A Phenomenological Treatment of Photomeson Production from Deuterons

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A phenomenological treatment of charged and neutral photomeson production from deuterons is presented which is independent of the details of the meson theory. It is shown that an experimental comparison of the deuteron and proton cross sections can be used to determine the nucleon spin flip probability.

I. INTRODUCTION

THE available experimental information about π-mesons produced by γ-rays incident on free protons[1] is not at present satisfactorily interpretable theoretically, even though this is one of the simplest meson reactions. The difficulty lies in current meson theory which is incapable of making quantitative predictions so long as the meson-nucleon coupling is not weak. For photomeson reactions in light complex nuclei, nevertheless,[2] it is possible to make a phenomenological analysis based on the impulse approximation. This paper presents such a treatment for the special case of the deuteron. The method is expected to be applicable no matter what theory of mesons and their coupling to nucleons finally turns out to be correct.

The production of mesons by photons on deuterium as contrasted to that from free protons is of interest because of the information it gives about the dependence of the reaction on the charge and spin of nucleons. The charge dependence is manifested in the positive-negative ratio, since only the neutron can serve as a source of negative mesons and only the proton for positives. The spin dependence, on the other hand, shows up in the relative number of positive mesons emitted from a proton bound in a deuteron compared to the number emitted when the proton is free. Feshbach and Lax[3,4] have pointed out that unless the proton spin changes in the emission process, the deuteron cross section for positive photomeson production will be diminished relative to the free proton cross section. This is because in the former case there are two neutrons left in a triplet spin state, and due to the Pauli principle they can have only odd orbital angular momentum. For γ-ray energies presently available from synchrotrons and betatrons, the relative energy of the two neutrons is likely to be of the order of 15 Mev, or even less when the meson is emitted forward. The exclusion of S states therefore can reduce the available phase space considerably. It will be shown in this paper that the exclusion principle effect may be expressed in terms of the spin change probability without reference to meson theory. The conclusion reached therefore, is that the ratio of deuteron to proton cross sections can be used as a direct and quantitative measure of the nucleon spin flip probability.

Neutral photomeson production from deuterons will also be considered. Both neutron and proton can act as source in this case, so there will be interference effects. These will be of the same order of magnitude as the exclusion principle effect in the charged meson production but of a sign which depends on the detailed meson theory.

II. GENERAL CONSIDERATIONS FOR CHARGED MESON PRODUCTION

The fundamental assumption of the impulse approximation[2] is that the meson production "amplitudes" from the various nucleons (in this case two) are linearly superposable to form the production amplitude for the whole nucleus. This is an assumption analogous to the use of Born approximation in the scattering of x-rays by many electron systems or the Fermi approximation in the scattering of slow neutrons by crystals. Sufficient conditions for validity[2] are: (1) Small individual amplitudes in comparison to the distance between sources. (2) Long mean free paths for both incoming and outgoing particles in comparison to the over-all dimensions of the system. (3) A "collision time" short compared to the period of the nuclear system. These conditions are well satisfied for our case. The smallness of the fine structure constant guarantees the small amplitudes and long photon mean free path. The condition on the outgoing meson mean free path can, in the case of the deuteron, be restated as a requirement that the meson-nucleon scattering cross section be small compared to the cross-sectional area of the deuteron. The latter is $\sim 10^{-24}$ cm^2, whereas the meson-nucleon cross section[5] is only $\sim 10^{-26}$ cm^2. The collision time seems certain to be sufficiently small, since the average extent to which energy

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[1] J. Steinberger and A. S. Bishop, Phys. Rev. **78**, 494 (1950); Bishop, Steinberger, and Cook, Phys. Rev. **80**, 291 (1950); Steinberger, Panofsky, and Steller, Phys. Rev. **78**, 802 (1950).
[2] G. F. Chew and G. C. Wick (to be published).
[3] H. Feshbach and M. Lax, Phys. Rev. **76**, 134 (1949).
[4] M. Lax and H. Feshbach, Phys. Rev. **81**, 189 (1951).
[5] Chedester, Isaacs, Sachs, and Steinberger, Phys. Rev. **82**, 958 (1951).

conservation is violated in the intermediate states must be of the order of the meson rest energy.

An additional possibility arises, when we consider nucleons bound in a nucleus, which could still invalidate the use of the superposition approximation. Strong nuclear forces may actually modify the character of the individual scattering centers. In other words, the charge-current distribution of meson-nucleon field which we call a free proton may become polarized when that "proton" is inside a complex nucleus, leading to "exchange" currents which have an electromagnetic interaction absent for free nucleons. This possibility is ignored in the present paper on the qualitative grounds that the deuteron is so weakly bound that the meson "clouds" surrounding the individual neutron and proton overlap only slightly. The question is receiving further study.

