From the values of the elastic constants obtained here the Debye characteristic temperature θ_D has been computed as a function of temperature, and as is to be expected, is constant over a wide temperature range. The computed values of θ_D are shown in Fig. 4, together with the results obtained by Kellermann[14] from a theoretical analysis of the NaCl lattice vibrations and with those obtained by Clusius, Goldmann, and Perlick[15] from specific heat measurements. Comparison of the three curves shows that the inadequacy of the Debye theory in providing an accurate description of the properties of the crystal may be attributed to the choice of the distribution function for the frequencies, rather than to anomalous behavior of the elastic constants. The value of θ_D at absolute zero temperature is 292.6±0.5°K.

[14] E. W. Kellermann, Trans. Roy. Soc. (London) **A238**, 513 (1940); Proc. Roy. Soc. (London) **A178**, 17 (1941).

[15] Clusius, Goldmann, and Perlick, Z. Naturforsch. **4A**, 424 (1949).

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Scattering of Protons by Alpha-Particles*

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Differential scattering cross sections for 5.10±0.10-Mev protons by helium have been determined at laboratory angles from 30° to 150°. Probable errors in the cross sections varied from 2.4 percent at 30° to 4.1 percent at 150°. The scattered protons were detected by a proportional counter. The apparatus was calibrated by a study of proton-proton scattering.

I. INTRODUCTION

THE scattering of protons by helium has previously been studied at energies up to 3.6 Mev.[1] In the present work the scattering of 5.10-Mev protons by helium has been studied at laboratory angles of 30°, 45°, 60°, 75° 58′, and 150°. The effect of a slight variation in energy on the scattering cross section at 30° was investigated.

The determination of absolute scattering cross sections is usually complicated by difficulties in measuring certain geometrical constants of the apparatus. In the present work, moreover, the capacitance of a standard condenser and associated circuit used in integrating the beam current presents a problem in measurement. These uncertain constants were here lumped into an "apparatus factor." This apparatus factor was then evaluated by an experiment on proton-proton scattering. The best value for the proton-proton scattering cross section based on all the available experimental data was used for this calibration.

II. THE SCATTERING APPARATUS

A schematic diagram of the scattering apparatus is presented as Fig. 1.

The beam from the Washington University cyclotron entered the apparatus through a collimator consisting of two 4-mm diameter slits separated from one another by about 34 cm. Four 6-mm diameter baffles were spaced at approximately equal intervals between the slits. A window of 0.25-mil rubber hydrochloride separated the scattering apparatus from the cyclotron.

The unscattered particles were collected by a faraday cage. The faraday cage, maintained at a pressure of about 4×10^{-5} mm of Hg, was separated from the scattering gas by a window of 0.5-mil polyethylene.[2] The charge was accumulated on a 1μf polystyrene condenser made by John E. Fast and Company of Chicago, Illinois. This condenser showed no appreciable leakage or polarization. The voltage across the condenser was measured by a null method using a Compton electrometer as the null indicator. The error in the charge measurement was estimated to be ±0.2 percent, the largest part of this error being due to slight drifts in the electrometer zero.

The scattered protons were defined by a slit system consisting of two 2-mm×4-mm slits spaced about 8.5 cm apart. The distance from the center of the scattering volume to the second slit, the counter slit, differed slightly at the various angles. All the data were normalized to a distance of 16.15 cm. Uncertainty in this distance contributed an estimated ±0.6 percent error to the final cross sections. The proton-proton

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[1] Freier, Lampi, Sleator, and Williams, Phys. Rev. **75**, 1345 (1949); N. P. Heydenburg and N. F. Ramsey, Phys. Rev. **60**, 42 (1941).

[2] The 0.5-mil and 0.25-mil polyethylene used in this work was kindly furnished by the Du Pont Company.

calibration procedure did not eliminate this one geometrical source of error, because some of the angles studied in the proton-alpha work could not be studied in the proton-proton work. The calibration procedure eliminated only those errors which remained constant at the various scattering angles, e.g., the slit sizes and distance between slits.

The slit system provided an angular resolution of about $\pm 1° 40'$ at all the angles studied (i.e., $\theta''-\theta=\theta, -\theta'=1° 40'$, where θ'', θ, and θ' are indicated in Fig. 1).

The scattered particles were detected by a proportional counter having an inside diameter of 1.50 inches, an inside length of five inches, and a center conductor of 10-mil tungsten wire. A potential difference of 1080 volts was used, corresponding to a filling of 19 cm of Hg of argon and one cm of Hg of carbon dioxide. The counter window was of 0.25-mil polyethylene.

