where V is the velocity of the cloud with respect to the earth.[3] In contrast to this must be put the energy radiated, (2),

$$-\Delta U\cong 4\times 10^{-15}H^2U^2\Delta t$$

where Δt is the collision time. If Δt is put at 0.1 light year, 3×10^6 sec,

$$-\Delta U/U\cong 1.2\times 10^{-20}U. \tag{5}$$

It is necessary that U be less than 10^{12} ev if this loss is not to be serious.[6]

Thus radiation caused by acceleration of electrons in interstellar magnetic fields might place an upper limit of 10^{12} ev on the energy of electrons in the primary cosmic radiation, if collisions with magnetohydrodynamic fields are the source of cosmic-ray energy. Other types of acceleration which also permit cosmic rays to travel throughout the galaxy would not have even this limitation if the permanent galactic magnetic field is any weaker than 10^{-5} gauss.

Close to the sun and the earth magnetic fields much stronger than this are eventually encountered. Travel times are ordinarily short, however, and Pomeranchuk[7] has shown, by integrating Eq. (2), that while electrons with energy initially above 10^{17} ev would be reduced to about 10^{17} ev by the time they struck the high atmosphere, electrons below 10^{17} ev would be little affected.

[6] The radiation loss is probably over-estimated because of the assumption that the electron is moving perpendicular to the field. Furthermore 10^{-5} gauss seems perhaps an order of magnitude too large for the average field during collision. The value of Δt chosen would appear to be an upper limit if the mean time between collisions is only 1.3 years.

[7] I. Pomeranchuk, J. Phys. (U.S.S.R.) 2, 65 (1940).

TABLE I. Energy radiated per second, P, and per year, P' for electrons in troichoidal orbits of energy U. Values are listed for two different assumptions concerning the solar dipole moment a. Also tabulated is the fractional energy lost per year, P'/U, and the time, t, required for the energy of an electron to decrease from U_0 to U, where $U_0 \gg U$, because of radiation in such orbits.

Sun's dipole moment	U ev	P ev sec^{-1}	P' ev yr^{-1}	P'/U yr^{-1}	t sec
$a = 1.0\times10^{34}$ gauss cm^3	10^9	3×10^{-9}	9×10^{-2}	9×10^{-11}	10^{17}
$P \cong 3\times10^{-54}U^5$	10^{10}	3×10^{-4}	9×10^{3}	9×10^{-7}	10^{13}
$t \cong 10^{53}U^{-4}$	10^{11}	3×10	9×10^{8}	9×10^{-3}	10^{9}
	10^{12}	3×10^{6}	9×10^{13}	9×10	10^{5}
	10^{13}	3×10^{11}	9×10^{18}	9×10^{5}	10
	10^{14}	3×10^{16}	9×10^{23}	9×10^{9}	10^{-3}
$a = 4.2\times10^{33}$ gauss cm^3	10^9	7×10^{-9}	2×10^{-1}	2×10^{-10}	
$P \cong 7\times10^{-54}U^5$	10^{10}	7×10^{-4}	2×10^{4}	2×10^{-6}	
	10^{11}	7×10	2×10^{9}	2×10^{-2}	
	10^{12}	7×10^{6}	2×10^{4}	2×10^{2}	
	10^{13}	7×10^{11}	2×10^{19}	2×10^{6}	
	10^{14}	7×10^{16}	2×10^{24}	2×10^{10}	

Radiation losses, on the other hand, become very important if the cosmic-ray particles and, in particular, electrons are forced to spend long times in orbits near the sun (or, for that matter, any star with no stronger dipole moment). This is particularly true for high energy electrons which in a dipole field occupy orbits in regions of high magnetic field strength. The relationship between electron energy, U, and the average radius, R, of a stable trochoid in the field of the sun is

$$R^2 = 300(3-2\sqrt{2})a/U \tag{6}$$

where a is the dipole moment of the sun in gauss cm^3 and U is in ev. Thus

$$H^2 = a^2R^{-6} = 7.3\times10^{-6}U^3/a \text{ gauss}^2 \tag{7}$$

and the power radiated by such electrons as a function of energy is, from Eq. (2)

$$P \cong 2.9\times10^{-20}U^5/a \text{ ev/sec.} \tag{8}$$

If $a \cong 1.0\times10^{34}$ gauss cm^3 this energy loss is enormous for the most energetic electrons possible in such orbits—that is, those of 10^{14} ev which travel just outside the sun. But below 2×10^{11} ev the radiation becomes feeble and is inadequate by about two orders of magnitude to account for the lack of electrons if the cosmic radiation originates and is accelerated in the solar system.

The power radiated by electrons in trochoidal orbits for two different assumptions about the solar dipole moment is tabulated in Table I. The fractional energy lost per year is also plotted in Fig. 2, where it may be compared with the fractional gain provided by Alfvén's mechanism for cosmic-ray acceleration. In any reasonable theory of solar or stellar origin these losses might account for a high energy cut off between 10^{11} ev and 10^{12} ev, but not any lower.