It will be sufficient to restrict consideration to nucleon recoil velocities which are small compared to the velocity of light, since for larger recoils the deuteron will behave simply as a pair of free nucleons. The outgoing meson, however, must be treated relativistically. Using units in which $\hbar=c=1$, and the meson rest mass is also unity, the incident photon energy will be denoted by ν_0, and its momentum by ν, the emitted meson energy and momentum by μ_0 and μ. We assume the deuteron to be initially at rest with binding energy $-\epsilon_i$ and represent the total momentum of the recoiling nucleon system by $\mathbf{D}$ and the internal (relative) energy by ϵ_f. The initial energy of the system as a whole is then $\nu_0+2M+\epsilon_i$, and the final energy is $\mu_0+2M+D^2/4M+\epsilon_f$. The neutron-proton mass difference will be ignored. Thus the cross section for meson emission into the momentum interval $d\boldsymbol{\mu}$, while the nucleus recoils into the interval $d\mathbf{D}$ and the internal state f, may be expressed in the form

$$d\sigma_f/d\boldsymbol{\mu}d\mathbf{D}=(2\pi)^{-2}|Q_f|^2\delta(\mu_0+\epsilon+D^2/4M-\nu_0) \quad (1)$$

where Q_f is the appropriate element of the scattering matrix, and $\epsilon=\epsilon_f-\epsilon_i$. According to our fundamental assumption, the scattering matrix is to be approximated by

$$T=T_1+T_2,$$

where T_j is the part of the scattering matrix which describes the photoproduction of, let us say, a positive meson by the jth nucleon acting alone. (The positive and negative meson production problems are obviously symmetrical.) The dependence of T_j on the nucleon position $\mathbf{r}_j$, spin $\boldsymbol{\sigma}_j$, and isotopic spin $\boldsymbol{\tau}_j$, may be made explicit by writing

$$T_j=(\boldsymbol{\sigma}_j\cdot\mathbf{K}+L)\exp[i(\boldsymbol{\nu}-\boldsymbol{\mu})\cdot\mathbf{r}_j]\tau_j^+. \quad (2)$$

This form is required by momentum conservation and the properties of the Pauli spin operator. If $\mathbf{K}$ and L are allowed to depend on the momenta of photon, meson, and nucleon, as well as on the polarization of the photon, then the form is completely general. (Even though we are treating the meson spin as if it were zero, most of the important results do not actually depend on this point.)

$|Q_f|^2$ must be summed over the final and averaged over the initial spin and isotopic spin states. The initial deuteron is a spin triplet and an isotopic spin singlet

$$|i\rangle=2^{-\frac{1}{2}}[p(1)n(2)-p(2)n(1)]^3\chi_m(2\pi)^{-\frac{3}{2}}u_i(\rho), \quad (3)$$

where $\boldsymbol{\rho}=\mathbf{r}_1-\mathbf{r}_2$ is the relative coordinate and ${}^3\chi_m$ is the triplet spin function, m being the z component of the total spin. The small amount of spin-orbit coupling in the deuteron is being neglected.

The final state consists of two neutrons, an isotopic triplet, with asymptotic relative momentum $\mathbf{k}_f$, so that $\epsilon_f=k_f^2/M$. The Pauli principle requires separate consideration of the two final states: one that is symmetric and one that is antisymmetric in space.

$$|f_e\rangle=(2\pi)^{-\frac{3}{2}}n(1)n(2)^1\chi_0\, u_{f,e}(\boldsymbol{\rho})\exp(i\mathbf{D}\cdot\mathbf{R}), \quad (4a)$$

$$|f_0\rangle=(2\pi)^{-\frac{3}{2}}n(1)n(2)^3\chi_m u_{f,o}(\boldsymbol{\rho})\exp(i\mathbf{D}\cdot\mathbf{R}). \quad (4b)$$

$\mathbf{R}=\frac{1}{2}(\mathbf{r}_1+\mathbf{r}_2)$ is the center-of-mass coordinate. We now introduce (2), (4a) or (4b) into Q_f. The integration over $\mathbf{R}$ leads to a momentum conservation requirement that only final states with $\mathbf{D}=\boldsymbol{\nu}-\boldsymbol{\mu}$ be allowed, and the matrix elements for the symmetric and antisymmetric cases become respectively (with the momentum delta-function factored out)

$$Q_{f,e}=2^{-\frac{1}{2}}\langle{}^1\chi_0|\mathbf{K}\cdot(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)|{}^3\chi_m\rangle E_f, \quad (5a)$$

$$Q_{f,o}=2^{-\frac{1}{2}}\langle{}^3\chi_m|\mathbf{K}\cdot(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)+2L|{}^3\chi_m\rangle O_f, \quad (5b)$$

where

$$E_f=\int u_{f,e}^*(\boldsymbol{\rho})\exp(i\mathbf{k}_0\cdot\boldsymbol{\rho})u_i(\rho)d\boldsymbol{\rho}, \quad (6a)$$

$$O_f=\int u_{f,o}^*(\boldsymbol{\rho})\exp(i\mathbf{k}_0\cdot\boldsymbol{\rho})u_i(\rho)d\boldsymbol{\rho}, \quad (6b)$$

if $\mathbf{k}_0=\frac{1}{2}\mathbf{D}$.

