Internal Pair Spectrum from $B^{10}+d$

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(Received August 20, 1951)

A magnetic lens pair spectrometer has been used to measure the energy spectrum of the internal pairs formed from the bombardment of B^{10} with deuterons. The spectrometer has been calibrated with the nuclear pair from $F^{19}+p$ with an energy of 6.05 ± 0.03 Mev. The B^{10} spectrum shows gamma-rays of energies 4.43 ± 0.07, 6.51 ± 0.13, 6.75 ± 0.13, 7.34 ± 0.20, and 8.93 ± 0.13 Mev. Unresolved lines appear to exist in the region 4.5 to 5.5 Mev. The 6.51 Mev line is tentatively assigned to C^{11*} while the other lines can be attributed to B^{11*}.

INTRODUCTION

THE detection of the internal pairs arising from a nuclear reaction is a means of determining the gamma-ray spectrum above about 3 Mev. The high energy spectrum from the deuteron bombardment of 96 percent separated B^{10} has been investigated by this method. The spectrum from the reaction has been shown by earlier cloud-chamber measurements of the pairs to contain three strong lines at approximately 4.5, 6.5, and 9.0 Mev.[1,2] Recent results with magnetic 180° pair spectrometers have confirmed the cloud-chamber determinations. Terrell and Phillips[3] found three lines at 4.52, 6.71, and 9.04 Mev, with an indication that the two lower energy lines are complex. Rutherglen[4] reports gamma-ray lines at 4.5, 6.5, 6.7, and 8.94 Mev. Recent measurements of the proton[5,6] and neutron[7,8] groups from the reaction have clarified the energy level schemes of the mirror nuclei B^{11} and C^{11} to such an extent to justify a careful investigation of the gamma-ray spectrum from $B^{10}+d$ and assignment of gamma-ray energies to the corresponding levels.

Apparatus

The spectrometer used in these experiments, shown in Fig. 1, is basically a double magnetic lens beta spectrometer. The two focusing coils contain about 150 turns each of $\frac{3}{8}$-inch copper tubing, and are cooled by water flowing through the conducting turns. The separation of the coils is completely adjustable in order to provide for changing the acceptance solid angle of the spectrometer. The current for the coils is supplied by a 24 kw 400 ampere generator which is stabilized electronically to better than one-half percent at the currents used.

No provision for eliminating the earth's magnetic field is provided for the spectrometer since electrons or positrons with energies less than 0.6 Mev have not been used. The baffle system of the spectrometer is relatively simple compared to the systems of normal beta-spectrometers. An aluminum-covered lead cylinder, $13\frac{1}{2}$ inches long and 4 inches in diameter serves as a center baffle. A disk baffle of Lucite slides along a thin aluminum tube fastened to the center baffle. This system has not been designed to given the optimum resolution and transmission characteristics for single electron spectra for reasons to be treated later.

The source end plate of the spectrometer is constructed to allow the spectrometer to be attached to the accelerating tube of the Rice Van de Graaff generator. In order to avoid the fringing field, the magnetic analyzer of the Van de Graaff was not used. Since only thick targets have been used and it is not necessary to know the bombarding energy to great accuracy, the electrostatic voltmeter gives a satisfactory determination of the accelerator potential. A small magnet about two meters from the spectrometer was used to analyze the beam into its mass components. A quartz plate with a $\frac{5}{16}$-inch round aperture was mounted in a fixed position 1 inch from the target. This arrangement allowed the beam of the accelerator to be positioned accurately, by viewing the fluorescence produced by the beam on the quartz. The beam current to the target was measured with a current integrator.

The counting arrangement consists of two Victoreen "Thyrode" 30 mg/cm² thin wall aluminum Geiger

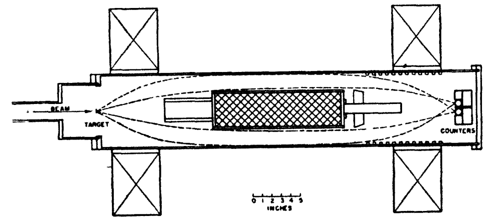

FIG. 1. Magnetic lens pair spectrometer in cross section. The accelerator beam passes through a quartz aperture to the target. Pairs produced in the source are focused and counted in coincidence in the counter assembly.

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† This paper is based in part on a Ph.D. thesis submitted by the first named author to the faculty of the Rice Institute in May, 1951. This work was supported by the joint program of the ONR and AEC, and by the Research Corporation.

