transition gamma-rays, while no significant number seem to be contributed by the higher two levels. This could be explained by the higher energy available for alpha decay for the two upper levels, and the consequent reduction in the lifetime for the breakup. Bennett, Roys, and Toppel[22] have found from the reaction $Li^7(\alpha, \gamma)B^{11}$ that the 9.28 level has a width of 6 kev, which would indicate that this level should break up by particle emission. The 8.93 and 9.19 levels were shown to be appreciably narrower than the 9.28 level, within instrumental fluctuations. The results of Bennett, *et al.*, suggest that the radiation from these three levels consists of about 15 percent emission straight to ground, the remaining radiation being due to cascade transitions.

A consistent interpretation of the gamma-ray intensities may include the following: The 9.28 level mainly breaks up by the emission of an alpha-particle. The 9.19 level breaks up by emission of an alpha-particle in competition with gamma-emission which mainly appears in the form of cascade radiation. The 8.93 level can decay by a radiative transition straight to ground or by cascade transitions, with about equal probabilities for the two processes. Alpha-emission may also arise from the 8.93 level, but most likely has a small probability of occurring. This interpretation could explain the discrepancies of the relative intensities of the 4.43, 6.75, and 8.93-Mev lines, as well as the absence of quanta from the two upper levels.

The authors wish to thank Professor T. W. Bonner for suggesting this work and for his aid in all phases of the work. We also wish to thank the Los Alamos Scientific Laboratory for the loan of the Po-Be source.

[22] Bennett, Roys, and Toppel, Phys. Rev. **82**, 20 (1951).

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Renormalization Theory of the Interactions of Nucleons, Mesons, and Photons

J. C. WARD*
Clarendon Laboratory, Oxford, England
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A general program for the removal of divergencies from the theory of the interactions of nucleons, mesons, and photons is formulated. It is shown that the procedure is equivalent to renormalization of the constants of the theory.

THE success of the concept of renormalization in giving a theory of spinor electrodynamics that is both finite[1] and in good agreement with experiment[2] has made the extension of the method to other field theory problems most desirable, particularly the extension to the problem of nuclear forces. The first step in this direction is to show that the renormalized S-matrix is finite. This problem has not been satisfactorily dealt with until recently because of certain mathematical difficulties. The main source of trouble has been the overlapping divergencies. A process will be outlined below that in principle enables the renormalized S-matrix to be calculated to any degree of approximation. Bound-state phenomena remain outside the scope of the present treatment, and a suitable approach to the bound state remains to be discovered.

We select for study the interaction of pseudoscalar charged mesons, nucleons, and photons, with pseudoscalar coupling between meson and nucleon. A few remarks on the possible inclusion of neutral mesons will be made later. The Feynman rules for the construction of the S-matrix have been given by several authors,[3] and may be derived from the interaction hamiltonian

$$H=\frac{ie}{\hbar c}\left(\varphi^*\frac{\partial\varphi}{\partial x_\mu}-\frac{\partial\varphi^*}{\partial x_\mu}\varphi\right)A_\mu-\left(\frac{ie}{\hbar c}\right)^2\varphi^*\varphi A_\mu A_\mu$$
$$+ie(\bar{\psi}\gamma_\mu\gamma_\mu\psi)+if\bar{\psi}\gamma_5(\tau_-\varphi^*+\tau_+\varphi)\psi+\delta\lambda(\varphi^*\varphi)^2,$$

where mass renormalization and surface-dependent terms have been neglected. The functions $\varphi(x)$, $\psi(x)$, and $A_\mu(x)$, denote the meson, nucleon, and electromagnetic fields, respectively. We briefly state the rules again:[4]

1. Each photon line gives a factor
$$(2\pi)^{-3}\hbar c\delta_{\mu\nu}\int D_F(p)d^4p.$$
2. Each meson line gives a factor $(2\pi)^{-3}\int\Delta_F(p)d^4p$.
3. Each nucleon line gives a factor $(2\pi)^{-3}\int S_F(p)d^4p$.
4. Each meson-photon 3-vertex gives a factor
$$ie(\hbar c)^{-2}(2\pi)^4(p_1+p_2)_\mu\delta(p_1-p_2+p_3).$$
5. Each meson-photon 4-vertex gives a factor
$$-ie^2(\hbar c)^{-3}(2\pi)^4\delta_{\pi\rho}\delta(p_1-p_2+p_3+p_4).$$

* Now at the Institute for Advanced Study, Princeton, New Jersey.