Since the final states u_f are either even or odd, it is conventional to extend any integrations which may be done only over half of $\mathbf{k}_f$ space, providing the normalization of the final states is taken to be:

$$\int u_{f,e}^*(\boldsymbol{\rho})u_{f',e}(\boldsymbol{\rho})d\boldsymbol{\rho}=\delta(\mathbf{k}_f-\mathbf{k}_f')+\delta(\mathbf{k}_f+\mathbf{k}_f')$$

$$\int u_{f,o}^*(\boldsymbol{\rho})u_{f',o}(\boldsymbol{\rho})d\boldsymbol{\rho}=\delta(\mathbf{k}_f-\mathbf{k}_f')-\delta(\mathbf{k}_f+\mathbf{k}_f').$$

One may already see from (5) that only $\mathbf{K}$ is effective in changing the spin state of the deuteron. It is also clear from (6) that O_f approaches zero at threshold ($k_f=0$) and also whenever the nuclear recoil is zero ($k_0=0$). Under either of these circumstances, then, the spin flip term will dominate.

In obtaining (5), the assumption has been made that $\mathbf{K}$ and L depend only on $\boldsymbol{\nu}$ and $\boldsymbol{\mu}$ and not at all on the nucleon momenta. For charged meson production, where the nucleon currents are relatively unimportant, this is certainly justified, and even for neutral production, it is not unreasonable. This is because the final momentum of the struck particle is usually not far from the value $\mathbf{D}=\boldsymbol{\nu}-\boldsymbol{\mu}$ which it would assume if the proton were free and at rest. One can understand $\mathbf{K}$ and L to be evaluated for zero initial nucleon momentum and a final nucleon momentum equal to $\boldsymbol{\nu}-\boldsymbol{\mu}$ and need neglect only the relatively small fluctuations about these values.

The expressions (5a) and (5b) are to be squared, summed over final nuclear spins and averaged over initial nuclear spins and photon polarizations. This yields

$$\langle|Q_f|^2\rangle_{\mathrm{Av}}=\tfrac{2}{3}\langle|\mathbf{K}^2|\rangle_{\mathrm{Av}}|E_f|^2 +[4/3\langle|\mathbf{K}^2|\rangle_{\mathrm{Av}}+2|L^2|]\langle|O_f|^2\rangle_{\mathrm{Av}} \quad (7)$$

with conservation of momentum understood. To go further we must decide what distribution function we should like to have. One may ask for the total number of mesons at a given angle, as a function of angle, or perhaps also the spectrum of mesons at each angle. For certain experiments one might even be interested in the distribution of nucleon recoils. The most natural theoretical distribution parameter turns out to be the square of the nuclear recoil, that is, D^2. We shall first discuss the distribution in this quantity and then show that it has a rather close connection to the meson angular distribution.

III. THE DISTRIBUTION OF NUCLEAR RECOILS

If the momentum conservation condition is used to eliminate the meson momentum as a degree of freedom in (1), the distribution of nuclear recoils is obtained:

$$d\sigma_f/d\mathbf{D}=(2\pi)^{-2}\langle|Q_f|^2\rangle_{\mathrm{Av}}\delta(\mu_0+\epsilon+D^2/4M-\nu_0) \quad (8)$$

where

$$\mu_0=((\boldsymbol{\nu}-\mathbf{D})^2+1)^{\frac{1}{2}}=(\nu_0^2+D^2-2\nu_0 D\cos\theta_D+1)^{\frac{1}{2}}.$$

The energy delta-function may next be used to eliminate θ_D, the angle between $\boldsymbol{\nu}$ and $\mathbf{D}$, leaving as free variables the magnitude of $\mathbf{D}$ and the azimuth of $\mathbf{D}$ with respect to $\boldsymbol{\nu}$. The cross section obviously does not depend on this angle for an unpolarized beam of photons, so we end up with the distribution in D^2 for a given final nuclear state f,

$$d\sigma_f/dD^2=(2\pi)^{-1}\langle|Q_f|^2\rangle_{\mathrm{Av}}(\nu_0-\epsilon-D^2/4M)/2\nu_0. \quad (9)$$

The limits on D^2 are determined by the equation,

$$(\nu_0-\epsilon-D^2/4M)^2=(\nu_0-D)^2+1, \quad (10)$$

which may be solved to give the maximum value of k_f^2 for a given value of D,

$$k_m^2=M[\nu_0-((\nu_0-D)^2+1)^{\frac{1}{2}}]-(\alpha^2+k_0^2), \qquad \alpha^2=-M\epsilon_i. \quad (11)$$

In Fig. 1, a plot is made of k_m^2 as a function of D^2 for several values of ν_0.

From a general point of view, the problem is now solved. Using the ground-state deuteron wave function and the continuum wave functions of the di-neutron, the overlap integrals E_f and O_f may be evaluated. The matrix elements K and L are to be given by the meson theory. From a practical point of view, however, it is important to make further approximations, both to avoid excessive lator and to circumvent the present lack of a reliable meson theory.