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[2] J. Halpern and H. R. Crane, Phys. Rev. 55, 415 (1939).
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[4] Report at the Harwell Conference, 1950.
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[6] W. O. Bateson, Phys. Rev. 80, 982 (1950).
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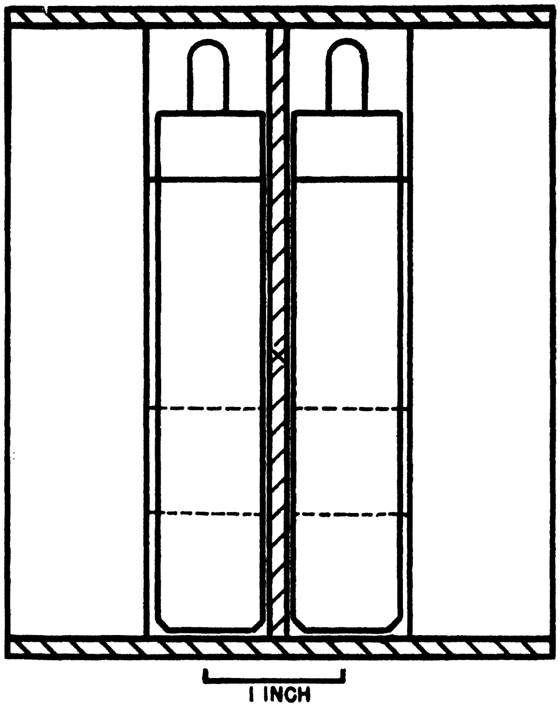

FIG. 2. Counter assembly consisting of two thin-wall Geiger tubes mounted on Lucite blocks. The assembly may be used fully open for high counting efficiency or with lead plates covering the counters with apertures located by the dotted lines.

counters, mounted on Lucite blocks. The counters are separated with $\frac{1}{8}$-inch lead strip to prevent spurious coincidences from single particles passing through both counters. Figure 2 shows a front view of the counter holder as seen when looking along the axis of the spectrometer from the source end. The counters may be covered with sheet lead containing apertures in the proper positions, or they may be used in the fully exposed condition shown in the figure. The cross locates the optical axis of the spectrometer, and the dotted lines locate the apertures which were used for some of the experiments. The counter leads are brought through Kovar seals in the endplate to a conventional coincidence circuit which had a resolution of about 0.5 microsecond.

COUNTING PAIRS

Several properties of magnetic lens spectrometers make them appear promising for use as pair spectrometers. The momentum of a focused particle is linear with focusing current. There is no necessity for collimation of the gamma-rays, which enables the use of a larger fraction of the gamma-rays produced in a reaction. The angular separation of the electron and positron of a pair does not affect the resolution of the instrument.

When a source of pairs is used as the source for the spectrometer, the electrons and positrons are both focused through the instrument in similar helical orbits, with opposite directions of rotation. When the electron and positron of a pair have equal energies, both members can be focused through the spectrometer at one value of the magnetic field strength. If the energies are not the same, both members cannot be focused. Thus, for a pair of given total energy, there is a unique value of focusing current at which both members of the pair can be focused. (More accurately, there is a range of currents for which both members can get through, since the spectrometer has a finite resolving power.) Therefore, if some method of knowing when a pair is focused through the spectrometer is available, then the spectrometer can be used for measuring high energy gamma-ray spectra.

Two methods for counting pairs in coincidence have been used. The first of these methods may be described as off axis separation, and has been reported by Siegbahn.[9] If the source of pairs is placed off the optical axis of the spectrometer, the electrons and positrons have separated focal points. By placing counters at the two focal points, the pair members can be counted in coincidence. The positions of the focal points for a given source position were located by the exposure of dental x-ray film placed in the region of the focal points, using the internal conversion line from an intense Cs^{137} source. This source served as a source of electrons and positrons by reversing the direction of the focusing current for the location of the positron focus.

The second method used can be described as statistical separation.[10] The envelopes of the trajectories drawn in Fig. 1 illustrate how statistical separation takes place. The source of pairs is placed on the optical axis of the spectrometer. One of the properties of a magnetic lens spectrometer is that a ring focus exists for monoenergetic particles leaving the source in a small angular range. As shown in the figure, all trajectories pass through a ring focus, but the position at which a particle strikes the counting area depends upon the angle at which it is ejected from the source. Since a great many pairs are ejected with a small but appreciable angular separation, it is to be expected that many of them will be separated when arriving at the detector. Pair members with slightly different energies may also be focused at different regions of the detector. Therefore, coincidences may be counted between two counters placed at the focal point as shown in Fig. 1.