[1] F. J. Dyson, Phys. Rev. **75**, 1736 (1949); J. C. Ward, Proc. Phys. Soc. (London) **A64**, 54 (1951).

[2] P. Kusch and H. M. Foley, Phys. Rev. **74**, 250 (1948). W. E. Lamb, Jr., and R. C. Retherford, Phys. Rev. **72**, 241 (1947).

[3] R. P. Feynman, Phys. Rev. **76**, 769 (1949); P. T. Matthews, Phys. Rev. **80**, 292 (1950); F. Rohrlich, Phys. Rev. **80**, 666 (1950).

[4] The similar rules for external lines have been omitted.

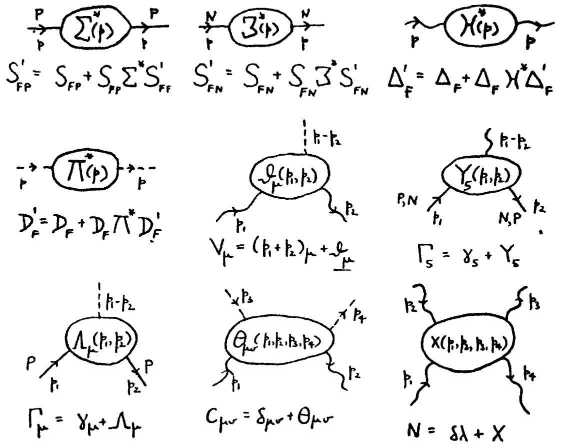

FIG. 1. The family of primitive divergent graphs, with their associated functions.

6. Each proton-photon vertex gives a factor

$$-ie\gamma_\mu\tau p(2\pi)^3\delta(p_1-p_2+p_3).$$

7. Each nucleon-meson vertex gives a factor

$$-if\gamma_5(2\pi)^4\delta(p_1-p_2+p_3).$$

Charge conservation is applied in drawing the possible graphs, and each graph has a weight which may be determined by inspection of the number of ways that it can appear in the S-matrix.

The condition for primitive divergence is $\frac{3}{2}E_n+E_m+E_{ph}<5$.[5] This gives rise to the family of graphs shown in Fig. 1. Because of the charge symmetry, Furry's theorem excludes $E_{ph}=1$ or 3, $E_N=E_m=0$. $E_m=1$ or 3, $E_{ph}=E_n=0$ is excluded by charge conservation. The neutron-photon interaction gives a finite graph,[6] as does the scattering of light by light,[7] when the effects of all graphs of the same order are considered. The scattering of mesons by mesons, however, is a true primitive divergent, which must be canceled by the introduction of the term $\delta\lambda(\varphi^*\varphi)^2$ into the hamiltonian. The effects of these divergent graphs are represented in the usual way by the insertion of functions at lines and vertices of the reduced graphs or skeletons. A list of these functions is given in Fig. 1.

The construction of the renormalized S-matrix is performed by successive approximation. We avoid dealing with self-energy parts explicitly by the introduction of the following method. Consider the function $\Sigma^*(p)$ which represents the effect of insertion of all possible proper self-energy parts into proton lines according to the equation $S_{FP}'=S_F+S_F\Sigma^*S_{FP}'$. This function obeys the relation[6]

$$-(1/2\pi)\partial\Sigma^*(p)/\partial p_\mu=\Lambda_\mu(p,p),$$

where $\Lambda_\mu(p_1,p_2)$ is the corresponding vertex function (see Fig. 1). We may rewrite this result in the form