Integration of Eq. (8) to obtain the time required for an electron to decline in energy from U_0 to U gives

$$t \cong 10^{53}(U^{-4} - U_0^{-4}). \tag{9}$$

If $U_0 \gg U$ the time required for an electron to reach an orbit associated with energy U is also given in Table I. Nearly 30 years are required to bring an electron out from the 10^{14} ev orbit to the 10^{11} ev orbit by radiation and 3×10^5 years to get it into the 10^{10} ev orbit. Even if there is no competing acceleration in these orbits the energy loss is too slow.

To argue that cosmic rays are accelerated near stars with considerably weaker dipole moment so that the radiation from Eq. (8) is still strong at 10^9 ev leads to two difficulties. First the flux transported by stellar beams is reduced, and the effectiveness of an Alfvén type accelerator becomes doubtful. Second, and more serious, radiation losses even from protons will grow excessive. For Eq. (2) and Eq. (8) become, for protons,

$$P \cong 2.5\times10^{-28}H^2U^2 \tag{9}$$

$$P \cong 1.8\times10^{-33}U^5/a. \tag{10}$$

To make the radiation from a 10^9 ev electron comparable to that from a 10^{11} ev electron which is near a dipole of moment 10^{34} gauss cm^3, it is necessary to reduce the dipole moment to 10^{24} gauss cm^3. But then the power radiated by a 10^{14} ev proton is, from Eq. (10), 1.8×10^{13} ev/sec, and, from a 10^{12} ev proton, 1.8×10^3 ev/sec. This is excessive competition if acceleration is simultaneously occurring and, if not, during the time a fast electron is being reduced to 2×10^9 ev, the protons will be drawn down to at most 2.5×10^{12} ev. This is close to two orders of magnitude too low to agree with the observed highest energy in the singly charged component of the cosmic radiation.

IV. COLLISIONS WITH THERMAL PHOTONS

Another type of energy loss can become almost catastrophic for electrons when their energy is very high, and the probability of its occurring also becomes significantly high in regions where the density of low energy photons is high. This type of loss occurs when a high energy electron collides with a photon of any energy, in particular with photons in the radiofrequency and thermal range.

A. General Theory

It was pointed out some time ago by Feenberg and Primakoff[8] that the collision between an electron whose energy is greater than a few Bev and a low energy photon in the black body radiation from the sun or some other star results in a reduction of the electron's energy in the rest frame of the observer which can be very great indeed. These authors, using Planck distribution functions at 6000°K for the photons and the appropriate approximations to the Klein-Nishina cross sections for photon-electron collision in the rest system of the electron, derived general expressions for the collision rates and energy loss, and applied these to several special cases—of an electron passing through the low density photon distribution in interstellar and

[8] E. Feenberg and H. Primakoff, Phys. Rev. **73**, 449 (1948); see also J. W. Follin, Phys. Rev. **72**, 743(A) (1947).

inter-galactic space and of an electron which comes from far in the galaxy directly to the earth against the stream of solar photons. They showed that, of these three cases, only when the electrons must traverse distances as great as those in inter-galactic space are photon encounters numerous enough to reduce seriously the high energy component of the primary electron spectrum.

Here, for the sake of completeness, the general scheme of low frequency photon-high energy electron encounters will be reviewed, and one other special case—that of an electron which remains for a comparatively long time close to the sun or any other similar star—will be considered in detail.

Consider an electron of energy U in the rest system of the earth passing through a region in which the density of photons is given as $n(\epsilon, \alpha)$, a function of ϵ, the photon energy and α, where $\alpha+\pi$ is the angle between the directions of motion of the electron and the photon. Then, denoting by $'$ quantities measured in the frame of reference in which the electron is initially at rest, the rate at which electrons are scattered is

$$\frac{dN}{dt}=\frac{1}{\gamma}\frac{dN}{dt'}=\frac{c}{\gamma}\int d\Omega'\int_0^\infty n'(\epsilon', \alpha')\sigma'(\epsilon')d\epsilon' \qquad (11)$$

where $\gamma=(1-\beta^2)^{-\frac{1}{2}}$, $\sigma'(\epsilon')$ is the Klein-Nishina total cross section for the scattering of a photon of energy ϵ' by an electron at rest,

$$\epsilon'=\gamma\epsilon(1+\beta\cos\alpha), \qquad (12)$$

and $n'(\epsilon, \alpha')d\epsilon'd\Omega'$ is the number of photons per cm^3 with energy between ϵ' and $\epsilon'+d\epsilon'$ travelling initially within a cone of solid angle $d\Omega'$. In virtue of the invariance of

$$[\epsilon n(\epsilon, \alpha)d\epsilon d\Omega]^{\frac{1}{2}}/\epsilon,$$

Eq. (11) may also be written

$$dN/dt=c\int d\Omega\int_0^\infty n(\epsilon, \alpha)(1+\beta\cos\alpha)\sigma'(\epsilon')d\epsilon. \qquad (13)$$

If the energy lost by the electron and gained by the photon in a collision is $(\epsilon_1-\epsilon)$ where ϵ_1 is the energy of the photon after the collision, the rate of energy loss by the electron is

$$-dU/dt=c\int d\Omega\int_0^\infty n(\epsilon, \alpha)(1+\beta\cos\alpha)\times\int\sigma'(\epsilon', \theta')(\epsilon_1-\epsilon)d\Omega'd\epsilon. \qquad (14)$$

Here $\sigma'(\epsilon', \theta')d\Omega'$ is the differential Klein-Nishina cross section, where θ' is the angle of scattering for the photon and Ω' the corresponding solid angle.