Both the statistical and off-axis methods have been used for counting pairs. The two methods are complementary for the following reasons. For a focusing coil separation of 8 inches, (inside to inside edges of the coils) the electron and positron focal points are separated by one inch if the source is off axis by one inch. For larger coil separations, the separation of the two foci becomes smaller. In order to increase the transmission of the spectrometer it is necessary to increase the coil separation, but this results in a very small

[9] K. Siegbahn and S. Johansson, Rev. Sci. Instr. 21, 442 (1950).
[10] S. J. Bame, Jr., and L. M. Baggett, Phys. Rev. 79, 415 (A) (1950).

separation or an overlapping of the positron and electron foci, so that two counters cannot be placed in position to count coincidences. For this reason the statistical method of separation becomes useful when large transmission is desired.

The single counting rates for a pair spectrometer of this type are quite high, because of several factors. Many pairs originating at the source have an energy or angular distribution such that only one member is focused to the counters. Lower energy gamma-rays produce photo- and Compton electrons which add to the single counting rate. In addition, many reactions of interest produce beta-emitters. This high single counting rate has been a limiting factor in the experiments described here, since Geiger counters are not suitable for extremely high counting rates, and small resolving times for coincidences.

Since the ratio of pairs counted to single counts is very small, it is important to reduce to a minimum spurious coincidences arising from single particles. A single electron or positron can cause a coincidence by counting in one counter and then suffering a double scattering in such a way as to set off the second counter. The number of these improbable events is of the same order of magnitude as true pair counts unless precautions are taken. Scattering from behind the counters is reduced by using a low Z material to back up the counters. In this case Lucite was used. All parts of the baffle system should be as far as possible from the counters and should be made of a material with a low Z. Thus, the baffle system of the spectrometer has been kept relatively simple, and the optimum system of baffles from the point of view of beta-ray spectroscopy has not been used. The disk baffle shown in Fig. 1 is made of Lucite, and aluminum rings are spaced on the inside of the tube to reduce scattering from the brass wall.

Three types of pair formation can be utilized by the spectrometer. These are the external, internal, and nuclear pair processes. External pairs[11] are formed by gamma-rays passing through a radiator. The spectrum of these pairs can be measured by placing a suitable converter in front of the target in the source position of the spectrometer. Internal pairs,[12–14] which are formed in the nuclear fields of the same nuclei which emit the quanta can be used with the spectrometer by placing the target in the source position without a radiator. In the same manner, the nuclear pair[15,16] from $F^{19}(p,\alpha)O^{16}$ can be detected.

The energy and angular distributions of external and internal pairs are similar, so that the spectrometer has nearly the same efficiency for counting either type of pair. The nuclear pair has a greater probability of being ejected with both members having the same energy, but the angular separation of the members is more likely to be large.[16–18] Experimentally, this spectrometer has proved to be roughly $\frac{1}{3}$ as efficient for nuclear pairs as for internal pairs.

For the measurement of gamma-ray spectra, internal pairs prove to have advantages over external pairs. In order to obtain as many external pairs from a radiator as are produced internally in the target, a lead radiator having an approximate thickness of 0.002 inch is required. While the target need be only as thick as necessary to stop the beam of charged particles, a radiator must be relatively thick. The consequent scattering and energy degradation of pairs reduce the efficiency and impair the resolution of the spectrometer. These considerations have prompted the use of the internally formed pairs whenever possible.

EXPERIMENTAL RESULTS

A. External Pairs from a Po-Be Source

The gamma-ray spectrum of a Po-Be source was determined, using statistical separation, in order to determine whether a spectrometer of this type might be adaptable for measuring weak intensity gamma-rays. The spectrum of this source contains a single line of energy 4.45 Mev from $Be^9(\alpha,n)C^{12}$, as determined and discussed by Terrell.[19] The Po-Be source has a diameter of about $\frac{7}{8}$ inch which makes it a poor source for a lens spectrometer, since small sources are necessary in order to obtain good resolution. The radiator for this run was 0.005 inch thorium, $1\frac{1}{2}$ inches by $1\frac{1}{2}$ inches square. The source was placed on the axis of the spectrometer with the radiator directly in front. The counting system was used with no apertures over the counters, and with the sensitive counting region centered on the optical axis. The coil separation for this case was 25 inches, inside to inside of the coils, and the radiator and counters were spaced 32 inches apart, symmetrically located within the coils. The disk baffle was kept against the center baffle for maximum transmission.

The spectrum in Fig. 3 consists of the completely uncorrected coincidence counting rate. The accidental coincidence rate was negligible, due to the weakness of the source. The background approximately follows the single counting rate, and is probably due to doubly scattered electrons, since the counting system is wide open. Knowing the original intensity of the source[19] and the half-life, it was calculated that the transmission of the spectrometer for this run was one pair count per 7.5×10^5 quanta.

