over most of the data. Each measurement, excepting the background, consisted of at least 10,000 counts, which were easily obtained owing to the high efficiency of the several rings of counters. The statistical accuracy of 3 percent was modified by systematic and random errors to an accuracy of 10 percent in cross section. The variations in neutron energy were the same as above.

V. RESULTS FOR MOLYBDENUM

The variations in the total neutron cross section of molybdenum, Fig. 4, are small owing to the several isotopes of the same relative abundance.[2] The nature and accuracy of the data reveal no structure, even at low energies. The cross section is practically constant at 9 barns from 10 to 250 kev and then decreases uniformly to 6 barns at 1230 kev.

VI. ACKNOWLEDGMENT

The author appreciates the advice and assistance of Drs. A. S. Langsdorf, Jr., J. M. Blair, and A. Wattenberg, and Messrs. J. Wallace and R. Nobles and other members of the Physics Division, Argonne National Laboratory.

PHYSICAL REVIEW VOLUME 84, NUMBER 5 DECEMBER 1, 1951

The Stopping Power of Metals and Semiconductors*

Z. H. Heller and D. J. Tendam
Purdue University, Lafayette, Indiana
(Received August 24, 1951)

Stopping power measurements have been made on Si, Ni, Cu, Ge, Zr, Rh, Ag, Sn, and Au. The reduction in range of the cyclotron deuteron beam produced by samples of these materials was determined by measurement of the ionization in air at the end of the beam path. To evaluate the ionization curves a semi-empirical curve, given here, which relates extrapolated ranges with mean number ranges was computed for large straggling parameters. A range-energy curve for deuterons in Ge is given as well as approximate air and Al equivalents for Si and Ge. Range-energy curves were measured for the metals.

The semiconductor results were compared with those for the metals to determine whether the difference in electrical properties produces a difference in energy loss. No difference was detected within the limits of precision of this experiment.

Combining data from this experiment with those of other investigations a study of the dependence of stopping power on velocity and atomic number has been made. An empirical formula derived from these data expresses the electronic stopping power relative to Al as a function of energy and atomic number over a wide range. A formula is provided for using this relation to calculate range-energy curves in any element with $Z>10$, and its validity is discussed.

I. INTRODUCTION

In the course of the investigations at this laboratory on the nuclear bombardment of semiconductors[1] it became necessary to know the stopping powers of Si and Ge in bulk and, since no experimental values were available, equipment to make these measurements was assembled. Since the electrical properties of a material are known to affect the rate of energy loss, it was decided to measure range-energy curves for several metals at the same time in order to compare their stopping powers with those of the semiconductors. The use of 9-Mev deuterons in this investigation also affords an opportunity to close a gap in the experimental studies of the velocity dependence of stopping power. As described below, an empirical relation which enables one to calculate approximate range-energy curves has been established from these data and those of other investigators.

II. APPARATUS

An apparatus similar to that described by Wilson[2] was used to measure the residual range of the deuteron beam of the Purdue cyclotron after transmission through sample specimens (Fig. 1). The monitor chamber was needed to follow fluctuations in beam current, and the end of the range in air was indicated by the current in the shallow chamber; both chamber voltages were adjusted to their current plateaus. The range was reduced stepwise by 0.001-inch Al foils and only the last few centimeters of the range were measured in air since the outward spreading of the beam prevented measuring the entire distance in air. An energy analyzer, employing three slits and the fringing flux of the cyclotron magnet to provide energy definition, also permitted convenient placing of the apparatus at some distance from the field. The two galvanometer arrangement employed by Wilson to record the chamber currents was replaced by the simple dual-triode bridge circuit shown in Fig. 2. The ratio of the grid resistances necessary to give zero

* Supported by the ONR.
[1] Lark-Horovitz, Bleuler, Davis, and Tendam, Phys. Rev. 73, 1256 (1948).
[2] R. R. Wilson, Phys. Rev. 60, 749 (1941).