In the rest frame of the electron the usual Compton relation

$$\epsilon_1'=\epsilon'\Big/\left[1+\frac{\epsilon'}{mc^2}(1-\cos\theta')\right] \qquad (15)$$

holds. By virtue of Eq. (12) and the fact that for $\beta\cong 1$, α' is a very small angle, this leads to

$$\epsilon_1=\frac{\gamma\epsilon'(1-\beta\cos\theta')}{1+(\epsilon'/mc^2)(1-\cos\theta')}. \qquad (16)$$

In those cases in which the energy of the photon in the electron's frame is very high, that is, because of (12),

$$\epsilon'=(U\epsilon/mc^2)(1+\beta\cos\alpha)\gg mc^2 \qquad (17)$$

the Klein-Nishina differential cross section reduces to

$$\sigma'(\epsilon', \theta')d\Omega'\cong\tfrac{1}{2}(e^2/mc^2)^2(\epsilon_1'/\epsilon')d\Omega'. \qquad (18)$$

The condition (17) can, of course, be satisfied because U is high, ϵ is high, or because α is sufficiently small. The value (18) for the cross section together with proper attention to (17) makes it possible to write for the contribution to Eq. (12) by photons and electrons satisfying the condition $\epsilon'\gg mc^2$

$$-\left[\frac{dU}{dt}\right]^*\cong c\int d\Omega\int_{\epsilon_a}^\infty n(\epsilon, \alpha)(1+\beta\cos\alpha)\times\left[2\pi r_0^2\frac{mc^2}{2\epsilon'}\ln\left(1+\frac{2\epsilon'}{mc^2}\right)\times U\left\{1-\frac{2\epsilon'/mc^2}{(1+2\epsilon'/mc^2)\ln(1+2\epsilon'/mc^2)}\right\}\right]d\epsilon \qquad (19)$$

where $\epsilon_a\gg(mc^2)^2/U(1+\cos\alpha)$ and r_0 is the classical electron radius.

Likewise, for the contribution of these collisions to dN/dt, the total cross section to this approximation

$$\sigma'(\epsilon')\cong 2\pi r_0^2(mc^2/2\epsilon')\ln(2\epsilon'/mc^2) \qquad (20)$$

must be used. Then

$$\left[\frac{dN}{dt}\right]^*\cong c\int d\Omega\int_{\epsilon_a}^\infty n(\epsilon, \alpha)(1+\beta\cos\alpha)\times\left[2\pi r_0^2\frac{mc^2}{2\epsilon'}\ln\frac{2\epsilon'}{mc^2}\right]d\epsilon. \qquad (21)$$

From (19) and (21) it is clear that the average energy lost by an electron in one collision is, in this case,

$$\Delta U\cong U\left\{1-\frac{2\epsilon'/mc^2}{(1+2\epsilon'/mc^2)\ln(1+2\epsilon'/mc^2)}\right\}. \qquad (22)$$

On the other hand for such photon, electron combinations that

$$\epsilon'=U\epsilon(1+\beta\cos\alpha)/mc^2\ll mc^2, \qquad (23)$$

the appropriate approximations to the Klein-Nishina cross sections are

$$\sigma'(\epsilon', \theta')d\Omega'\cong\tfrac{1}{2}r_0^2(1+\cos^2\theta')d\Omega' \qquad (24)$$

and

$$\sigma'(\epsilon')\cong(8\pi/3)r_0^2. \quad (25)$$

The contributions of these low energy collisions to (13) and (14) are

$$\left[\frac{dN}{dt}\right]^{**}\cong c\int d\Omega\int_0^{\epsilon b} n(\epsilon,\alpha)(1+\beta\cos\alpha)\left[\frac{8\pi}{3}r_0^2\right]d\epsilon \quad (26)$$

and

$$-\left[\frac{dU}{dt}\right]^{**}\cong c\int d\Omega\int_0^{\epsilon b} n(\epsilon,\alpha)(1+\beta\cos\alpha)\times\frac{8\pi}{3}r_0^2\frac{\gamma\epsilon'}{1+\epsilon'/mc^2}d\epsilon. \quad (27)$$

The average energy lost in one collision is thus, clearly,

$$\Delta U\cong\gamma\epsilon'/[1+(\epsilon'/mc^2)]. \quad (28)$$

B. Cosmic Rays Confined to the Solar System Near the Earth

Four special cases of interest will now be considered. One is that of an electron executing a circular orbit about the sun at a distance equal to that of the earth from the sun. The energy lost by such an electron will be roughly equivalent to that lost in a stable trochoid of the same dimensions. In this case the angle between the photon and electron trajectories will be fixed at $\pi/2$. These results will then be extended to the case of electrons moving in the trochoids properly belonging to the energy U. The two other cases were treated by Feenberg and Primakoff.[8] One is that of an electron passing through a sea of isotropically moving photons of low density. This is equivalent to the state of affairs in interstellar space. The other case is that of an electron which comes into the earth from far out in the galaxy under the condition $\alpha=0$.