The largest possible pulses are produced when the particle to be counted just stops in the counter. In order to secure this optimum pulse size, slowing foils appropriate to the type and angle of scattering under study were inserted at the counter slit. Securing the largest possible pulses from the counter served to minimize the background counts caused by stray neutrons in the cyclotron room. The neutron-induced counts were of all sizes, hence the use of a discriminator circuit with level set just low enough to include the desired pulses eliminated all small background pulses. Shielding of the counter by the use of borax and paraffin further reduced the background.

The slowing foils used to obtain optimum size pulses also served to eliminate all counts due to the recoil alpha-particles in the proton-alpha work. The thickness of the slowing foil used was, at the least, six times the range of the recoil alpha-particles.

The counter pulses were first amplified by an Atomic Instrument Company model 205-A preamplifier, next conducted through 150 feet of coaxial cable to the control room, and thereupon fed to an Atomic Instrument Company model 204-B amplifier and discriminator. The output pulses from the discriminator were counted by a Nuclear Instrument Company scaling circuit.

The scattering angle to be studied was selected by attaching the counter and associated slit system to any one of the five ports located in the walls of the gas chamber. These ports were accurately placed to within about 10′ of arc. The system could be pumped out within one hour after changing angle. It was not necessary to admit air to the counter during the change.

The pressure of the scattering gas was determined by a U-tube manometer filled with Apiezon oil B. One side of the manometer was always connected to the pumps. The error introduced by inaccurate readings of this manometer was estimated to be ± 0.3 percent. Gas pressures of about 2.2 cm of Hg were used in the work. One series of runs was taken with about half the normal pressure. The data so obtained were consistent with those at the higher pressures usually used.

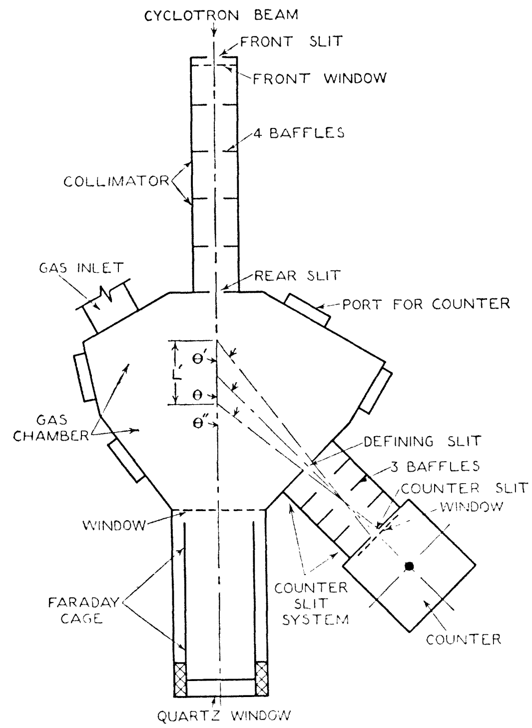

FIG. 1. Schematic diagram of the scattering apparatus (top view).

The temperature of the scattering gas was measured by a thermometer located some distance from the chamber. This thermometer was found to read within 0.5°C of another thermometer placed at the chamber. Making a liberal allowance for possible error due to poor ventilation with resultant fluctuations in the temperature, it is felt that a precision of ± 0.7 percent may safely be assigned to this measurement.

The calibration procedure used made it necessary to obtain only relative temperature and pressure measurements of high precision. It was necessary to know the absolute values only approximately.

All vacuum seals in the system were made by the use of O rings.

III. RUN PROCEDURE

The chamber was pumped out to a pressure of less than 4×10^{-5} mm of Hg. A background run was then made in which all conditions were identical to those in the scattering runs except that the chamber contained no gas. The background counts were determined at the pulse height discriminator settings that were expected to be correct for the scattering runs.

The chamber was then filled with hydrogen or helium of 99.5 percent purity, furnished in standard cylinders by the Ohio Chemical Company. The gas pressure and temperature were recorded.

The scattering data were then taken in a succession of runs. A run was commenced by simultaneously con-

TABLE I. Best values for the proton-proton scattering cross sections at 5.10 Mev, based on the best values of Jackson and Blatt for the expansion coefficients. θ Rs the center-of-mass scattering angle and σ Rs the center-of-mass scattering cross section.

θ	σ(barn)
60°	0.0955±0.0020
90°	0.0982±0.0020
120°	0.0955±0.0020

necting the cable from the faraday cage to the standard condenser and turning on the scaling circuit to record pulses registered in the proportional counter. When the voltage across the standard condenser reached about one volt the run was ended by disconnecting the cable from the condenser and turning off the scaling circuit. After the desired number of counts had been accumulated, the temperature and pressure were again recorded. In no case did the before-and-after temperature readings vary by more than 2°C, or the pressure readings by more than 0.3 percent.