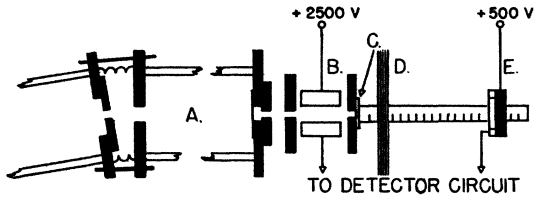

FIG. 1. Schematic of range apparatus. (A) magnetic analyzer. (B) monitor chamber. (C) sample mount. (D) aluminum foils. (E) collector chamber.

galvanometer current is proportional to the ordinate of the range ionization curve at a given point and very large fluctuations in beam current may occur without altering a balance setting. Heater fluctuations and positive ion grid current effects are canceled by having the two triodes in one envelope and by using equal grid resistors, but the plate potentiometer and the galvanometer zero rheostat are necessary to balance the tube characteristic differences. The balance readings were taken at 5–10 μa monitor current for which the sensitivity was more than adequate.

Foils of the metals Al, Ni, Cu, Zr, Rh, Ag, Sn, and Au were readily available. Their thicknesses were determined by weighing on a sensitive balance; the uniformity of the foils was checked by comparing the mass per unit area of the small strips cut for use with that of the parent foils. The samples of Si and Ge were ground by hand between parallel glass plates uniformly separated by tungsten spacers. Because these materials fractured easily and irregularly in the grinding process, their areas were found by projecting images of the samples at known magnification and tracing the enlarged outlines with a planimeter.

III. EVALUATION OF IONIZATION CURVES

The use of ionization chambers gave the well-known Bragg-type curve from which the extrapolated range was found. Because the range-energy curves for air which were used[3] are given in terms of mean number range and because the large range straggling in this

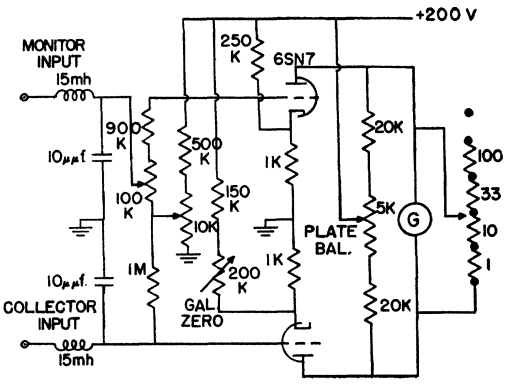

FIG. 2. Current detector circuit.

[3] M. S. Livingston and H. A. Bethe, Revs. Modern Phys. 9, 261 (1937).

experiment (due to energy spread, foil inhomogeneities, and the natural collision statistics) made standardization necessary, all ranges were converted to mean number ranges. The straggling was assumed to be gaussian and the method which Livingston and Holloway[4] applied to the range of natural alpha-particles was extended to the case of large straggling. These authors characterized the end of the ionization curve by the numbers α, the straggling parameter, y, the distance between the extrapolated range point and the intercept of the extrapolation line with the ordinate of maximum ionization, and x, the distance from the mean range point to the extrapolated range point. The observed ionization curve is given by

$$I(\xi)=\int_{\xi}^{\infty} F(\xi')S(\xi'-\xi)d\xi', \qquad (1)$$

where ξ is the distance from the mean range point, $F(\xi)$ is the range distribution function and $S(\xi'-\xi)$ is the ionization of a single particle at distance $\xi'-\xi$ from its endpoint. For a given $F(\xi)$, the integration may be performed numerically for several points ξ, and the

TABLE I. Conversion from extrapolated range to mean range for various straggling parameters.