A study of the matrix elements Q_f reveals that for a recoil, $D=2k_0$, the important final nuclear states are restricted to a range of energies whose half-width is of the order $3\alpha k_0/M$, while from (11), it is seen that k_0 cannot be greater than ν_0. The average energy transfer, ϵ, corresponding to a recoil, D, is approximately $D^2/4M$. (This last point can be understood qualitatively by going to the limit $\alpha=0$, so that the proton behaves as if completely free, absorbing the entire recoil.) Now in

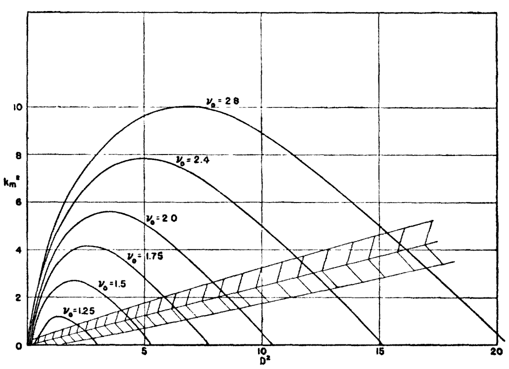

FIG. 1. A diagram of k_m^2, the upper limit of k_f^2, as a function of ν_0 and D^2. The shaded region includes the most important contributions.

(9), the only dependence on f, except for the matrix element Q_f, occurs in the term, ϵ. If the variation in ϵ is only $\sim 3(\alpha/M)\nu_0$ for a given value of D, the variation may be neglected, since $\alpha/M\approx 1/20$. Inserting (7) and replacing by its approximate average, $D^2/4M$, one may rewrite (9) as

$$d\sigma_f/dD^2=(2\pi)^{-1}\{2(\tfrac{1}{3}|E_f|^2+\tfrac{2}{3}|O_f|^2)\langle|\mathbf{K}^2|\rangle_{\mathrm{Av}} +2|O_f|^2\langle|L|^2\rangle_{\mathrm{Av}}\}(\nu_0-D^2/2M)/2\nu_0. \quad (12)$$

The matrix elements $\mathbf{K}$ and L depend on the final nuclear state through their dependence on the meson momentum $\boldsymbol{\mu}=\boldsymbol{\nu}-\mathbf{D}$ (which depends on the angle θ_D and thus on ϵ_f). We shall assume, however, that the variation is small over the range of directions of D which can occur with appreciable probability for a given magnitude D. This assumption is more difficult to justify quantitatively than the preceding one and will be better for some meson theories than for others. It leaves only E_f and O_f as functions of the final nuclear state f, and formula (12) should now be compared

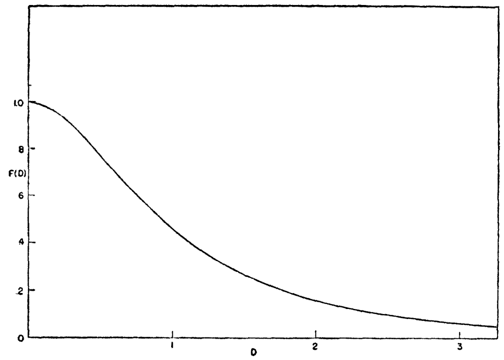

FIG. 2. The exclusion principle function $F(D)$ according to formula (A3).

with the corresponding formula for a free proton,

$$d\sigma_p/dD^2=(2\pi)^{-1}\{\langle|\mathbf{K}^2|\rangle_{\text{Av}}+\langle|L|^2\rangle_{\text{Av}}\}(\nu_0-D^2/2M)/2\nu_0$$
$$=d\sigma_K/dD^2+d\sigma_L/dD^2. \quad (13)$$

The "spin flip" and the non-"spin flip" terms have been separated. In this notation, formula (12) becomes

$$d\sigma_f/dD^2=2(\tfrac{1}{3}|E_f|^2+\tfrac{2}{3}|O_f|^2)d\sigma_K/dD^2 +2|O_f|^2d\sigma_L/dD^2, \quad (12')$$

and we see that if the individual spin flip and non-spin flip cross sections are known for the free proton, as a function of the proton recoil D, then by the calculation of the overlap integrals E_f and O_f one may obtain the deuteron cross section, without any reference to meson theory. Conversely, an experimental knowledge of both deuteron and proton cross sections allow a direct determination of the separate spin flip and non-spin flip terms, since these terms occur with different weights in the two processes.

The evaluation of E_f and O_f, once the deuteron and di-neutron wave functions are known, is straightforward but tedious. In a forthcoming paper, Feshbach, Goldberger, and Villars[6] will describe the calculation and present some results, using the wave functions of the Hulthèn potential. A principal feature which can be obtained only from such a calculation is the "resonance" in the cross section due to the "almost binding" of the singlet di-neutron. This causes E_f to increase sharply for values of ϵ_f less than 1 Mev, the effect being spectacular for small values of D. Unless the spin flip term, $d\sigma_K/dD^2$, is very small, this resonance will dominate the behavior of the cross section near threshold.