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[15] W. A. Fowler and C. C. Lauritsen, Phys. Rev. **56**, 840 (1939); E. P. Tomlinson, Phys. Rev. **60**, 159 (A) (1941).
[16] J. R. Oppenheimer and J. S. Schwinger, Phys. Rev. **56**, 1066 (1939); J. R. Oppenheimer, Phys. Rev. **60**, 164 (A) (1941).
[17] S. Devons and G. R. Lindsey, Nature **164**, 539 (1949).
[18] Rasmussen, Hornyak, Lauritsen, and Lauritsen, Phys. Rev. **77**, 617 (1950).
[19] J. Terrell, Phys. Rev. **80**, 1076 (1950).

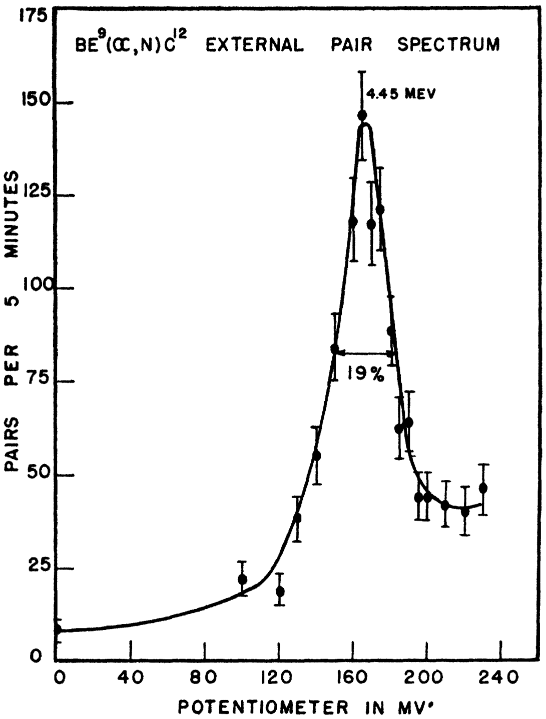

FIG. 3. External pair spectrum from a Po-Be source using a 0.005-inch thorium radiator with the spectrometer arranged for high efficiency detection. The counting rate at the peak corresponds to one pair count per 7.5×10^5 quanta.

B. Nuclear Pairs

The nuclear pairs from $F^{19}(p, \alpha)O^{16}$ offer a strong source of pairs for testing and calibrating a magnetic lens pair spectrometer. As a calibration for the run on $B^{10}+d$, the nuclear pair spectrum was taken using the off-axis method for pair separation. The coil separation for this arrangement was 10 inches. The source and counting system were symmetrically located with respect to the coils, in the same positions as shown in Fig. 1. The source was located $\frac{7}{8}$ inch below the optical axis, and the source diameter, as limited by the quartz aperture was $\frac{5}{16}$ inch. The counters were covered with $\frac{1}{8}$ inch lead plates with $\frac{3}{4}$ inch square apertures in the positions shown by the dotted lines in Fig. 2. The center of the apertures was $\frac{3}{4}$ inch below the optical axis.

This arrangement was tested with the internal conversion line from a Cs^{137} source. The results of this test, shown in Fig. 4, were obtained by taking the spectrum in both counters, for both directions of focusing current. The electron and "positron" lines have resolutions of 8 and 9 percent. To indicate the resolution attainable with the off axis separation method, Fig. 5 shows the internal conversion line taken with smaller apertures, the disk baffle in position to restrict the electron trajectories, and with a 14-inch coil separation. Of course, very strong gamma-sources would be necessary to make use of resolutions of this order.

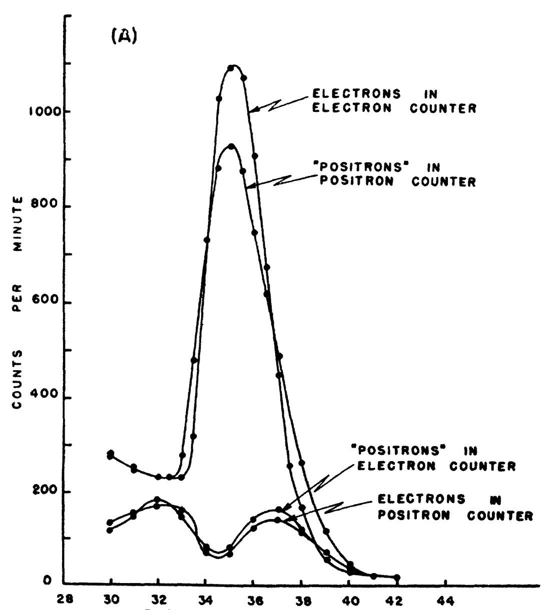

FIG. 4. Cs^{137} internal conversion line. This spectrum was a test of the off axis separation of pairs technique. The "positron" source was obtained by reversing the focusing current. Coil separation 10 in.; $\frac{1}{4}$ in. source $\frac{7}{8}$ in. off axis; exit apertures $\frac{3}{4}$ in. square, baffle system set for maximum transmission.