α, cm	x, cm	y, cm
0.000	0.00	0.06
0.354	0.30	0.63
0.707	0.69	1.19
1.414	1.47	2.14
2.121	2.17	3.02
2.828	2.98	4.16

ionization curve drawn. Using their single particle curve, they found that for small α the relation between x and y was linear, $x=0.43y-0.081$ cm. We used the single particle curve given for deuterons by Livingston and Bethe[3] extrapolated to zero energy, and folded it as indicated with the gaussian function for several values of α. The resulting curves yielded the values of x and y given in Table I. The plot of x *vs* y is linear for small values of α and then curves upward. The numerical integration was performed very carefully so that these values are as accurate as the single particle ionization curve. Errors in this would produce a second order effect in the experimental results. The ranges found in air were corrected to 15°C, and 760 mm Hg (the correction for water vapor was negligible), and converted to ranges in Al using the data of Smith.[5]

The major recognized source of error in the measurements was the unavoidable shifting of the energy of the particles used. Such shifts occurred because the analyzer employed the fringing flux of the cyclotron magnetic field which had to be adjusted for any changes in oper-

[4] M. G. Holloway and M. S. Livingston, Phys. Rev. 54, 18 (1938).

[5] J. H. Smith, Phys. Rev. 71, 32 (1947).

ating conditions. Small energy changes caused appreciable current changes at the end of the range where the Bragg curve descends steeply and where the extrapolation line must be drawn. These shifts were most significant for small residual ranges where the resulting uncertainties in range were of the order of 5 percent. However, the errors in the calculated quantities (Al equivalents, relative electronic stopping powers, etc.) are not that large. Reproducibility of the curves was satisfactory. Analysis of all the known sources of error yields a value of about 2 percent.

IV. RESULTS

The Range-Energy Curves for Metals

Curves of absorber thickness *vs* residual range in Al were measured for the seven metals, covering the beam range, 77.2 mg/cm² Al, in three steps with the Ni, Zr, Ag and Au foils, in four steps with Cu and Rh, and in five steps with the Sn foils. Several residual range curves were repeated to check reproducibility. Using the data of Smith[5] these could be converted to range-energy curves. Figure 3 is the range-energy curve for Rh. This was needed for use in stacked foil excitation experiments with Rh. The results of the metal measurements were reorganized as discussed subsequently.

Range-Energy Relations for Si and Ge

Two samples of Ge of 64.4 mg/cm² and 84.6 mg/cm², and one of Si of 34.0 mg/cm² thickness all having high resistivity were studied. Two residual range curves were run for each sample with different portions in front of the defining hole to average the effects of the slight sample inhomogeneities. Then Al foils were placed before the thinner Ge sample and different portions of the range in Ge were measured. The latter data were converted to the range-energy curve for Ge shown in Fig. 4. The following figures were measured for approximate range calculations: 1.44 mg/cm² Ge=1 mg/cm² Al; 2.18 mg/cm² Ge=1 cm air; 0.985 mg/cm² Si=1 mg/cm² Al, and 1.50 mg/cm² Si=1 cm air.

The Stopping Power of Semiconductors

The well-known energy loss equations of Bohr,[6] Bethe,[7] and Bloch[8] were developed for isolated stopping atoms, rather than for atoms in the solid state. Von Weizsäcker,[9] dealing with the effects of the electrical properties, arrived at an energy loss which depended on the resistivity. However, Gerritsen[10] showed experimentally that the stopping power of Al was unchanged as it was made superconducting. Kramers[11] explained this negative result by showing theoretically that the

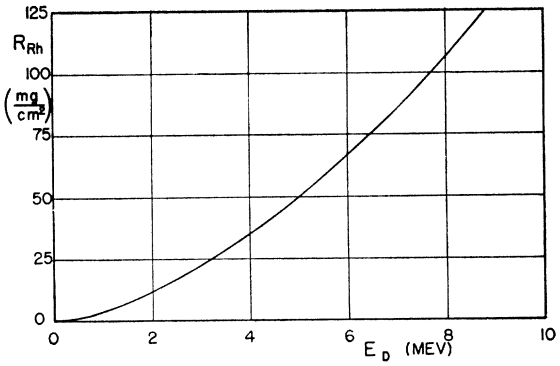

FIG. 3. Range-energy curve for deuterons in Rh.

major part of the stopping was due to the inner electron shells and is affected only slightly by the electrical properties of the medium. Subsequently, Madsen and Venkateswarlu[12] measured the stopping power of Be, for which the conduction electrons form a more significant portion of the total number, and found a difference between stopping in the solid state and that calculated for separate atoms. The theory of Bohr,[13] accounting for the free electron effects, was in excellent agreement with their results.