The existence of the neutron-neutron interaction, which tends to favor final states with low values of ϵ_f, actually helps make valid a very useful further approximation. Suppose one wishes to know the total cross section for a given recoil D, but for all allowable final states f. If the sum is extended to all values of k_f (i.e., ϵ_f), even those not allowed by the energy conservation condition (11) (see Fig. 1), then one is in a position to employ closure theorems. This overestimates the cross section, but if the energetic upper limit on ϵ_f is sufficiently greater than the "average" final nuclear energy, $D^2/4M$, the contribution from the forbidden states will not be important. The $n-n$ interaction helps keep the spread in energies ϵ_f on the low side of the above value.

The closure theorems which we need may be stated in the following form,

$$\int u_{f,e}^*(\boldsymbol{\rho})u_{f,e}(\boldsymbol{\rho}')d\mathbf{k}_f=\delta(\boldsymbol{\rho}-\boldsymbol{\rho}')+\delta(\boldsymbol{\rho}+\boldsymbol{\rho}'),$$

$$\int u_{f,o}^*(\boldsymbol{\rho})u_{f,o}(\boldsymbol{\rho}')d\mathbf{k}_f=\delta(\boldsymbol{\rho}-\boldsymbol{\rho}')-\delta(\boldsymbol{\rho}+\boldsymbol{\rho}_i).$$

Applying these formulas to (12′) in summing over all states f (which means integrating over half of $\mathbf{k}_f$ space), we find

$$d\sigma/dD^2\approx[1-\tfrac{1}{3}F(D)]d\sigma_K/dD^2 +[1-F(D)]d\sigma_L/dD^2, \quad (14)$$

where

$$F(D)=\int\exp(i\mathbf{D}\cdot\boldsymbol{\rho})u_i^2(\rho)d\rho. \quad (15)$$

The result (14) is extremely instructive in showing the effect of the exclusion principle. The function $F(D)$ (plotted in Fig. 2, for the Hulthèn deuteron function [see Appendix]) is equal to unity when $D=0$ and falls off with increasing D at a rate determined by the zero point motion of the deuteron. It reaches its half-maximum near $D=1$. For small D the non-spin flip term is thus suppressed, while $\frac{2}{3}$ of the spin flip contribution is allowed. Experiments designed to measure the nucleon spin dependence of the photomeson production clearly must concentrate on small values of D. For large values, F approaches zero and no new information from the deuteron can be obtained.

For large ν_0 the limits on the allowable range of D for the deuteron can be shown to effectively approach those for the proton. Thus the obvious requirement that deuteron and proton total cross sections shall asymptotically become equal as $\nu_0\rightarrow\infty$ is satisfied by (14), because the fraction of the total range of D^2 where $F(D)$ is significant becomes smaller and smaller as ν_0 increases. However, that portion of the range where $D\gtrsim 1$ will always have a substantially smaller cross section for the deuteron than for the proton, regardless of the size of ν_0.

It is unfortunate that the nuclear recoil D is not always an easily measurable quantity, since in comparing the properties of complex nuclei to those of free nucleons in high energy reactions, it is by far the most significant single parameter. In charged meson production experiments, the most easily measured quantities

[6] Feshbach, Goldberger, and Villars, private communication.

happen to be the energy and angle of the emitted meson. Before considering the theoretical distribution in these variables from a systematic standpoint, it should be pointed out that the total number of mesons emitted at a fixed laboratory angle is closely, although not exactly, related to the number of nuclear recoils of a given magnitude. Thus the results of this section may be applied roughly to the meson angular distribution.

The general connection between the meson angle, θ_μ, the nuclear recoil, D, and the nuclear excitation energy, ϵ, is as follows:

$$\cos\theta_\mu=\frac{\nu_0^2+(\nu_0-\epsilon-D^2/4M)^2-1-D^2}{2\nu_0[(\nu_0-\epsilon-D^2/4M)^2-1]^{\frac{1}{2}}}. \tag{16}$$

Except very close to threshold, we may argue, as we did for (9), that formula (17) is well approximated by replacing ϵ by $D^2/4M$. One may therefore use the approximate relation,

$$\cos\theta_\mu\approx\frac{\nu_0^2+(\nu_0-D^2/2M)^2-1-D^2}{2\nu_0[(\nu_0-D^2/2M)^2-1]^{\frac{1}{2}}}, \tag{17}$$

to connect a nuclear recoil D with a meson angle θ_μ. Since this connection is the same as for a free proton, it follows that a formula of the type (14) applies roughly for the meson angular distribution as well as for the distribution of recoils. In other words, if we write the cross section for meson production into the solid angle $d\Omega_\mu$, from a free proton at rest, as

$$d\sigma_p/d\Omega_\mu=d\sigma_K/d\Omega_\mu+d\sigma_L/d\Omega_\mu, \tag{18}$$

separating the spin flip and non-spin flip terms as before, then the corresponding deuteron cross section in the closure approximation is

$$d\sigma/d\Omega_\mu=[1-\tfrac{1}{3}F(D)]d\sigma_K/d\Omega_\mu+[1-F(D)]d\sigma_L/d\Omega_\mu, \tag{19}$$

where D is connected to θ_μ through (17).

