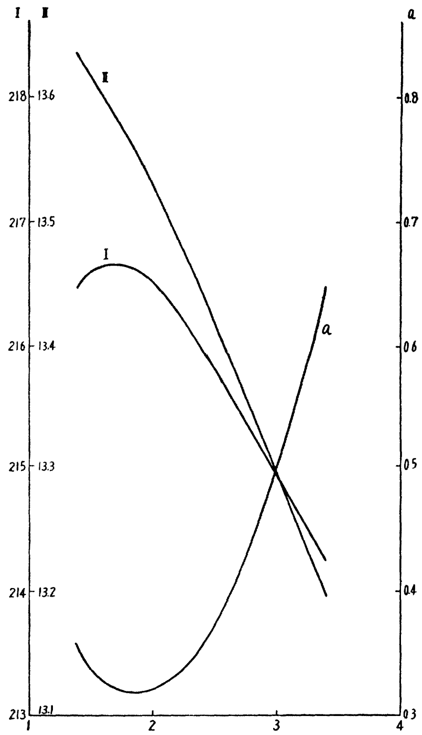

FIG. 3. The first-forbidden correction factors in the tensor interaction, to be applied to the beta-spectrum of RaE. The curves a, I, and II represent the correction factors corresponding to B_{ij}, $\int\beta\boldsymbol{\sigma}\times\mathbf{r}$, and $\{(\int\beta\boldsymbol{\sigma}\times\mathbf{r})^*\times\int\beta\boldsymbol{\alpha}+\text{c.c.}\}$, respectively.

RaE: In 1941, Konopinski and Uhlenbeck[3,7] explained the deviation of the spectrum of RaE from the allowed form by using mixtures of the matrix elements involved in the Fermi theory with vector or tensor interactions. The present analysis, however, leads to a criticism of their viewpoint, for the following two reasons: (1) According to Konopinski and Uhlenbeck, the beta-decay of RaE is classified as a second-forbidden transition. Since the decay of $_{83}\text{RaE}^{210}$ to the ground state of $_{84}\text{Po}^{210}$ is, on the current version of the shell model ($g_{9/2}$ or $i_{11/2}$, $h_{9/2}$)→(even, even) and therefore involves a parity change, one would expect that the transition is first-forbidden. (2) The conventional ft value of RaE is, according to the table of Feingold,[8] 1.1×10^8. On the other hand, the recent analysis of the second-forbidden beta-decay of Cl^{36},[9] Tc^{99}, Sb^{124},[10] and Cs^{137g},[11] by means of a linear combination of the matrix elements involved in the tensor or vector interactions, indicates that the conventional ft values for these beta-decays are $10^{12}\sim10^{13}$. When the ft values[12] are corrected by the forbidden f-functions, they do not change very drastically. This speaks strongly in favor of the classification of RaE as first-forbidden, not second-forbidden. In fact, for the beta-decays which have an "a" type spectrum and obey the selection rule $\Delta J=\pm2$, parity change yes, Taketani[12] and Davidson[5] found $10^7\approx10^8$ for the ft values, with little Z dependence. Although Konopinski and Uhlenbeck had obtained negative results, we reinvestigated, therefore, the first-forbidden cases for RaE, since the exact expansion of the coulomb correction in the same way as that of Tm^{170} may introduce some alterations.

It is shown that among the transitions involving parity change, (1) $\Delta J=\pm2$ is excluded. As is shown in Fig. 3, the correction factor of "a" type is uniquely determined and the predicted shape is evidently at variance with experiment. (2) $\Delta J=0$ is also rejected. Since the ground state of Po^{210} is presumably spin 0 and even, the nuclear matrix element allowed in the transition is determined to be $\int\beta\boldsymbol{\sigma}\cdot\mathbf{r}$. The correction factor corresponding to $\int\beta\boldsymbol{\sigma}\cdot\mathbf{r}$ is almost energy-independent, which cannot be fitted to experiment. (3) $\Delta J=\pm1$ involves a linear combination of several matrix elements, i.e., $\int\beta\boldsymbol{\sigma}\times\mathbf{r}$ and $\int\beta\boldsymbol{\alpha}$ in the tensor interaction, and $\int\mathbf{r}$ and $\int\boldsymbol{\alpha}$ in the vector interaction. Any real value for the ratio of the nuclear matrix elements, k_{1T} or k_{1V}, cannot yield the required shape of RaE (see Fig. 3).

[1] J. S. Fraser, Phys. Rev. **76**, 1540 (1949).
[2] H. M. Agnew, Phys. Rev. **77**, 655 (1950); see also D. Saxon and J. Richards, Phys. Rev. **76**, 186 (1949).
[3] E. J. Konopinski and E. Uhlenbeck, Phys. Rev. **60**, 308 (1941). See page 315.
[4] J. Davidson, Phys. Rev. **82**, 48 (1951).
[5] J. Davidson, Phys. Rev. **82**, 48 (1951).
[6] Nakamura, Shima, and Kobayashi, J. Phys. Soc. **4**, 166 (1949).
[7] E. J. Konopinski, Revs. Modern Phys. **15**, 209 (1943).
[8] A. M. Feingold, Revs. Modern Phys. **23**, 10 (1951).
[9] C. S. Wu and L. Feldmen, Phys. Rev. **82**, 457 (1951); H. W. Fulbright and J. C. D. Milton, Phys. Rev. **82**, 274 (1951).
[10] Nakamura, Umezawa, and Takebe, Phys. Rev. **83**, 1273 (1951); also Prog. Theoret. Phys. **6**, No. 4 (1951).
[11] L. M. Langer and R. J. D. Moffat, Phys. Rev. **82**, 635 (1951).
[12] Taketani, Nakamura, Umezawa, Ono, and Yamaguchi, Phys. Rev. **82**, 441 (1951).