$$\Sigma^*(p)-\Sigma^*(p')=-2\pi\int_0^1 d\lambda(p_\mu-p_\mu')\Lambda_\mu(p^\lambda,p^\lambda),$$

where $p^\lambda=p\lambda+p'(1-\lambda)$. The second term $\Sigma^*(p')$ can be made to play the part of the mass renormalization term if we agree to replace p'^2 and $p_\mu'\gamma_\mu$ by $-M^2$ and iM after the integration on λ, where M is the nucleon mass. (No distinction will be made between proton and neutron mass, although this could be done if desired.) This is sufficient, since p' can only appear in these invariant combinations. The function $\mathcal{H}^*(p)$, which represents the effect of insertion of all possible proper meson self-energy parts into meson lines according to the equation $\Delta_F'=\Delta_F+\Delta_F\mathcal{H}^*\Delta_F'$, may be treated in the same fashion. It obeys the equation relating it to the meson-photon 3-vertex function[6] $\vartheta_\mu(p_1,p_2)$

$$-(1/2\pi i)\partial\mathcal{H}^*(p)/\partial p_\mu=\vartheta_\mu(p,p),$$

which may be written as

$$\mathcal{H}^*(p)-\mathcal{H}^*(p'')=-2\pi i\int_0^1 d\lambda(p_\mu-p_\mu'')\vartheta_\mu(p^\lambda,p^\lambda).$$

With the convention that we write $p''^2=-m^2$, where m is the meson mass, after the integration on λ, the second term $\mathcal{H}^*(p'')$ can again be made to play the part of the mass renormalization. An identical procedure will be followed for the remaining self-energy parts, although it is now necessary to define functions $\mathfrak{T}_\mu(p)$ and $T_\mu(p)$ by the equations

$$-(1/2\pi)\partial\mathfrak{Z}^*(p)/\partial p_\mu=\mathfrak{T}_\mu(p),$$
$$-(1/2\pi i)\partial\Pi^*(p)/\partial p_\mu=T_\mu(p).$$

$\Pi^*(p)$and $\mathfrak{Z}^*(p)$ are the photon and neutron self-energy functions defined in Fig. 1. It is important to observe that the above definitions by differentiation are really implicit definitions, when the functions $\mathfrak{T}_\mu(p)$ and $T_\mu(p)$ are considered as calculated by the use of irreducible graphs, with appropriate insertions in lines and vertices. In fact, it is necessary to define now the functions

$$W_\mu=2p_\mu+T_\mu;\quad U_\mu=\gamma_\mu+\mathfrak{T}_\mu,$$

which may occur in the integral equations. Using these definitions, we have the results:

$$\Pi^*(p)-\Pi^*(0)=-2\pi i\int_0^1 d\lambda T_\mu(p\lambda)p_\mu,$$

$$\mathfrak{Z}^*(p)-\mathfrak{Z}^*(p')=-2\pi\int_0^1 d\lambda(p_\mu-p_\mu')\mathfrak{T}_\mu(p^\lambda).$$

[5] It should be observed that the direct application of the concept of primitive divergence may not suffice where a change in the usual perturbation method is desirable.

[6] A. Salam, Phys. Rev. **79**, 910 (1950).

[7] J. C. Ward, Phys. Rev. **77**, 293 (1950). The generalization of this kind of argument to all gauge-invariant interactions is immediate.

$\Pi^*(0)$ vanishes if calculated sufficiently carefully, paying attention to the gauge invariance. $\Sigma^*(p')$ becomes a mass renormalization term with the above conventions for p'.