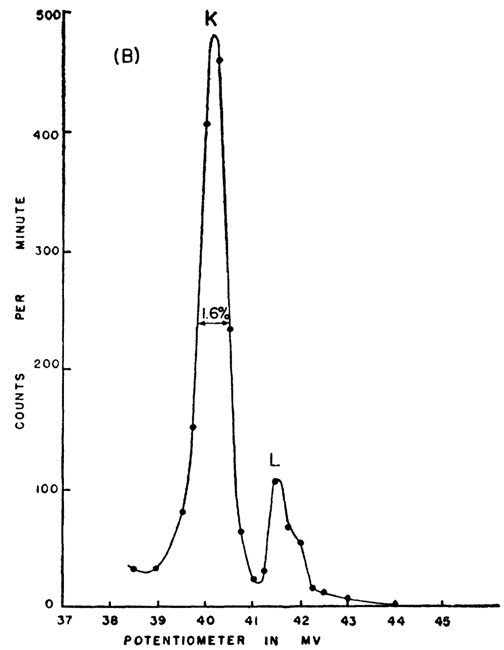

FIG. 5. Cs^{137} internal conversion line obtained with the source $\frac{7}{8}$ inch off axis. The spectrum was a test of the resolution attainable with this pair separation method. Coil separation 14 in.; $\frac{1}{4}$ in. source $\frac{7}{8}$ in. off axis; exit aperture $\frac{3}{4}$ in. diameter; baffle system set for good resolution.

The nuclear pair spectrum was taken with this arrangement, using a 9-mg/cm² calcium fluoride target evaporated on a 2-mg/cm² aluminum foil. The target was bombarded with 1.4-Mev protons, and pair counts were taken against a standard number of counts of a gamma-ray monitor and the current integrator. Figure 6 shows the nuclear pair spectrum obtained, with a resolution of 5.1 percent in contrast with the Cs^{137} calibration resolutions of 8 and 9 percent, illustrating the property of improved resolution for pairs to be expected with a spectrometer of this type. This spectrum has been corrected for a zero focusing current pair background and the accidental coincidence rate. This correction amounted to 80 counts per point, contrasted with 1600 counts at the peak. The nuclear pair total energy has been given by Chao, *et al.*[20] as 6.05±0.03 Mev, so this value has been used as a pair calibration for the spectrometer.

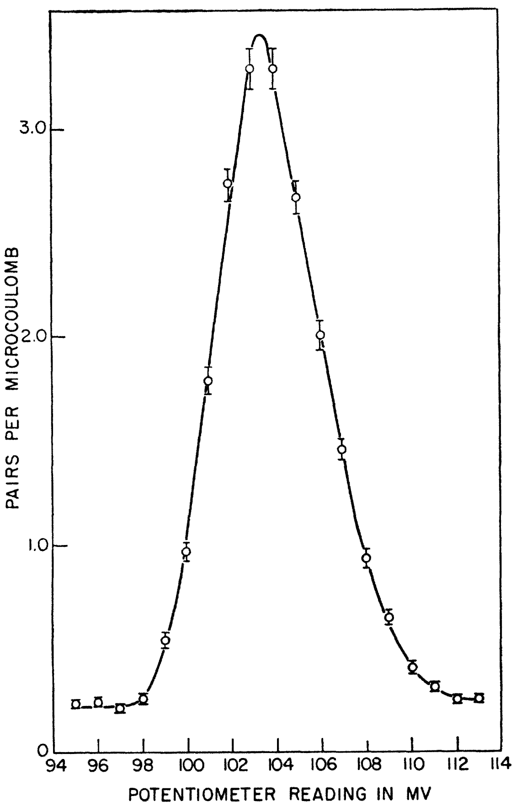

FIG. 6. Nuclear pair calibration line of total energy 6.05 Mev. The off axis separation technique was used with 1.4-Mev deuterons and a 9 mg/cm² calcium fluoride target.