The relationship (17) is plotted in Fig. 3 for several values of ν_0. It is seen that, in general, small angles of meson emission correspond to small values of the nuclear recoil and therefore to a large exclusion principle effect. It follows that experiments to determine the nucleon spin dependence must be done at small angles of meson emission.

Nearly all the approximations made in evaluating formula (9) can be systematically improved by a method due to Placzek.[7] This consists of an expansion in powers of $\epsilon-D^2/4M$, of which the zero'th order terms are the results given here by our closure approximation. This approach also yields the total cross section in a simple form. A subsequent paper will discuss the "Placzek corrections" to the closure approximation.

[7] G. Placzek, private communication.

IV. THE ENERGY AND ANGULAR DISTRIBUTION OF CHARGED MESONS

If one eliminates from (1) the nucleon recoil D and fixes the final state f by energy conservation, then the distribution of final meson momenta $\boldsymbol{\mu}$ is obtained. After the same approximations which led to (9), one finds

$$d\sigma/d\boldsymbol{\mu}=(2\pi)^{-2}\{(\tfrac{1}{3}I_e+\tfrac{2}{3}I_o)\langle|\mathbf{K}^2|\rangle_{\text{Av}}+I_o\langle|L|^2\rangle_{\text{Av}}\},$$

where

$$\begin{aligned} I_e&=\int d\mathbf{k}_f|E_f|^2\delta(\mu_0+\epsilon+(\boldsymbol{\nu}-\boldsymbol{\mu})^2/4M-\nu_0),\\ I_o&=\int d\mathbf{k}_f|O_f|^2\delta(\mu_0+\epsilon+(\boldsymbol{\nu}-\boldsymbol{\mu})^2/4M-\nu_0). \end{aligned} \tag{20}$$

The integrals I_e and I_o can be carried out for the Hulthèn wave functions,[6] but the results are so cumbersome that they have been evaluated at only a few selected points. In a paper by Feshbach and Lax[8] a tractable result is obtained by neglecting the final state interaction between the two neutron. As explained above, this cannot properly represent the upper end of the meson spectrum, where ϵ_f is small and the neutrons interact strongly. For large ϵ_f the approximation is presumably adequate. For reasons of economy we shall here go no further in the discussion of this approach to the meson energy distribution but refer the reader to the work of Feshbach and Lax, who have studied it thoroughly.

V. NEUTRAL MESON PRODUCTION

The phenomenological treatment of neutral meson production is similar to that for charged meson produc-

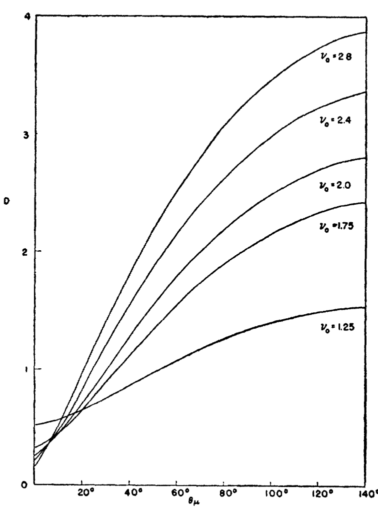

FIG. 3. The relationship between the recoil, D, of a free nucleon and the angle of meson emission, θ_μ (in the laboratory system), for various values of ν_0. Except quite close to threshold, this relationship is also approximately true for deuteron recoils.

[8] H. Feshbach and M. Lax (to be published).

tion. All arguments used for the charged meson case apply equally well here, with the possible exception of the assumption that the matrix elements do not vary in an important way with the nucleon momenta. The cross section in the laboratory system takes the form,

$$d\sigma_{f,o}/d\boldsymbol{\mu}d\mathbf{D}=(2\pi)^{-2}|Q_f|^2\delta(\mu_0+\epsilon+D^2/4M-\nu_0), \quad (21)$$

where

$$\begin{aligned} Q_f&=\langle f|T_n+T_p|i\rangle,\\ T_n&=(\boldsymbol{\sigma}_n\cdot\mathbf{K}_n+L_n)\exp[i(\boldsymbol{\nu}-\boldsymbol{\mu})\cdot\mathbf{r}_n], \\ T_p&=(\boldsymbol{\sigma}_p\cdot\mathbf{K}_p+L_p)\exp[i(\boldsymbol{\nu}-\boldsymbol{\mu})\cdot\mathbf{r}_p].\end{aligned} \quad (22)$$

The notation permits the matrix elements for meson production on free neutron targets to differ from those with free proton targets. The final relative energy ϵ_f of the neutron proton pair takes the value k_f^2/M, if the two particles recede with relative momentum k_f, and the value $+\epsilon_i$, if the final state is the unmodified deuteron bound state.