This treatment of self-energy parts is designed to avoid the trouble of overlapping divergencies, which occurs most frequently in self-energy terms. It is also a useful means of including the mutual dependence of the divergent parts of self-energy and vertex terms in the renormalization. There are no overlaps in Λ_μ or Υ_μ; but T_μ, ϑ_μ, $\theta_{\mu\nu}$, and X still have an ambiguous construction from lower order parts. This may be dealt with in the following manner, which is the simplest way of dealing with overlapping or "b" divergencies. Let $M=\mu m$, and regard μ and m as independent variables. Then the functions T_μ, ϑ_μ, $\theta_{\mu\nu}$, and X will be considered as defined by the equations

$$T_\mu=-\int_m^\infty T_{\mu,m'}dm', \quad \vartheta_\mu=-\int_m^\infty \vartheta_{\mu,m'}dm',$$

$$\theta_{\mu\nu}=-\int_m^\infty \theta_{\mu\nu,m'}dm', \quad X=-\int_m^\infty X_{,m'}dm'.$$

Here we use the notation $\partial A/\partial m=A_{,m}$, and the functions to be inserted in the integral equations may now possibly be the original functions differentiated with respect to m as well. The reader may find it worth while to see for himself how differentiation with respect to the mass reduces the degree of divergence of integrals and sorts out the overlapping divergencies. The essential idea behind this is that "b" divergencies present a multiplicative effect and that differentiation can transform a multiplicative effect into an additive one. Illustrations of the effect of this process on simple graphs containing overlaps are given in Fig. 2. It should be noticed in passing that in the case of the calculation of T_μ it is also necesssary to substitute $\partial\vartheta_\mu/\partial p_\nu$ for the effects of corresponding insertions in the graphs defining $T_\mu(p)$.

CONSTRUCTION OF THE FINITE FUNCTIONS

It is now possible to define the scheme for construction, by repeated approximation, of the finite functions which replace the infinite functions in the calculation of the renormalized S-matrix. We use the notation $\langle F\rangle$ to imply that the finite functions to some order of approximation have been inserted in the irreducible graphs used in calculating F. The following equations define the finite functions:

$$N_c=\delta\lambda_1-\int_m^\infty [\langle X_{,m'}\rangle-\langle X_{,m'}(p'',p'',p'',p'')\rangle]dm', \quad (1)$$

where the finite function N_c is indeterminate by a finite constant $\delta\lambda_1$, which must be regarded as an empirical constant;

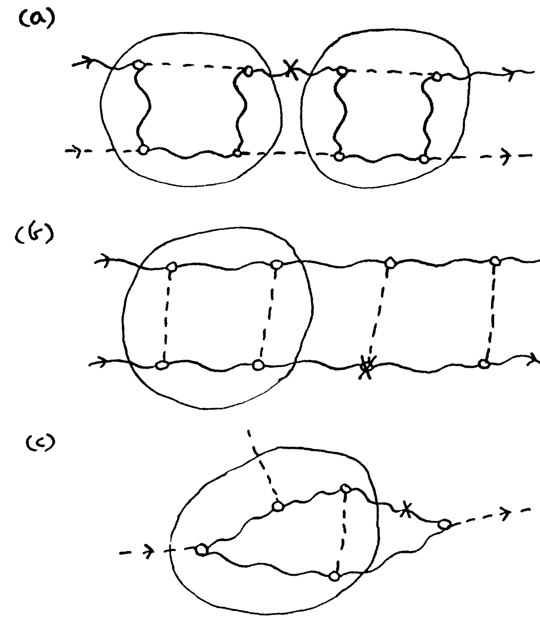

FIG. 2. Simple examples of the method of substitution into irreducible graphs. (a) A typical C-part graph. X represents the action of $\partial/\partial m$. All possible C-parts have to be inserted in the enclosures, giving factors $Z_1^{-1}C_{\mu\nu c}$. (b) A typical M-part graph. All possible M-parts have to be inserted in the enclosure, giving a factor $Z_1^{-2}N_c$. (c) Example of a term which appears in the calculation of T_μ. $Z_1^{-1}\partial V_{\nu c}/\partial p_\mu$ is the effect of all possible parts in the enclosure.

$$C_{\mu\nu c}=\delta_{\mu\nu}-\int_m^\infty [\langle\theta_{\mu\nu,m'}\rangle-\tfrac{1}{4}\delta_{\mu\nu}\langle\theta_{\lambda\lambda,m'}(p'',p'',0,0)\rangle]dm'. \quad (2)$$