C. Internal Pairs from $B^{10}+d$

The gamma-ray spectrum from $B^{10}+d$ was investigated with the off axis arrangement described in the previous section. A 20-mg/cm² target of 96 percent separated metallic B^{10} on a 1-mg/cm² aluminum foil was bombarded with 1.4-Mev deuterons. With a source of this thickness, approximately 95 percent of the pairs formed will be internal pairs. The resulting spectrum is shown in Fig. 7 plotted on an energy scale. The spectrum has been corrected by subtracting a zero

[20] Chao, Tollestrup, Fowler, and Lauritsen, Phys. Rev. **79**, 108 (1950).

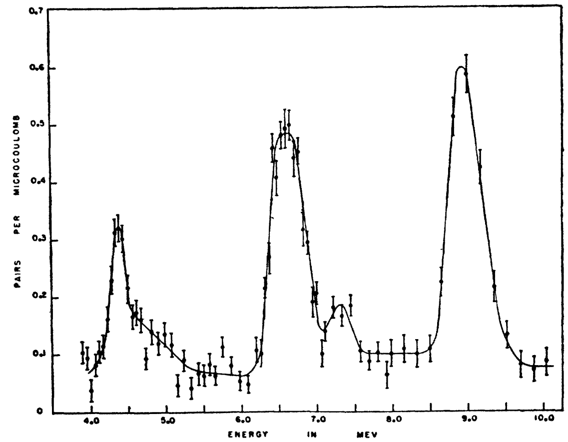

FIG. 7. Internal pair spectrum from $B^{10}+d$ with $E_d=1.4$ Mev, using a 20 mg/cm² metallic boron target. The energy scale is obtained using the 6.05-Mev nuclear pair calibration.

magnetic field background and the accidental coincidence rate. The total correction was about 30 percent of the counting rate. There appears to be a background remaining after the corrections have been made. This background can be due to unresolved, low intensity gamma-ray lines, to inaccuracies in making the corrections, and to coincidences due to annihilation radiation and doubly scattered electrons.

The gamma-ray lines found from this reaction are shown in Table I. The energies were determined on the basis of the nuclear pair calibration, taking the loss of energy in the calcium fluoride target to be 20 kev per particle and 33 kev in the B^{10} target. The probable errors assigned to the energy values were determined by inspection of the nuclear pair and internal pair spectra with regard to how the peaks may be shifted within the statistics.

From the high energy tail on the 4.43-Mev line, it is evident that one or more low intensity lines are present between 4.5 and 5.5 Mev. The resolution of the wide line is 8.8 percent, and enough data were taken to show that this line is essentially flat on top, in opposition to the sharp peak of the nuclear pair. The two energies given for the broad central peak are found by assuming this peak to be due to two nearly equal intensity lines

TABLE I. Gamma-ray energies determined from the internal pair spectrum of $B^{10}+d$, shown in Fig. 7. The yields, taken from Terrell and Phillips,* are used to calculate the spectrometer efficiency, which is defined as the number of pair counts per quantum.

Line	Gamma-ray energy	Resolution (%)	Assignment	Yield,* quanta per microcoulomb	Spectrometer efficiency
1	4.43±0.07	6.8	B^{11*}	34×10^6	0.77×10^{-8}
2	6.51±0.13	8.8	C^{11*}		
3	6.75±0.13		B^{11*}		
4	7.34±0.20		B^{11*}		
5	8.93±0.13	5.8	B^{11*}	12×10^6	4.3×10^{-8}

* See reference 3.

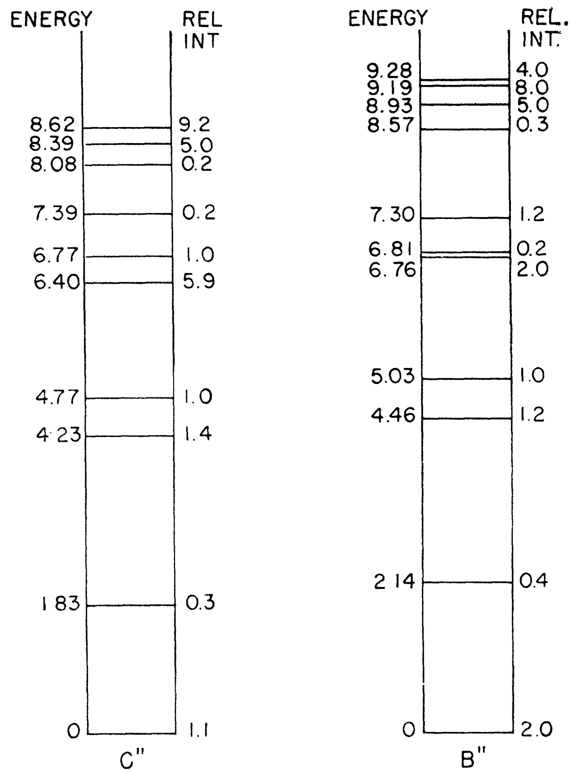

FIG. 8. $C^{11}-B^{11}$ energy levels from the neutron (see reference 7) and proton (see reference 5) groups. The neutron groups were taken at 0° to the beam at $E_d=3.64$ Mev. The proton groups were taken at 90° with $E_d=1.51$ Mev.