At a distance R_e from the sun, where R_e is the radius of the earth's orbit, the density of photons from the sun in the energy interval between ϵ and $\epsilon+d\epsilon$ is

$$n(\epsilon)d\epsilon=C\epsilon^2d\epsilon/(e^{\epsilon/kT}-1) \quad (29)$$

where

$$C=(15/\pi^4)(\eta/c(kT)^4) \quad (30)$$

and $\eta=1.94$ cal/min cm² is the solar constant at the earth. The total number of photons per cm³ at R_e is, therefore,

$$n_{R_e}=\int_0^\infty n(\epsilon)d\epsilon=2\pi^3c(kT)^3/25.8=1.9\times10^7\text{ cm}^{-3} \quad (31)$$

for $T=6000°$K, $kT\cong0.52$ ev. The energy density is

$$\rho_{R_e}=\eta/c=2.7\times10^7\text{ ev/cm}^3 \quad (32)$$

and the average energy of these photons is

$$\bar{\epsilon}_{R_e}=\rho_{R_e}/\eta_{R_e}=1.42\text{ ev}=2.73kT. \quad (33)$$

To compute the energy which a fast electron of energy U may be expected to lose because of collisions with sunlight when it is circling the sun in an orbit of radius R_e we set $\alpha=\pi/2$ in (11) and (13). In this case, from (12), a photon of energy ϵ will have, in the rest frame of the electron, an energy

$$\epsilon'=\gamma\epsilon=U\epsilon/mc^2. \quad (35)$$

For the purpose of being definite, the maximum photon energy to be considered in the case $\epsilon'\ll mc^2$ will be $\epsilon^*/4$ where

$$\epsilon^*=\epsilon^*(U)=(mc^2)^2/U. \quad (36)$$

Since, for an electron of $U=2.5\times10^{10}$ ev

$$\epsilon^*/4=0.1\text{ ev}$$

practically all the photons present will be accepted for $U<2\times10^{10}$ ev.

In this case, Eq. (26) becomes

$$\left[\frac{dN}{dS}\right]^{**}\cong\frac{8\pi}{3}r_0^2\int_0^{\epsilon=(mc^2)^2/4U} n(\epsilon)d\epsilon$$

$$\cong6.67\times10^{-25}n_{R_e}\left\{1-\frac{25.8}{2\pi^3}\sum_{j=1}^{\infty}\left[\left(\frac{1.3\times10^{11}}{U}\right)^2\frac{1}{j}+\frac{2.6\times10^{11}}{U}\frac{1}{j^2}+\frac{2}{j^3}\right]\exp(-1.3\times10^{11}j/U)\right\} \quad (37)$$

where $s=ct$.

The subtractive term within the brackets remains important down to $U\cong2\times10^{10}$ ev, where it has the value 0.055. At $U=10^{11}$ ev this term is 0.8. When U is sufficiently low that the entire photon distribution may be used

$$dN/dS\cong(8\pi/3)r_0^2\int_0^\infty n(\epsilon)d\epsilon\cong6.67\times10^{-25}n_{R_e}$$

$$\cong1.3\times10^{-17}\text{ cm}^{-1}. \quad (38)$$

Thus above 2×10^{10} ev, Eq. (37) does not give the total collision rate but only the contribution of those photons in the distribution whose energy is less than $\epsilon^*/4$. For 2×10^{10} ev electrons these are all photons less than 3.1 volts, but for 10^{11} ev electrons, only those below 0.6 volts.

In each of these collisions the average energy loss is, from (28),

$$\Delta U\cong U^2\bar{\epsilon}_{R_e}/(mc^2)^2\cong5.7\times10^{-12}U^2. \quad (39)$$

At the other extreme are those collisions in which $\epsilon'\gg mc^2$, which here will be taken as those in which the photon's energy in the earth rest frame is not less than

$$4\epsilon^*=4(mc^2)^2/U. \quad (40)$$

For electron energies higher than 4×10^{12} ev, all of the sun's radiation except that in the extreme infrared below 5μ qualifies.

Now (21) becomes

$$\left[\frac{dN}{dS}\right]^* \cong \int_{4\epsilon^*}^{\infty} \pi r_0^2 C \frac{\epsilon\epsilon^*}{e^{\epsilon/kT}-1}\ln\frac{2\epsilon}{\epsilon^*}d\epsilon$$
$$\cong \pi r_0^2 C (kT)^3 u^* \int_{4kTu^*}^{\infty} \left(u\ln\frac{2u}{u^*}\right)\sum_{j=1}^{\infty} e^{-ju}du \quad (41)$$

where $u=\epsilon/kT$.