Despite their relatively high number of inner electrons, Si and Ge were compared with the metals to observe whether any stopping differences due to the electrical properties exist. The stopping power of a semiconductor can be compared with that which a metal of the same Z would have by using a "Bloch plot" for a guide.

The energy loss formula developed by Bethe and by Bloch is

$$-dE/dx=(4\pi z^2e^4/mv^2)NZ\ln(2mv^2/I), \qquad (2)$$

where e and m are the electronic charge and mass, v and z the velocity and charge number of the incident particle, and Z, I, and N are the atomic number, mean ionization potential, and the number per cm³ of the

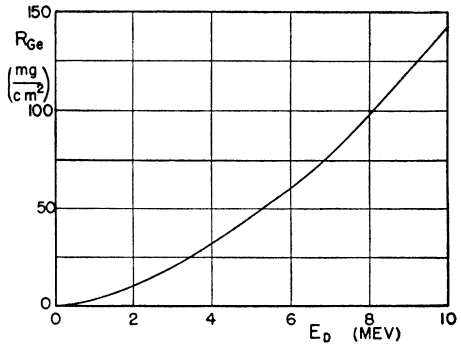

FIG. 4. Range-energy curve for deuterons in Ge.

[6] N. Bohr, Phil. Mag. **25**, 10 (1913).
[7] H. A. Bethe, Ann. Physik **5**, 325 (1930).
[8] F. Bloch, Ann. Physik **16**, 285 (1933).
[9] C. F. von Weizsäcker, Ann. Physik **17**, 869 (1933).
[10] A. N. Gerritsen, thesis, University of Leiden, 1948.
[11] H. A. Kramers, Physica **13**, 401 (1947).
[12] C. B. Madsen and P. Venkateswarlu, Phys. Rev. **74**, 648 (1948).
[13] A. Bohr, Kgl. Danske Videnskab. Selskabs Mat.-fys. Medd. **XXIV**, 19.

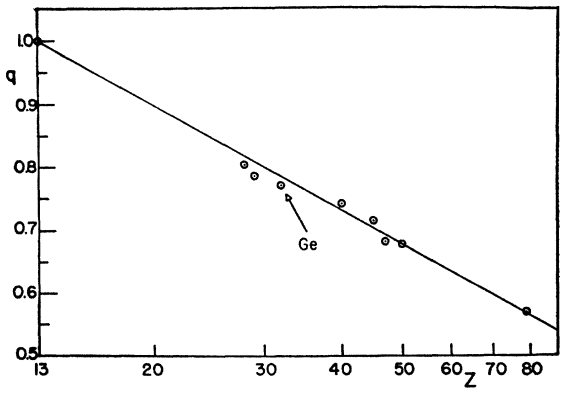

FIG. 5. Stopping power per electron relative to Al *vs* ln$Z$ for absorber thicknesses equivalent to 84.6 mg/cm² Ge.

stopping atoms. Bloch,[14] using a statistical atomic model, showed that I/Z is approximately a constant. Thus a graph of q, the stopping power per electron relative to Al, *vs* ln$Z$ for equal particle velocities (called here Bloch plots) should give a straight line according to the relation,

$$q=\frac{-1}{NZ}\left(\frac{dE}{dx}\right)_Z\Big/\left[\frac{-1}{N_{\mathrm{Al}}13}\left(\frac{dE}{dx}\right)_{\mathrm{Al}}\right]$$
$$=C-[\ln Z/(\ln 2mv^2-\ln I_{\mathrm{Al}})], \quad (3)$$

where C is a constant.

The linearity of this type of plot has been verified by Mano,[15] Wilson,[2] Kelly,[16] and Teasdale[17] even at lower velocities than the statistical model requires (see below). The position of the point for a semiconductor relative to such a line drawn for thicknesses of metals equivalent in stopping to the given semiconductor sample (to insure that the average velocities in the samples were equal) should indicate any systematic differences which may exist.