having line shapes similar to the nuclear pair line. That this line is actually two lines has been reported by Rutherglen,[4] who obtained energy values of 6.5 and 6.7 Mev in good agreement with the values obtained with this spectrometer. The statistics on the weak 7.34-Mev line are too poor to justify calculation of the resolution. This line has been observed in the spectra obtained from three independent runs with the spectrometer.

The yields for lines 1 and 5 shown in Table I were determined by Terrell and Phillips.[3] The efficiencies for this spectrometer were determined using these yields and the values for integrated beam current which were taken along with the gamma-ray monitor counts.

DISCUSSION

Several reactions which may account for the gamma-rays from $B^{10}+d$ are: (1) $B^{10}(d,\alpha)Be^8$, $Q=17.81$ Mev; (2) $B^{10}(d,p)B^{11}$, $Q=9.24$ Mev; and (3) $B^{10}(d,n)C^{11}$, $Q=6.53$ Mev. Reaction (1) yields Be^8 which has excited levels at 4.8, 7.0, and 9.8 Mev in addition to other levels. However, a lack of gamma-alpha coincidences from $B^{10}(d,\alpha)$ suggests that this reaction is not involved.[21] The reactions (2) and (3) have recently been studied by means of the proton[5,6] and neutron groups.[7,8]

[21] J. Thirion, Compt. rend. **229**, 1007 (1949).

The energy level schemes of the mirror nuclei B^{11} and C^{11} obtained by measurement of these groups are shown in Fig. 8, along with the observed relative intensities. The neutron groups giving the levels shown for C^{11} were measured at 0° by Johnson using 3.64-Mev deuterons. The proton groups, measured at 90° to the beam, were measured by Van Patter, *et al.*, using 1.51-Mev deuterons. In the work reported here, the levels in C^{11} above 7.93 Mev were not excited, since 1.4-Mev deuterons were used.

The broad, flat-topped line shown in Fig. 7 is attributed to the 6.76-Mev level of B^{11*} and the 6.40-Mev level of C^{11*}. The intensities of the two lines are assumed to be nearly equal, so that by inspection of the relative intensities of the neutron and proton groups, it appears that gamma-lines from any other of the C^{11*} levels will have small intensities in comparison with lines from B^{11*}.

Terrell and Phillips[3] have found that the low energy line at 4.5 Mev arises from the bombardment of both B^{10} and B^{11}. The line from the B^{11} reaction is attributed to $B^{11}(d,n)C^{12}$. The line observed in this work at 4.43 Mev comes primarily from B^{11*}. A small part is due to the B^{11} content of the target and possibly the 4.23-Mev level in C^{11*}.

The statistics on the high side of the 4.43-Mev line do not justify assignment of any gamma-ray energies, although it is probable that several lines are present. It appears likely that the 5.03-Mev level of B^{11*} is contributing to this broad distribution. In addition, the 4.77 level in C^{11*} and cascade quanta in both nuclei may be contributing.

The 7.34-Mev line is probably due to a transition to ground from the 7.30 level of B^{11*}. Finally, the 8.93-Mev line can be attributed to a ground transition from the 8.93 level of B^{11*}. This line is slightly broad, and within statistics, it is possible that there is a 9.19-Mev line having an intensity from 0–10 percent of that of the 8.93 line. The probable assignments of the gamma-ray lines are shown in Table I.

By comparing the relative intensities of the proton groups forming the B^{11*} levels, it appears that some explanation is needed for the absence of an appreciable amount of radiation from the upper two levels as well as for the relative intensities of the gamma-ray lines. From the yields given in Table I, the 4.43 line has 2.8 times the intensity of the 8.93 line. Assuming a twofold increase of the spectrometer efficiency between 6.75 and 8.93 Mev, the 6.75 line has approximately the same intensity as the 8.93-Mev line. These relative intensities do not agree with those for the proton groups.