When $U\geqq 4\times 10^{12}$ ev the lower limit may be safely set at 0. Then

$$\frac{dN}{dS}\cong \pi r_0^2 C(kT)^3 u^* \sum_{j=1}^{\infty}\left[\frac{\Gamma'(2)}{j^2}+\frac{1}{j^2}\ln\frac{2}{u^*}-\frac{\ln j}{j^2}\right]$$
$$\cong \pi^3 r_0^2 C(kT)^2\frac{(mc^2)^2}{U}\left[\tfrac{1}{6}\ln\frac{2UkT}{(mc^2)^2}-0.030\right]$$
$$\cong\frac{9.43\times 10^{-6}}{U}\left[\tfrac{1}{6}\ln\frac{4U}{10^{12}}-0.030\right]. \quad (42)$$

On the other hand, where U is so small that a significant range in ϵ does not contribute to the collision rate in the higher energy region (40), a nonvanishing lower limit of integration must be set in (41). Then

$$\left[\frac{dN}{dS}\right]^*\cong \pi r_0^2 C(kT)^2\frac{(mc^2)^2}{U}\sum_{j=1}^{\infty}\left\{\left[\frac{1}{j}\left(\frac{4(mc^2)^2}{U}+\frac{1}{j}\right)\right.\right.$$
$$\left.\times\ln 8kT+\frac{1}{j^2}\right]\exp[-4j(mc^2)^2/U]$$
$$\left.-\frac{1}{j^2}\,\mathrm{Ei}[-4j(mc^2)^2/U]\right\}$$
$$\cong\frac{9.57\times 10^{-7}}{U}\sum_{j=1}^{\infty}\left\{\left[\frac{1}{j}\left(\frac{10^{12}}{U}+\frac{1}{j}\right)\ln 4.16+\frac{1}{j^2}\right]\right.$$
$$\left.\times\exp(-10^{12}j/U)-\frac{1}{j^2}\,\mathrm{Ei}(-10^{12}j/U)\right\} \quad (43)$$

where

$$-\mathrm{Ei}(x)=\int_x^{\infty}e^{-t}dt/t. \quad (44)$$

For large values of U (and sufficiently small values of j)

$$-\mathrm{Ei}(-10^{12}j/U)\sim 1/\ln(\delta 10^{12}j/U)$$

where

$$\ln\delta\cong 0.577$$

and (43) then reduces to

$$\left[\frac{dN}{dS}\right]^*\cong\frac{9.57\times 10^{-7}}{U}\sum_{j=1}^{\infty}\left[\frac{1}{j^2}\ln\frac{16U}{10^{12}}+\frac{1-\ln\delta}{j^2}-\frac{\ln j}{j^2}\right] \quad (45)$$

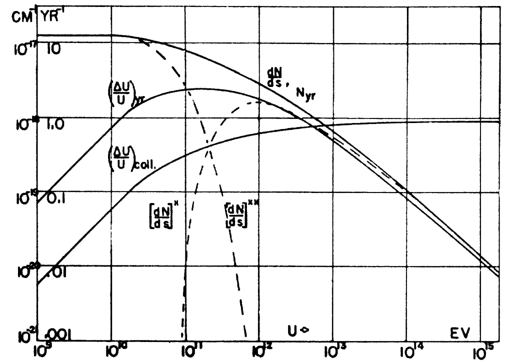

FIG. 1. Collision rate and energy loss for encounters between thermal photons from the sun and an electron of energy U constrained to a circular orbit about the sun of radius equal to that of the orbit of the earth. The solid curve for dN/ds and N_{yr} gives to a good approximation the total collision rate per cm and per year for thermal photons of any energy, calculated from Eq. (38) for $U<2\times 10^{10}$ ev and Eq. (42) for $U>4\times 10^{10}$ ev. The broken curves are for $[dN/ds]^{**}$ and $[dN/ds]^*$, Eq. (37) and Eq. (43), and give the contribution of collisions in which $\epsilon'\ll mc^2$ and $\epsilon'\gg mc^2$.

The average fractional energy loss per collision $\Delta U/U=dU/Udn$ is computed from (39) and (46) for $U<2\times 10^{10}$ ev and $U>4\times 10^{12}$ ev. A smooth connecting curve is assumed.

The average fractional loss per year $(\Delta U/U)_{\mathrm{yr}}$ is computed by multiplying $3\times 10^7 c dN/ds=N_{\mathrm{yr}}$ and $dU/UdN=\Delta U/U$.

which is equivalent to (42). At 10^{14} ev the collision rate calculated from (45) is 9.15×10^{-20} cm^{-1} or 2.75×10^{-9} sec^{-1}.

The average energy lost by an electron in one collision is now, from (22),

$$\Delta U\cong U\{1-[\ln 2U\epsilon R_e/(mc^2)^2]^{-1}\}$$
$$\cong U\{1-[\ln(U/10^{11})]^{-1}\}. \quad (46)$$

Hence, for electrons of energy no greater than 2×10^{10} ev, (38) and (39), and for electrons above 10^{13} ev, (42) and (46) represent the collision frequency and the energy loss per collision with the entire photon output of the sun to an approximation as valid as are the approximations to the Klein-Nishina cross sections. In the range $10^{13}>U>2\times 10^{10}$ ev, neither (37) nor (43), nor their sum gives the total number of collisions per cm. The range of photon energies between $(mc^2)^2/4U$ and $4(mc^2)^2/U$ which there contains a large number of photons is not covered by either one. In this region the full expression for the Klein-Nishina cross sections must be used in (13) and (14), since there ϵ' is of the order of mc^2.

Instead of carrying out such a calculation it is simpler to use the fact that the full expression for the cross sections does not lead to oscillations or discontinuities in (13) and (14) as functions of U and connect with a smooth curve the graphical representations of (38) and (42) for dN/ds as function of U, and of (39) and (46) for $\Delta U/U$ as a function of U. This is done in Fig. 1.