For neutral meson production, the distribution of nuclear recoils may actually be experimentally observable. In any case one can arrive at a formula, analogous to (12):

$$\begin{aligned} d\sigma_{f,o}/dD^2=(2\pi)^{-1}\{&\tfrac{2}{3}\langle|\mathbf{A}_f^2|\rangle_{\text{Av}}+\langle|B_f|^2\rangle_{\text{Av}}\\ &+\tfrac{1}{3}\langle|\mathbf{C}_f^2|\rangle_{\text{Av}}\}(\nu_0-D^2/2M)/2\nu_0, \end{aligned} \quad (23)$$

where

$$\begin{aligned} \mathbf{A}_f&=(u_f^3(\boldsymbol{\rho}),[\mathbf{K}_n\exp(i\mathbf{k}_0\cdot\boldsymbol{\rho})+\mathbf{K}_p\exp(-i\mathbf{k}_0\cdot\boldsymbol{\rho})]u_i(\rho))\\ B_f&=(u_f^3(\rho),[L_n\exp(i\mathbf{k}_0\cdot\boldsymbol{\rho})+L_p\exp(-i\mathbf{k}_0\cdot\boldsymbol{\rho})]u_i(\rho))\\ \mathbf{C}_f&=(u_f^1(\rho),[\mathbf{K}_n\exp(i\mathbf{k}_0\cdot\boldsymbol{\rho})-\mathbf{K}_p\exp(-i\mathbf{k}_0\cdot\boldsymbol{\rho})]u_i(\rho)).\end{aligned} \quad (24)$$

By expressing the deuteron wave functions u_i, u_f^1, and u_f^3 in momentum space, a dependence of $\mathbf{K}_{n,p}$, and $L_{n,p}$ on the nucleon momenta can in principle be taken into account in evaluating (24). In order to obtain a tractable result, however, we shall continue to neglect this dependence.

Formula (23) has already been summed over final spins, but there is one special case in which the spin of the final nuclear system can be observed, that is, when the deuteron remains bound. This case may be very interesting from an experimental point of view, so a formula is now given separately for it:

$$d\sigma_{00}/dD^2=(2\pi)^{-1}\{\tfrac{2}{3}\langle|A_0|^2\rangle_{\text{Av}}+\langle|B_0|^2\rangle_{\text{Av}}\}(\nu_0-D^2/4M)/2\nu_0, \quad (25)$$

where

$$\begin{aligned} \langle|A_0|^2\rangle_{\text{Av}}&=(\langle|\mathbf{K}_n|^2\rangle_{\text{Av}}+\langle|\mathbf{K}_p|^2\rangle_{\text{Av}}+2\,\mathrm{Re}\langle\mathbf{K}_n^*\cdot\mathbf{K}_p\rangle_{\text{Av}})F^2(k_0)\\ \langle|B_0|^2\rangle_{\text{Av}}&=(\langle|L_n|^2\rangle_{\text{Av}}+\langle|L_p|^2\rangle_{\text{Av}}+2\,\mathrm{Re}\langle L_n^*L_p\rangle_{\text{Av}})F^2(k_0).\end{aligned} \quad (26)$$

Interference effects in (25) obviously may be strong, and the result cannot be completely expressed in terms of the free proton and free neutron cross sections since it depends on the relative phases of $\mathbf{K}_n$ to $\mathbf{K}_p$ and L_n to L_p. Notice that again the important parameter is $k_0=D/2$. The "elastic" meson production falls off rapidly as the nuclear recoil becomes large.

As before, we may use closure to obtain the total cross section, "elastic" plus "inelastic" for a given recoil D. The formula analogous to (14) is

$$\begin{aligned}\frac{d\sigma_0}{dD^2}\approx\frac{1}{2\pi}\{&\langle|\mathbf{K}_n^2|\rangle_{\text{Av}}+\langle|\mathbf{K}_n^2|\rangle_{\text{Av}}+\langle|L_n|^2\rangle_{\text{Av}}+\langle|L_p|^2\rangle_{\text{Av}}\\ &+2F(D)\,\mathrm{Re}[\tfrac{1}{3}\langle\mathbf{K}_n^*\cdot\mathbf{K}_p\rangle_{\text{Av}}+\langle L_n^*L_p\rangle_{\text{Av}}]\}\\ &\times\frac{\nu_0-D^2/2M}{2\nu_0},\end{aligned} \quad (27)$$

which is seen to be the free proton cross section plus the free neutron cross section plus interference terms, whose importance depends on the size of D.

Interpretation of neutral meson production from deuterons is complicated by the interference terms, but there is the advantage that for the "elastic" part, which under certain conditions will not be at all negligible, the calculation of the nuclear overlap integrals can be carried out without approximation. Experimentalists are recommended to concentrate on this part of the investigation.

VI. CONCLUSIONS

The photoproduction of mesons in deuterium has been discussed phenomenologically on the basis of the following assumptions. (1) The production "matrix" is the sum of the free neutron and proton production matrices, evaluated between initial and final nuclear states. (2) The matrix elements, except for the momentum conservation factor, do not vary appreciably from their value in free nucleon photoproduction. It has been shown that the result for charged mesons is then expressible in terms of the free nucleon spin flip and non-spin flip meson production cross sections and certain overlap integrals which have nothing to do with meson theory. These integrals can be evaluated if the deuteron bound state wave function and the di-neutron continuum wave functions are given. Neutral meson production from deuterium involves the same integrals but also interference terms, the size of which do depend on meson theory.