A possible explanation for the absence of the high energy lines is that these levels usually break up with the emission of alpha-particles. The excitation energy necessary for a breakup of the B^{11*} into Li^7+He^4 is about 8.64 Mev, so the upper three levels possess enough energy to break up in this manner. The pair spectrum shows that the 8.93-Mev level emits ground transition gamma-rays, while no significant number seem to be contributed by the higher two levels. This could be explained by the higher energy available for alpha decay for the two upper levels, and the consequent reduction in the lifetime for the breakup. Bennett, Roys, and Toppel[22] have found from the reaction $Li^7(\alpha, \gamma)B^{11}$ that the 9.28 level has a width of 6 kev, which would indicate that this level should break up by particle emission. The 8.93 and 9.19 levels were shown to be appreciably narrower than the 9.28 level, within instrumental fluctuations. The results of Bennett, *et al.*, suggest that the radiation from these three levels consists of about 15 percent emission straight to ground, the remaining radiation being due to cascade transitions.

A consistent interpretation of the gamma-ray intensities may include the following: The 9.28 level mainly breaks up by the emission of an alpha-particle. The 9.19 level breaks up by emission of an alpha-particle in competition with gamma-emission which mainly appears in the form of cascade radiation. The 8.93 level can decay by a radiative transition straight to ground or by cascade transitions, with about equal probabilities for the two processes. Alpha-emission may also arise from the 8.93 level, but most likely has a small probability of occurring. This interpretation could explain the discrepancies of the relative intensities of the 4.43, 6.75, and 8.93-Mev lines, as well as the absence of quanta from the two upper levels.

The authors wish to thank Professor T. W. Bonner for suggesting this work and for his aid in all phases of the work. We also wish to thank the Los Alamos Scientific Laboratory for the loan of the Po-Be source.

[22] Bennett, Roys, and Toppel, Phys. Rev. 82, 20 (1951).

PHYSICAL REVIEW VOLUME 84, NUMBER 5 DECEMBER 1, 1951

Renormalization Theory of the Interactions of Nucleons, Mesons, and Photons

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(Received March 5, 1951)

A general program for the removal of divergencies from the theory of the interactions of nucleons, mesons, and photons is formulated. It is shown that the procedure is equivalent to renormalization of the constants of the theory.

THE success of the concept of renormalization in giving a theory of spinor electrodynamics that is both finite[1] and in good agreement with experiment[2] has made the extension of the method to other field theory problems most desirable, particularly the extension to the problem of nuclear forces. The first step in this direction is to show that the renormalized S-matrix is finite. This problem has not been satisfactorily dealt with until recently because of certain mathematical difficulties. The main source of trouble has been the overlapping divergencies. A process will be outlined below that in principle enables the renormalized S-matrix to be calculated to any degree of approximation. Bound-state phenomena remain outside the scope of the present treatment, and a suitable approach to the bound state remains to be discovered.

We select for study the interaction of pseudoscalar charged mesons, nucleons, and photons, with pseudoscalar coupling between meson and nucleon. A few remarks on the possible inclusion of neutral mesons will be made later. The Feynman rules for the construction of the S-matrix have been given by several authors,[3] and may be derived from the interaction hamiltonian

$$H=\frac{ie}{\hbar c}\left(\varphi^*\frac{\partial\varphi}{\partial x_\mu}-\frac{\partial\varphi^*}{\partial x_\mu}\varphi\right)A_\mu-\left(\frac{ie}{\hbar c}\right)^2\varphi^*\varphi A_\mu A_\mu$$
$$+ie(\bar{\psi}\gamma_\mu\gamma_p\psi)+if\bar{\psi}\gamma_5(\tau_-\varphi^*+\tau_+\varphi)\psi+\delta\lambda(\varphi^*\varphi)^2,$$

where mass renormalization and surface-dependent terms have been neglected. The functions $\varphi(x)$, $\psi(x)$, and $A_\mu(x)$, denote the meson, nucleon, and electromagnetic fields, respectively. We briefly state the rules again:[4]

1. Each photon line gives a factor
$$(2\pi)^{-3}\hbar c\delta_{\mu\nu}\int D_F(p)d^4p.$$
2. Each meson line gives a factor $(2\pi)^{-3}\int\Delta_F(p)d^4p$.
3. Each nucleon line gives a factor $(2\pi)^{-3}\int S_F(p)d^4p$.
4. Each meson-photon 3-vertex gives a factor
$$ie(\hbar c)^{-2}(2\pi)^4(p_1+p_2)_\mu\delta(p_1-p_2+p_3).$$
5. Each meson-photon 4-vertex gives a factor
$$-ie^2(\hbar c)^{-3}(2\pi)^4\delta_{\pi\rho}\delta(p_1-p_2+p_3+p_4).$$

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[4] The similar rules for external lines have been omitted.