A third curve in Fig. 1 gives the average fractional loss of energy per year and is obtained by multiplying the number of collisions per year, $N_{\mathrm{yr}}=3\times 10^7 c dn/ds$ and $\Delta U/U=dU/Udn$.

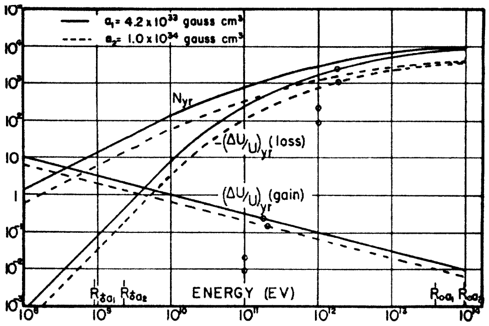

FIG. 2. Number of collisions per year with thermal photons from the sun by electrons with energy U traveling in the bound orbits belonging to that energy. Average fractional energy loss per year owing to these collisions: $-(\Delta U/U)_{\mathrm{yr}}$ (loss) and fractional energy gain per year by the Alfvén acceleration process. The circles are points on the straight line curve giving the fractional loss because of radiation by electrons going in these orbits. Curves are given for two assumptions concerning the dipole moment of the sun. (The lower of each pair of circles corresponds to the lower value of the dipole moment.) The energies corresponding to orbits at the earth's distance $R_{♁}$ and at the sun's rim $R_{\odot}$ are marked.

The collision rate remains high, at 1.27×10^{-17} per cm or 11.4 per year up to a little above 10^{10} ev. For higher energies it drops steadily to 5 per year at 2.4×10^{11} ev, 1 per year at 4.0×10^{12} ev, and only one every 100 years at 1.26×10^{15} ev. On the other hand the average fractional energy lost in each collision rises with U and approaches unity as $U\rightarrow\infty$. Thus at 10^9 ev it is only 5.7×10^{-13}, at 10^{10} ev, 5.7×10^{-2}, at 10^{11} ev, 0.302, at 10^{13} ev, 0.793, and at 10^{15} ev, 0.893. The combination of these two sends the average fractional energy lost per year by an electron forced to rotate about the sun near the earth through a maximum of 2.5 at 1.26×10^{11} ev. It is greater than unity from 1.4×10^{10} ev to 2.4×10^{12} ev, but falls to 7.2×10^{-2} at 10^9 ev and 1.06×10^{-2} at 10^{15} ev.

The contribution of high energy collisions, $[dN/ds]^*$ and of low energy collisions $[dN/ds]^{**}$ from (43) and (37) are also plotted in Fig. 1.

C. Cosmic Rays in Stable Trochoids About the Sun

Acceleration of cosmic radiation in the solar system according to the proposal of Alfvén[5] is accomplished while the particles are trapped in trochoidal orbits in the dipole field of the sun. The radius of these orbits is inversely proportional to the square root of the particle energy according to Eq. (6). Alfvén shows that if the mechanism of acceleration is the betatron action of the magnetic flux transported by solar beams the fractional energy gain per year (neglecting losses) is

$$\Delta U/U \cong 2.08\times10^{-13}R \qquad (47)$$

where $a=4.2\times10^{33}$ gauss cm^3 is taken as the dipole moment of the sun. Thus, orbits in the neighborhood of the earth, where $R=1.45\times10^{13}$ cm and $U\cong10^9$ ev, have a fractional gain of 3 per year, and at $R=4.7\times10^{12}$ cm with $U=10^{10}$ ev the gain is still 1 per year. The most energetic singly charged particles possible in this view are those which move along the solar disk with $R=R_s\cong7\times10^{10}$ cm and $U\cong4.5\times10^{13}$ ev. Here $\Delta U/U$ is only about 0.015 per year.

In the case of electrons this accretion of energy must be contrasted with the rate at which electrons lose energy. It has already been shown that radiation competes ineffectively below 2×10^{11} ev. But the higher the electron energy the closer it moves to the sun. The expressions above for the collision rate and energy losses of electrons encountering sunlight must be modified because the density of photons increases as R^{-2}. Thus, in (29), where R_e is replaced by R,

$$C=C(R)=(R_e/R)^2C_{R_e}=(U/U_0)C_{R_e} \qquad (48)$$

where

$$U_0=9.63\times10^8 \text{ ev}.$$

Thus the expressions for dN/ds and $(\Delta U/U)_{\mathrm{yr}}$ must be multiplied by

$$U/U_0=1.04\times10^{-9}U$$

to give the true collision rate and fractional energy loss per year for electrons in stable trochoidal orbits. Hence (38) and (42) become

$$dN/dS \begin{cases} \cong1.32\times10^{-26}U, \quad U<2\times10^{10} \text{ ev} & (49) \\ \cong1.63\times10^{-15}[\ln(4U/10^{12})-0.030], \quad U>10^{13} \text{ ev} & (50) \end{cases}$$

with similar expressions for (37), (43), and (45).