Exact evaluation of the nuclear overlap integrals is in general tedious, but an approximation has been described which should be adequate to give the meson angular distribution except near threshold. Near threshold the final state nuclear interaction dominates the behavior of the cross section. In one special case, the "elastic" production of neutrals, the influence of the interaction is easily calculable. In other cases, when the final state is in the continuum, the formulas are complicated, but have been evaluated in a few cases by another group of workers.

In general, two-body effects are important when the nuclear recoil is of the same order or smaller than the momenta occurring in the zero point motion of the deuteron. This means that if information on the nucleon spin dependence of the matrix elements is to be derived from a comparison of deuteron and proton cross sections, the measurement must be carried out at small angles of meson emission and not too high an incident photon energy.

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Close communication with H. Feshbach and M. Lax, who are also publishing a paper on this subject, was of great assistance to the authors. Much of the contents of the two papers is similar, and indeed an effort was made to employ the same notation. However, the Feshbach-Lax paper will be seen to emphasize different distribution variables than are here employed. For example they give the differential meson energy distribution, but do not obtain the total intensity at a given angle in a closed form. Also, since the validity of the different approximations made in the two papers is not entirely clear, it was felt worthwhile to publish the two separately.

APPENDIX

The Hulthèn Wave Function for the Ground State of the Deuteron

A convenient representation of the deuteron ground state is given by the Hulthèn function,

$$u_i(\rho)=\left[\frac{\alpha}{2\pi(1-\alpha\rho_1)}\right]^{\frac{1}{2}}\frac{(e^{-\alpha\rho}-e^{-\beta\rho})}{\rho}, \tag{A1}$$

where ρ_1 is the triplet neutron-proton effective range (1.74×10^{-13} cm), connected to the parameter, β, by the relation $\rho_1=4/(\alpha+\beta)-1/\beta$. The corresponding momentum space wave function is

$$c(\mathbf{k})=\left[\frac{\alpha}{\pi^2(1-\alpha\rho_1)}\right]^{\frac{1}{2}}\frac{(\beta^2-\alpha^2)}{(\alpha^2+k^2)(\beta^2+k^2)}. \tag{A2}$$

The Integral $F(k)$

From (15) and (A1), one easily finds

$$F(k)=\int\exp(i\mathbf{k}\cdot\boldsymbol{\rho})u_i^2(\rho)d\boldsymbol{\rho}=\frac{1}{1-\alpha\rho_1}\left(\frac{2\alpha}{k}\right)\times\left\{\tan^{-1}\left(\frac{k}{2\alpha}\right)+\tan^{-1}\left(\frac{k}{2\beta}\right)-2\tan^{-1}\left(\frac{k}{\alpha+\beta}\right)\right\} \tag{A3}$$

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Half-Life and Alpha-Particle Energy of U^{236}

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Several samples containing U^{236} were examined for measurement of the specific activity and the alpha-particle energy. The samples contained U^{234}, U^{235}, U^{236}, and U^{238}, the isotopic composition being known from mass spectrometer data. The samples were "weighed" through fission counting comparison with standards containing known amounts of pure U^{235}. The alpha-activity due to U^{236} was determined by ionization chamber energy analysis. The same instrument was used for the energy determination.

The half-life of U^{236} was found to be 2.46×10^7 years and the alpha-particle energy 4.499 Mev.

WORK at Los Alamos[1] in 1943 showed that some of the neutrons absorbed by U^{235} led to capture rather than to fission, with the presumable formation of the isotope U^{236}. It was later observed in highly irradiated U^{235} both mass-spectrographically[2] and with the alpha-energy pulse analyzer.[3] From the alpha-activity detected with the latter instrument, from the U^{235} capture cross section, and from the estimated neutron flux, the half-life was calculated to be approximately 2×10^7 years. The measured alpha-particle energy was 4.5 Mev.

U^{236} has been prepared recently in a more concentrated form by extensive neutron-irradiation of U^{235} followed by electromagnetic separation. Several of these samples have been made available,[4] which has made it possible for us to make more accurate measurements of both the half-life and alpha-particle energy of U^{236}.

The determination of the U^{236} specific activity involved the measurement of the weight of U^{236} in each sample and the alpha-particle disintegration rate due

TABLE I. Composition of U^{236} samples (mole percent).

Sample no.	U^{234}	U^{235}	U^{236}	U^{238}
I	0.3	59.0±0.20	22.15±0.25	18.55±0.25
II	0.4	54.5±0.24	22.0±0.30	23.1±0.30
III	0.65±0.02	58.58±0.19	37.62±0.18	3.17±0.03

[1] Unpublished Los Alamos work.
[2] D. Williams and P. Yuster, Los Alamos Report LAMS-195 (1945) (unpublished).
[3] Ghiorso, Brittain, Manning, and Seaborg, Phys. Rev. 82, 558 (1951).
[4] We are indebted for these samples to Dr. R. S. Livingston of the Electromagnetic Research Laboratory, Carbide and Carbon Chemicals Division, Oak Ridge, Tennessee.