The integral $\int_m^\infty\langle\theta_{\mu\nu,m'}\rangle dm'$ is divergent only for $\mu=\nu$, and only the diagonal parts need a subtraction;

$$V_{\mu c}=(p_1+p_2)_\mu-\int_m^\infty [\langle\vartheta_{\mu,m'}\rangle-\tfrac{1}{4}(p_1+p_2)_\mu\langle\theta_{\lambda\lambda,m'}(p'',p'',0,0)\rangle]dm', \quad (3)$$

where we have used the identity[8] $\frac{1}{2}(\partial\vartheta_\mu/\partial p_\nu)(p,p)=\theta_{\mu\nu}(p,p,0,0)$;

$$\Gamma_{\mu c}=\gamma_\mu+\langle\Lambda_\mu(p_1,p_2)\rangle-\langle\Lambda_\mu(p',p')\rangle; \quad (4)$$

$$U_{\mu c}=\gamma_\mu+\langle\Upsilon_\mu(p)\rangle-\langle\Upsilon_\mu(p')\rangle; \quad (5)$$

$$\Gamma_{5c}=\gamma_5+\langle Y_5(p_1,p_2)\rangle-\langle Y_5(p',p')\rangle; \quad (6)$$

$$W_{\mu c}=2p_\mu-\int_m^\infty [\langle T_{\mu,m'}\rangle-p_\nu(\langle\partial T_{\mu,m'}\rangle/\partial p_\nu)_0]dm'. \quad (7)$$

With the aid of these definitions the finite self-energy

[8] See reference 6. Strictly speaking, this identity should not be used until we discuss the renormalization proof. The reader will verify that the use then is legitimate.

functions can be constructed:

$$\Delta_c=\Delta_F+2\pi i\Delta_F\int_0^1 d\lambda\int_m^\infty dm'[\langle\vartheta_{\mu,m'}(p^\lambda,p^\lambda)\rangle -\tfrac{1}{2}p_\mu{}^\lambda\langle\theta_{\lambda\lambda,m'}(p'',p'',0,0)\rangle](p_\mu-p_\mu'')\Delta_c, \quad (8)$$

$$D_c=D_F+2\pi iD_F\int_0^1 d\lambda\int_m^\infty dm'[\langle T_{\mu,m'}(p\lambda)\rangle -\lambda p_\nu(\langle\partial T_{\mu,m'}\rangle/\partial p_\nu)_0]p_\mu D_c, \quad (9)$$

$$S_{Pc}=S_{PF}-2\pi S_{PF}\int_0^1 d\lambda[\langle\Lambda_\mu(p^\lambda,p^\lambda)\rangle -\langle\Lambda_\mu(p',p')\rangle](p_\mu-p_\mu')S_{Pc}, \quad (10)$$

$$S_{Nc}=S_{NF}-2\pi S_{NF}\int_0^1 d\lambda[\langle\Upsilon_\mu(p^\lambda)\rangle -\langle\Upsilon_\mu(p')\rangle](p_\mu-p_\mu')S_{Nc}, \quad (11)$$

The above equations, taken together, constitute a system of integral equations that define the finite functions.

JUSTIFICATION OF THE SUBTRACTION PROCEDURE IN TERMS OF RENORMALIZATION

It will now be proved that the following relations hold between the infinite and finite functions:

$$\left.\begin{array}{ll} C_{\mu\nu}=Z_1^{-1}C_{\mu\nu c}(e_1,f_1); & S_P'=Z_6S_{Pc}(e_1,f_1);\\ V_\mu=Z_1^{-1}V_{\mu c}(e_1,f_1); & \Gamma_\mu=Z_6^{-1}\Gamma_{\mu c}(e_1,f_1);\\ \Delta'=Z_1\Delta_c(e_1,f_1); & S_N'=Z_7S_{Nc}(e_1,f_1);\\ N=Z_1^{-2}N_c(e_1,f_1); & U_\mu=Z_7^{-1}U_{\mu c}(e_1,f_1);\\ W_\mu=Z_3^{-1}W_{\mu c}(e_1,f_1); & \Gamma_5=Z_5^{-1}\Gamma_{5c}(e_1,f_1);\\ D'=Z_3D_c(e_1,f_1); & \end{array}\right\}\ \mathrm{A}$$