The frequency of collision rises now to a very high value as U increases toward 10^{14} ev and the electron moves closer and closer to the sun. At 10^{12} ev when $R=4.65\times10^{11}$ cm collisions occur at the rate of 1830 per year. Just outside the sun the rate is about 7750 per year.[9] Since $\Delta U/U$ per collision remains as it is given in Fig. 1, this increasing collision rate forces the fractional energy loss per year to increase steadily with energy. Both the number of collisions per year and the average yearly fractional decrease in energy are plotted in Fig. 2 as a function of the energy in these stable orbits. They are shown for two values of the dipole moment, 4.2×10^{33} gauss cm^3 the value given by Hale and others[10] and 1.0×10^{34} gauss cm^3 the value indicated from considerations of the low energy cosmic-ray cut off at the earth. The energies belonging to stable trochoids at the earth's distance R_e and at the edge of the sun R_s are marked on the abscissa. The rate of energy loss can be compared with the rate of gain given by the betatron mechanism which is also drawn in this figure. The loss exceeds the gain for all energies higher than 4.4 Bev in the case of the lower dipole moment and 5 Bev in the

[9] Because of the failure of the inverse square law near the sun, this is a rather poor approximation, of course.

[10] Hale, Seares, Van Maanen, and Ellerman, Astrophys. J. **47**, 206 (1918); Thiessen, Observatory **36**, 230 (1946).

case of the higher. For energies very little above these values, in fact, the loss is overwhelming so that there is an altogether negligible chance that any electron can penetrate into these energy regions.

On the basis of this simplified scheme of acceleration and degradation the critical energy, 5 Bev is in the band from 2.3 Bev to 13.6 Bev in which we can assume that only particles in bound orbits can arrive at the earth.[11,12] But, whatever is the explanation of this cut off, it should apply to electrons as well as to heavier primaries. Thus, according to this scheme of cosmic-ray production, there are very few electrons in the primary cosmic radiation. Whatever ones there are would lie in the energy interval between the general cosmic-ray cut off and the energy at which degrading collisions with the photons of the sun neutralize the slow gain in the solar betatron. This band is undoubtedly narrow, but there seems to be a definite possibility of electrons incident on the earth at very high latitudes in a very narrow band above 2.2 Bev. Such electrons would perhaps have been missed in experiments so far performed[1,2] which were sensitive only for the electrons incident above 5 Bev. However, the losses are probably underestimated in this treatment, which ignores radio-frequency radiation from the sun as well as the fact that the angle α between the electron and photon trajectories is not a constant. The rate of gain is almost certainly overestimated.[13] Any mechanism which requires electrons to spend a long time close to the sun, whether they are accelerated there or elsewhere, so long as the simultaneous acceleration is less than 0.2 U at 2.3×10^9 Bev, will lead to a complete absence of electrons at the earth.

The cross section of the earth for cosmic rays[12] is far too small to compete with a cross section which allows 11 collisions every year, each of them removing 0.02 U of the electron's energy.

High energy quanta resulting from these encounters certainly should be incident on the earth, but since they cannot be trapped in the solar system, the intensity is expected to be very low.

D. Cosmic Rays Confined Only to the Galaxy

If the cosmic radiation is considered to fill up the entire galaxy, the electrons present should collide in interstellar space with thermal photons from the stars. Feenberg and Primakoff[8] have shown in effect that despite the isotropy of radiation there the photon density is far too low to provide a serious drain even on electron energy.

In the calculation of the effects of Compton collision processes in interstellar space it is most simple to take the usual value for the energy density of radiation in the galaxy[14]

$$\rho_G = 0.3 \text{ ev cm}^{-3} \tag{51}$$

and consider all of these photons to come from sources at 6000°K. Then the density of photons in the galaxy becomes

$$n_g = \rho_g/\bar{\epsilon} \cong 0.3/1.42 = 0.21 \text{ cm}^{-3} \tag{52}$$

and, because of the isotropy,

$$n(\epsilon, \alpha) = n_g(\epsilon)/4\pi.$$

A cosmic-ray electron will suffer collisions with these photons at a rate given by (11), whence, for $\epsilon' \ll mc^2$

$$\begin{aligned} dN/ds &\cong \tfrac{1}{4}[(8\pi/3)r_0^2]n_g, \quad U<2\times10^{10} \text{ ev} \\ &\cong 3.3\times10^{-26} \text{ cm}^{-1}. \end{aligned} \tag{53}$$

One collision with a loss of energy given by (39) every 3×10^7 years does not exceed the collision rate given by (1) for $p-p$ meson producing collisions in interstellar space.

When $\epsilon' \gg mc^2$ the time between collisions is, naturally, even longer. In fact

$$\begin{aligned} \left[\frac{dN}{dS}\right]^* \cong \int_0^{2\pi} d\phi \int_0^{\pi} \int_{4\epsilon^*}^{\infty} \frac{n_g(\epsilon)}{4\pi}(1+\beta\cos\alpha) \\ \times \sin\alpha \left[2\pi r_0^2 \frac{mc^2}{2\epsilon'} \ln\frac{2\epsilon'}{mc^2}\right] d\epsilon d\alpha. \end{aligned} \tag{54}$$

When $U \geqq 10^{13}$ ev this may be written

$$\begin{aligned} \frac{dN}{dS} &\cong \frac{1}{2}\frac{C}{C_{R_e}}\left(\frac{dN}{dS}\right)_{R_e,\,\pi/2}(1-1/\ln[2UkT/(mc^2)^2]) \\ &\cong \frac{4\times10^{-14}}{U}\left(\ln\frac{4U}{10^{12}}-1\right) \end{aligned} \tag{55}$$

where

$$C/C_{R_e} = n_g/n_{R_e} \cong 10^{-7}. \tag{56}$$

The collision rate decreases steadily with U. At $U=4\times10^{12}$ ev it is one in 6×10^7 years, at 4×10^{13} ev, one in 3×10^8 years. In the galaxy these collisions are no more serious for electrons than other types of collisions for the other constituents of the primary cosmic radiation.