The metal thicknesses equivalent in energy reduction to the various semiconductor samples were taken from the residual range curves, and q was calculated for each case and plotted. Figures 5 and 6 show the results for the thick Ge sample and the Si sample; the thin Ge sample gave a similar result. It is seen that within the limits of accuracy the semiconductor points fall on the straight lines given by the metals. This confirms the relative unimportance of the conduction or dielectric properties of the medium for $Z\simeq 13$ or larger. Because Si is the elementary semiconductor having smallest atomic number for which such measurements could be made, no further conclusions could be drawn. The poor mechanical properties of graphite made it unsuitable for use in this experiment, and the possibility of preparation of thin diamond samples is being considered.

[14] F. Bloch, Z. Phys. **81**, 363 (1933).
[15] G. Mano, Ann. physique **1**, 407 (1934).
[16] E. L. Kelly, Phys. Rev. **75**, 1006 (1949).
[17] J. G. Teasdale, Tech. Report 3, U.C.L.A., December, 1949.

An Empirical Stopping Power Formula

Combining data from this experiment with those of other authors,[2,15–18] an empirical formula for computing range-energy relations which is valid for a very large range of Z and v values has been developed. We compared the data taken at various velocities by plotting the reciprocals of the slopes of the Bloch plots *vs* ln$\beta^2(\beta=v/c)$ which should again give a straight line according to (4).

$$-(d\ln Z/dq)=\ln(2mc^2/I_{\mathrm{Al}})+\ln\beta^2. \quad (4)$$

Values for $-(d\ln Z/dq)$ at $v=1.5$, 2.0, and 2.5×10^9 cm/sec were taken from our experiment. The resulting graph, Fig. 7, shows linearity over a very large range with small deviations at low velocities where the measurements of Mano and the present authors are subject to larger errors. The measurements of Wilson are seen to be inconsistent with the others. The value of β^2 at which Wilson's point is plotted is the mean taken for all of his samples. The agreement of the remaining points with the straight line may be deceptive since there were several steps admissive of error in the determination of $d\ln Z/dq$, and the mean velocities in the various experiments were difficult to fix. The encircled point of Bakker and Segrè[18] is plotted against lnβ^2 and that in the square against $\ln 2[(1-\beta^2)^{-\frac{1}{2}}-1]$ which is proportional to the kinetic energy in the relativistic limit. The proper relativistic correction to apply, that of Bethe[19] and Møller[20] which here gives $\ln[\beta^2\exp(-\beta^2)/(1-\beta^2)]$ as the abscissa to be used, gives the point indicated by the triangle. No correction is needed for the lower points. While the point in the square fits the curve joining the low velocity points better, the other is also to be considered as fitting the curve within the limits of precision.

From the velocity dependence found in Fig. 7 the relative electronic stopping power can be expressed as

$$q=1.00-1.18(\log Z-1.114)/(\log E_p+1.63), \quad (5)$$

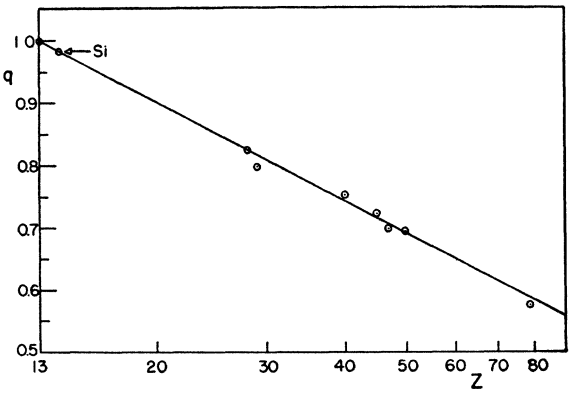

FIG. 6. Stopping power per electron relative to Al *vs* ln$Z$ for absorber thicknesses equivalent to 34.0 mg/cm² Si.