Relativistic Corrections to the Lamb Shift

MICHEL BARANGER
Cornell University, Ithaca, New York
(Received June 18, 1951)

THE α^6 corrections to the Lamb shift have been partly calculated, using a decomposition of terms due to Feynman. The one-photon part of the Lamb shift corresponds to the diagrams of Fig. 1, and can be written, according to Feynman's standard methods,[1]

$$\Delta E=e^2\int\bar{\psi}_0(2)\gamma_\mu K_+{}^V(2,1)\gamma_\mu\psi_0(1)\delta_+(s_{21}{}^2)d^3\mathbf{x}_1d^3\mathbf{x}_2d(t_2-t_1)-\Delta m\int\bar{\psi}_0(1)\psi_0(1)d^3\mathbf{x}_1.$$

By going to momentum space, and making an algebraic rearrangement of terms, this can be decomposed in the following way:

$$\begin{aligned}\Delta E=&-(e^2/\pi)\int\bar{\phi}_0(\mathbf{p}_2)\langle Va\rangle_\mu(p_2,p_2+s_2-k)K_+{}^V\\&\times(E_0-\omega;\mathbf{p}_2+\mathbf{s}_2-\mathbf{k},\mathbf{p}_1+\mathbf{s}_1-\mathbf{k})\\&\times\langle Ve\rangle_\mu(p_1+s_1-k,p_1)\phi_0(\mathbf{p}_1)k^{-2}d^4k_Fd^3\mathbf{p}_1d^3\mathbf{p}_2d^3\mathbf{s}_1d^3\mathbf{s}_2\\&+(e^2/\pi i)\int\bar{\phi}_0(\mathbf{p}_2)(2p_{2\mu}-\gamma_\mu\mathfrak{k})\mathfrak{V}(\mathbf{p}_2-\mathbf{p}_1)(2p_{1\mu}-\mathfrak{k}\gamma_\mu)\phi_0(\mathbf{p}_1)\\&\times(k^2-2p_2\cdot k)^{-1}(k^2-2p_1\cdot k)^{-1}k^{-2}d^4k_Fd^3\mathbf{p}_1d^3\mathbf{p}_2\\&-(e^2/\pi i)\int\bar{\phi}_0(\mathbf{p}_2)\mathfrak{V}(\mathbf{p}_2-\mathbf{p}_1)(2p_{1\mu}-\gamma_\mu\mathfrak{k})(2p_{1\mu}-\mathfrak{k}\gamma_\mu)\phi_0(\mathbf{p}_1)\\&\times(k^2-2p_1\cdot k)^{-2}k^{-2}d^4k_Fd^3\mathbf{p}_1d^3\mathbf{p}_2\\&+(e^2/\pi i)\int\bar{\phi}_0(\mathbf{p})(2p_\mu-\gamma_\mu\mathfrak{k})\gamma_\mu\phi_0(\mathbf{p})(k^2-2p\cdot k)^{-1}k^{-2}d^4k_Fd^3\mathbf{p}\\&-\Delta m\int\bar{\phi}_0(\mathbf{p})\phi_0(\mathbf{p})d^3\mathbf{p}.\end{aligned}$$

We are using the following notations: $\mathbf{a}$ represents a 3-vector, a represents a 4-vector, and $\mathfrak{a}$ represents $\Sigma_\mu a_\mu\gamma_\mu$. $\phi_0(\mathbf{p})$ is the normalized momentum wave function of the level under study, whose unperturbed energy is E_0. k is the 4-vector $(\omega,\mathbf{k})$. p, p_1, p_2 are the 4-vectors $(E_0,\mathbf{p})$, $(E_0,\mathbf{p}_1)$, $(E_0,\mathbf{p}_2)$. s_1, s_2 are the 4-vectors $(0,\mathbf{s}_1)$, $(0,\mathbf{s}_2)$. $\mathfrak{V}(\mathbf{q})$ is the Fourier transform of the potential energy times γ_t,

$$\mathfrak{V}(\mathbf{q})=(2\pi)^{-3}\int\mathfrak{V}(\mathbf{x})\exp(-i\mathbf{q}\cdot\mathbf{x})d^3\mathbf{x}.$$