with $e_1=Z_3^{\frac{1}{2}}e$, $f_1=(Z_1Z_6Z_7)^{\frac{1}{2}}Z_5^{-1}f$, where Z_1, Z_3, Z_5, Z_6, and Z_7 are suitably chosen infinite constants. It is easy to see that these equations are sufficient to enable the divergent constants to disappear from the S-matrix expressed in terms of the renormalized coupling constants e_1 and f_1.[9,10]

"Z_1" Functions

The result of inserting expressions A into the integral equations defining V_μ is to give

$$Z_1V_{\mu c}=(p_1+p_2)_\mu-Z_1^{-1}\int_m^\infty\langle\vartheta_{\mu,m'}(p_1,p_2,e_1,f_1)\rangle dm',$$

which is the equation defining $V_{\mu c}$ as long as

$$Z_1=1+\tfrac{1}{4}\int_m^\infty dm'\langle\theta_{\lambda\lambda,m'}\rangle.$$

Substitution into Compton part irreducible graphs gives

$$Z_1^{-1}C_{\mu\nu c}=\delta_{\mu\nu}-Z_1^{-1}\int_m^\infty\langle\theta_{\mu\nu,m'}\rangle dm'.$$

This is the equation defining $C_{\mu\nu c}$ if

$$Z_1=1+\tfrac{1}{4}\int_m^\infty dm'\langle\theta_{\lambda\lambda,m'}\rangle,$$

which agrees with the value of Z_1 found above. Finally, substitution into the definition of Δ' gives

$$Z_1\Delta_c=\Delta_F+2\pi i\Delta_F\int_0^1 d\lambda\int_m^\infty dm'(p_\mu-p_\mu'')\times\langle\vartheta_{\mu,m'}(p^\lambda,p^\lambda)\rangle\Delta_c,$$

which again agrees with the definition of Δ_c, with the same value of Z_1 as before.

"Z_3" Functions

Insertion of the expressions A into the graphs defining W_μ gives

$$Z_3^{-1}W_{\mu c}=2p_\mu-\int_m^\infty dm'Z_3^{-1}\langle T_{\mu,m'}\rangle.$$

This is the defining equation for $W_{\mu c}$ if $Z_3-1=\frac{1}{2}C$, where

$$C\delta_{\mu\nu}=\int_m^\infty(\langle\partial T_{\mu,m'}\rangle/\partial p_\nu)_0dm'.$$

The photon self-energy equation then becomes

$$Z_3D_c=D_F+2\pi iD_F\int_m^\infty dm'\int_0^1 d\lambda p_\mu\langle T_{\mu,m'}(\lambda p)\rangle D_c,$$

which is the correct equation for D_c with the same value of Z_3.

"Z_5, Z_6, and Z_7" Functions

Insertion of A into the definition of Γ_5 gives

$$Z_5^{-1}\Gamma_{5c}=\gamma_5+Z_5^{-1}\langle Y_5\rangle.$$

Comparison with the equation for Γ_{5c} gives for Z_5

$$\gamma_5(Z_5-1)=-\langle Y_5(p',p')\rangle.$$

The effect of substitution into the proton-photon vertex function is to give the equation

$$Z_6^{-1}\Gamma_{\mu c}=\gamma_\mu+Z_6^{-1}\langle\Lambda_\mu\rangle,$$

[9] The treatment of the external lines and the associated wave function renormalization will not be given here. The plausible argument given by Dyson (reference 1) can be justified by an approach which goes outside the S-matrix framework.