[18] C. J. Bakker and E. Segrè, Phys. Rev. **81**, 489 (1951).
[19] H. A. Bethe, Z. Physik **76**, 293 (1932).
[20] C. Møller, Ann. Physik **14**, 531 (1932).

where E_p is the energy (in Mev) of a proton having the same velocity as the particle considered. The range of validity of (5) in terms of Z and E_p is rather extensive. At the low energy end the straggling is probably due to errors, since the mechanism of stopping seems to vary smoothly down to $\beta^2 \sim 0.0005$ (200-kev protons) when the capture of electrons starts. The formula is here valid for $Z > 10$ since all the investigators found linear Bloch plots down to Al (Kelly observed slight deviations for Th and U). Pending further measurements, the upper limit of validity seems to be fixed only by the threshold for radiation loss which varies with the optical properties of the medium, and requires analysis for the particular case studied. Bakker and Segrè working with protons of high enough energy (340 Mev) for all electrons to be effective found that, for $Z > 26$, I/Z is a constant, but that below $Z = 26$ it increases. The slope for Fig. 7 was taken from their data for $Z > 26$; hence the formula becomes less reliable for $Z < 26$ as the velocity increases. Evidently more information is needed in this region. Since Mano found no systematic differences in stopping between gases and metals, and since we found none for semiconductors, the formula may be applied to all physical phases and to compounds whenever the above conditions apply. In the lighter elements the formula can only be used as a very rough guide. The errors involved are difficult to evaluate but from the agreement of the measurements Eq. (5) should be correct to within 5 percent.

From (4) and Smith's data approximate range-energy curves can be calculated using the relation,

$$R(E, Z) = \frac{13A}{27.0Z}\left\{\frac{R_{\mathrm{Al}}(E)}{q(E, Z)} - \int_0^E R_{\mathrm{Al}}(E')\frac{d(1/q)}{dE'}dE'\right\}, \quad (6)$$

where $R(E, Z)$ is the range of protons of energy E in mg/cm^2 of the element of atomic number Z and atomic weight A, and $R_{\mathrm{Al}}(E)$ is the range of protons in Al. The first term is the larger and can be considered a useful approximation; the second term is a correction to this and can be evaluated using

$$\frac{d(1/q)}{dE} = \frac{0.573 - 0.512 \log Z}{E(\log E - 1.18 \log Z + 2.95)^2}, \quad (7)$$

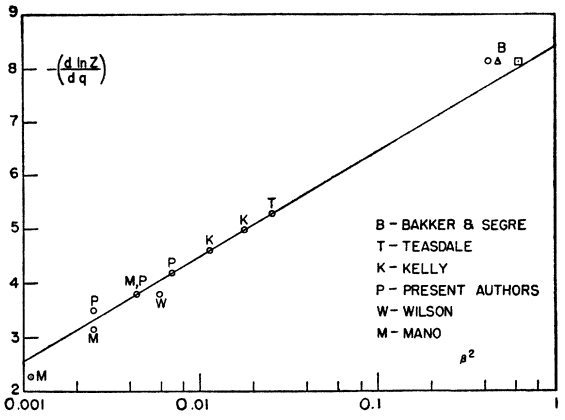

FIG. 7. Graph of $-(d \ln Z/dq)$ *vs* $\ln\beta^2$, using data from various experiments.

and integrating graphically. Equation (7) diverges for $E \sim 0$ but the integrand function should be cut off at about $E \sim 100$ kev and extrapolated to zero; the error involved is negligible. Range-energy relations for deuterons, alpha-particles, and mesons can be obtained from those for protons in the usual manner.

The equation of the line in Fig. 7 is

$$-(d \ln Z/dq) = 8.41 + 0.848 \ln\beta^2. \quad (8)$$

The slope is not unity as required by (4) and the value of I/Z found from the intercept is 17.4 ev, in disagreement with the best experimental value,[18] 9.4 ev. The disagreement is due in part to the fact that the mean ionization potential is a function of v in that the number of electrons effective in stopping depends on v. Moreover, the statistical model fails for $Z < 26$. In the experiments at low velocities, however, the linear Bloch plots were obtained using the uncorrected Z values, and were linear for elements whose effective Z values were less than 26 (i.e., all the lines pass through $q = 1$ at $Z = 13$).

The authors extend their appreciation for the help of K. Lark-Horovitz who proposed the problem, and of E. Bleuler who suggested many of the methods of carrying out the measurements and analyzing the data.