$K_+{}^V(E;\mathbf{p}_1,\mathbf{p}_2)$ is the Fourier transform of the propagation kernel

$$K_+{}^V(E;\mathbf{p}_1,\mathbf{p}_2)=(2\pi)^{-3}\int K_+{}^V(2,1)\exp[-i(\mathbf{p}_2\cdot\mathbf{x}_2-\mathbf{p}_1\cdot\mathbf{x}_1)+iE(t_2-t_1)]d^3\mathbf{x}_2d^3\mathbf{x}_1d(t_2-t_1).$$

$\langle Ve\rangle_\mu(p_f-k,p_i)$ is the "modified potential" for emission of a photon of polarization μ,

$$\langle Ve\rangle_\mu(p_f-k,p_i)=\mathfrak{V}(\mathbf{p}_f-\mathbf{p}_i)(2p_{i\mu}-\mathfrak{k}\gamma_\mu)(k^2-2p_i\cdot k)^{-1}-(2p_{f\mu}-\mathfrak{k}\gamma_\mu)\mathfrak{V}(\mathbf{p}_f-\mathbf{p}_i)(k^2-2p_f\cdot k)^{-1},$$

and makes transitions from momentum p_i to momentum $p_f - k$, p_i and p_f having the same time component E_0. $\langle Va\rangle_\mu(p_f, p_i - k)$ is the modified potential for absorption

$$\langle Va\rangle_\mu(p_f, p_i-k) = -\mathfrak{V}(\mathbf{p}_f-\mathbf{p}_i)(2p_{i\mu}-\gamma_\mu \mathfrak{k})(k^2-2p_i\cdot k)^{-1} + (2p_{f\mu}-\gamma_\mu \mathfrak{k})\mathfrak{V}(\mathbf{p}_f-\mathbf{p}_i)(k^2-2p_f\cdot k)^{-1}$$

and makes transitions from $p_i - k$ to p_f.

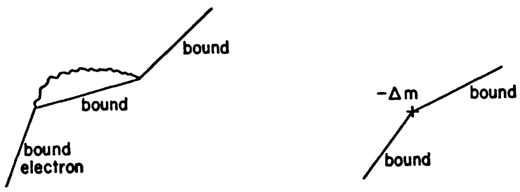

FIG. 1. Feynman diagrams for the one-photon part of the Lamb shift.

The last 4 terms in ΔE are identical with the "first-order Lamb shift," evaluated by several authors,[2] except for the fact that the momenta are not free, p_1^2, p_2^2, $p^2 \neq m^2$, and $\mathfrak{p}\phi_0 \neq m\phi_0$. This fact, when taken into account, introduces a term proportional to the square of the potential, and therefore not gauge-invariant. It has been shown by Bethe that this term is compensated by an equal and opposite one coming from the first term in ΔE, or "second-order Lamb shift."

After subtraction of this nongauge-invariant part, the second-order Lamb shift has been calculated by taking a comparison operator

$$R_\mu = (m\omega)^{-1}\nabla_\mu V \quad \text{for} \quad \mu = 1, 2, 3$$

and writing: Second-order Lamb shift $= (\langle Va\rangle_\mu, \langle Ve\rangle_\mu)$, the intermediary state being a relativistic free state, and with electron at rest initially and finally; $-(R_\mu{}^*, R_\mu)$, the intermediary state being a nonrelativistic free state, electron at rest initially and finally; $+(R_\mu{}^*, R_\mu)$, all states being nonrelativistic bound states. It has been found that this is a sufficient approximation if only terms of order α^6 are desired. The last part is nothing but Bethe's Lamb shift[3]

$$(2e^2/3\pi m^2)\Sigma_n' |(\phi_n|\mathbf{p}|\phi_0)|^2(E_n-E_0)[\ln(\lambda/2|E_n-E_0|)+5/6].$$

The first two parts we have now calculated; they give a correction of

$$(1+11/128-\ln 2/2)\alpha^4\,\mathrm{Ry} = 6.894 \text{ megacycles}$$

to the 2s state of hydrogen.

This brings the total Lamb shift[4] to 1058.3 Mc, when vacuum polarization terms are included. The best experimental value is 1061±2 Mc.[5]

The α^6 corrections to the first-order Lamb shift have not yet been investigated. However, in the summer of 1950 Kroll thought that there could be none, the highest corrections being of order $\alpha^7 \ln\alpha$. There are several other α^6 corrections, coming from the vacuum polarization term, and from the two-photon term, but they are probably rather small.

This problem was suggested by Professors H. A. Bethe and R. P. Feynman, and the author is greatly indebted to them for continued guidance throughout the work.

[1] R. P. Feynman, Phys. Rev. **76**, 749 (1949); Phys. Rev. **76**, 769 (1949).
[2] N. H. Kroll and W. E. Lamb, Phys. Rev. **75**, 388 (1949); J. B. French and V. F. Weisskopf, Phys. Rev. **75**, 1240 (1949); R. P. Feynman, Phys. Rev. **76**, 777 (1949); J. Schwinger, Phys. Rev. **76**, 790 (1949).
[3] H. A. Bethe, Phys. Rev. **72**, 339 (1947).
[4] Bethe, Brown, and Stehn, Phys. Rev. **77**, 370 (1950).
[5] R. C. Retherford and W. E. Lamb, Jr., Phys. Rev. **75**, 1325 (1949).