[10] It is an essential step in the following argument that the infinite constants are independent of m. Strangely enough, this is a consequence of the absence of a fundamental length.

which is the same as

$$\Gamma_{\mu c} = \gamma_\mu + \langle \Lambda_\mu \rangle - \langle \Lambda_\mu(p', p') \rangle$$

if

$$\gamma_\mu(Z_6 - 1) = -\langle \Lambda_\mu(p', p') \rangle.$$

From this, the self-energy treatment follows as usual:

$$Z_6 S_{Pc} = S_{PF} - 2\pi S_{PF} \int_0^1 d\lambda \langle \Lambda_\mu(p^\lambda, p^\lambda) \rangle (p_\mu - p_\mu') S_{Pc}$$

is the same equation as

$$S_{Pc} = S_{PF} - 2\pi S_{PF} \int_0^1 d\lambda [\langle \Lambda_\mu(p^\lambda, p^\lambda) \rangle - \langle \Lambda_\mu(p', p') \rangle](p_\mu - p_\mu') S_{Pc}$$

with the above value of Z_6. Finally, from the definition U_μ, we derive the equation

$$Z_7^{-1} U_{\mu c} = \gamma_\mu + Z_7^{-1} \langle \Upsilon_\mu \rangle$$

and this is identical with $U_{\mu c} = \gamma_\mu + \langle \Upsilon_\mu \rangle - \langle \Upsilon_\mu(p') \rangle$ if

$$(Z_1 - 1)\gamma_\mu = -\langle \Upsilon_\mu(p') \rangle.$$

The self-energy treatment is, as before:

$$Z_7 S_{Nc} = S_{NF} - 2\pi S_{NF} \int_0^1 d\lambda \langle \Upsilon_\mu(p^\lambda) \rangle (p_\mu - p_\mu') S_{Nc}$$

is the same as

$$S_{Nc} = S_{NF} - 2\pi S_{NF} \int_0^1 d\lambda [\langle \Upsilon_\mu(p^\lambda) \rangle - \langle \Upsilon_\mu(p') \rangle](p_\mu - p_\mu') S_{Nc}$$

with the above value of Z_7.

Möller Parts

Insertion of A into graphs defining N gives

$$Z_1^{-2} N_c = \delta\lambda - Z_1^{-2} \int_m^\infty \langle X_{,m'} \rangle dm'.$$

This is the same as

$$N_c = \delta\lambda_1 - \int_m^\infty [\langle X_{,m'} \rangle - \langle X_{,m'}(p'', p'', p'', p'') \rangle] dm'$$

with

$$Z_1^2 \delta\lambda = \delta\lambda_1 + \int_m^\infty \langle X_{,m'}(p'', p'', p'', p'') \rangle dm'.$$

This completes the proof of the relations A.

CONCLUDING REMARKS

The formalism presented above shows, with some degree of rigor, that it is possible to remove the divergencies that arise in the mutual interactions of nucleons, photons, and pseudoscalar charged mesons, by the use of renormalization. The proof was restricted to charged mesons only, for the sake of simplicity, the problem being already sufficiently complex. The introduction of neutral mesons presents no new difficulty in principle, since certain selection rules facilitate the introduction of neutral pseudoscalar mesons.[11] In fact, the combined effect of charge symmetry, charge conservation, and parity is to exclude all possible primitive divergent graphs not considered above, with the exception of neutral meson self-energy graphs, Möller interactions including neutral mesons, and neutral meson-nucleon vertices. Compton parts for neutral mesons, and the decay of a neutral meson into two photons, are convergent. Hence, the only effect of neutral mesons on the theory is to require the introduction of four new renormalized coupling constants. Whether such a theory can be made to fit the observed facts concerning nuclear forces is a question that can only be settled by much more detailed calculations.

The author wishes to thank Mr. A. Salam for stimulating discussions and for reviving the author's interest in this subject.

[11] These remarks have since been made in greater detail by P. T. Matthews, Phys. Rev. 81, 936 (1951).