E. Electron Traveling to Earth from Far Out in the Galaxy

An electron which strikes the earth after travelling from a point distant R from the sun directly counter to the sun's rays does not make enough collisions with this light to disappear from the high energy spectrum. Feenberg and Primakoff[8] also calculate the loss in this case by setting $\alpha=0$ and obtaining expressions as func-

[11] H. Alfvén, Phys. Rev. 72, 88 (1947).
[12] E. O. Kane, T. J. B. Shanley, J. A. Wheeler, Revs. Modern Phys. 21, 51 (1949).
[13] H. Alfvén, Phys. Rev. 77, 379 (1950).
[14] T. Dunham, Proc. Am. Phil. Soc. 81, 277 (1939).

tions of s, the instantaneous distance of the electron from the sun. Thus

$$n(\epsilon, s) = n_{R_e}(\epsilon)(R_e/s)^2 \tag{57}$$

and

$$dN/ds \cong (16\pi/3)r_0^2 n_{R_e}(R_e/s)^2, \quad U<4\times10^{10}\text{ ev}$$
$$N \cong 2.5\times10^{-17}[R_e-(R_e/R)^2] \tag{58}$$

$$\cong 3.7\times10^{-4} \qquad \text{for} \quad R\gg R_e. \tag{59}$$

In each collision the energy lost, from (28)

$$\Delta U \cong 2U^2\bar{\epsilon}/(mc^2)^2 \cong 11U^2/10^{12}. \tag{60}$$

This is a large loss. At 10^{10} ev, it is 1.1×10^9 ev. But the probability of collision is still too low to produce a discernible effect on the electron spectrum in this energy region.

Similarly

$$\frac{dN}{dS} \cong \left(\frac{8\times10^{-7}}{U}\ln\frac{8U}{10^{12}}\right)\left(\frac{R_e}{s}\right)^2, \quad U>10^{13}\text{ ev} \tag{61}$$

$$N \cong (8\times10^{-7}/U)R_e \ln(8U/10^{12}). \tag{62}$$

For $U=10^{13}$ ev this is only 5×10^{-6} collisions.

Thus cosmic-ray electrons would suffer Compton losses sufficient to remove them from the incident primary radiation at the earth if all, or almost all, the electrons can reach it only after travelling for a long time in a succession of bound orbits inside the earth's, occupying each long enough to suffer a degrading collision. At 10^{14} ev the time between collisions is (Fig. 2) $3\times10^7/3660=8\times10^3$ sec and the period 20 $R/c=140$ sec. So after 57 revolutions the electron would suffer a collision which reduces it to 1.5×10^{13} ev. If it should go then into the proper orbit it will suffer another collision in 1.3×10^4 sec after 360 revolutions and go out to the 3.3×10^{12} ev orbit. Finally in the 5×10^9 ev orbit the collision time is only up to 0.1 year. The time involved is quite short compared to the average life time of cosmic rays in the solar system, $\sim$5000 years.[12]

But there are two serious objections. One is obvious: for electrons as well as heavy particles all orbits from infinity are permitted to reach the earth for energies greater than 13.6 Bev. There is no apparent reason that these orbits should not produce, as is now believed, by far the greater contribution to the observed primary intensity at the earth. Even those high energy electrons which reach the earth after passing close to the sun have a chance of less than 1/57 of meeting a photon near the sun's disk. The second objection is that if cosmic rays are forced to come to the earth only after entering bound orbits then protons as well as electrons must suffer this restriction. Protons are, of course, observed. So, therefore, should electrons, and these in large numbers even if they are crowded into the low energy end of the spectrum. The low energy cut off mechanism should not be applicable if the electrons are forced to reach the earth from far inside its orbit.

V. CONCLUSIONS

Energy losses among high energy electrons which are specific for them alone appear to be on a scale large enough to account for the absence of high energy electrons incident among cosmic ray particles on the earth only when cosmic rays have a chance to encounter large numbers of low energy photons during their life time. This restricts theories of the origin of the cosmic radiation on the one hand to the entire universe, so that large intergalactic distances will usually have been traversed by particles which come to the earth (or perhaps high photon densities encountered at the beginning of the universe) or, on the other hand, it restricts them to the immediate vicinity of the sun, or perhaps other stars where the cosmic rays must at least receive the principal part of their acceleration. In particular, the loss in energy by electrons in their encounters with sunlight is sufficient to counteract the gain in energy caused by the action of the solar betatron proposed by Alfvén as the agent of acceleration in a theory for the purely solar origin of the cosmic radiation.