The gas was then pumped out and the background counting rate again determined. The before-and-after background counts were always found to be in agreement. The background data were highly consistent throughout the experiment. Based on the internal consistency of the data and the number of background counts recorded, it is felt that the probable error in the background correction is not more than ±10 percent.

The scattering data were corrected for the background counts and then normalized to uniform conditions of temperature and pressure, to 1 volt on the standard condenser, to a distance of 16.15 cm from the scattering center to the counter slit, and multiplied by the sine of the scattering angle. This latter factor arises from the increase in length of the scattering region (see Fig. 1) as the scattering angle becomes smaller.

IV. CALIBRATION USING PROTON-PROTON SCATTERING

Jackson and Blatt[3] have given a thorough analysis of the low energy proton-proton scattering data. They give an expansion in terms of the incident proton energy (the expansion for K in their paper) from which the phase shift and thus the scattering cross sections may be determined. They also give the best experimentally determined values for the two arbitrary constants in the expansion for K. Four recent precision experiments[4] on proton-proton scattering have been treated according to the method of Jackson and Blatt and all found to be in excellent agreement with their best values, the probable errors in the experiments overlapping the values given by Jackson and Blatt. Further, no systematic trend away from the Jackson and Blatt values is indicated.

The best proton-proton scattering cross sections at 5.10 Mev have, accordingly, been determined from Jackson and Blatt's best values for the expansion coefficients. These best cross sections at the angles of interest are listed in Table I. The errors quoted are based on the limits of error assigned by Jackson and Blatt to the best values for the expansion coefficients. These error limits are consistent with the additional data mentioned above.

The observed counts are given by

$$N_n = G\sigma,$$

where N_n is the number of observed counts corrected for background and normalized to uniform conditions of temperature, pressure, voltage on the condenser, value of $\sin\theta = 1$, and distance from counter slit to scattering region of 16.15 cm; σ is the differential scattering cross section in the laboratory; and G is the apparatus factor involving various geometrical factors, the capacitance of the standard condenser, the temperature and pressure of the scattering gas, and certain physical constants.

Proton-proton scattering was studied at laboratory angles of 30°, 45°, and 60°. Over 30,000 counts were recorded. The average apparatus factor, the value used in all later calculations, was found to be 1043×10^{24} cm^{-2} (±2 percent). The probable error includes errors in the measurement of voltage, pressure, temperature, distance from counter slit to scattering volume, scattering angle, the statistical probable error in the number of counts observed, and the estimated probable error in the background correction. The ratio of real counts to background counts was 11:1 at 60°, 25:1 at 45°, and 29:1 at 30°.

G was also calculated on the basis of direct measurements of all the factors involved and found to be 1016×10^{24} cm^{-2} (±5 percent). This satisfactory agreement between the two independent determinations of G serves as a check on any gross errors in the experimental procedure.

V. RESULTS OF THE PROTON-ALPHA SCATTERING

The scattering of 5.10±0.10-Mev protons by alpha-particles was studied at five angles. The cyclotron beam was then slowed down to 4.80±0.10 Mev by interposing a 1-mil aluminum foil ahead of the collimator. Data were taken at an angle of 30° with this lower energy beam of protons.

The normalized and corrected proton-alpha data along with the differential scattering cross sections are listed in Table II. The probable errors quoted for N_n include errors in the measurement of pressure, temperature, voltage on the standard condenser, the distance from the counter slit to the scattering volume, the scattering angle, the statistical probable error in the observed counts, and the probable error in the back-

[3] Jackson and Blatt, Revs. Modern Phys. **22**, 77 (1950).

[4] Bondelid, Ph.D. thesis, Washington University, 1950; Mather, Phys. Rev. **82**, 133 (1951); Meagher, Phys. Rev. **78**, 666 (1950). Meagher's results are listed by Jackson and Blatt, although excluded from their evaluation of the best values for the expansion coefficients; Zimmerman and Kruger, Phys. Rev. **83**, 218 (1951).

ground correction. The probable errors quoted for the scattering cross sections include the error in N_n and the ± 2 percent probable error in the proton-proton scattering value for G. It should be noted that the G for proton-alpha scattering is one-half the G for proton-proton scattering, because helium is a monatomic gas while hydrogen is a diatomic gas.

VI. IMPURITY SCATTERING

Impurity scattering is not believed to affect the results of this work seriously. No evidence of the building up of impurities in the gas chamber was noted. Neither an increase in pressure with time nor a change in the observed counts with time were noticed. In the worst case, proton-proton scattering at 30°, a 1-percent contaminant of air would have produced only about a 1-percent error in the observed number of counts. Moreover, tests were conducted showing that particles scattered from a heavy impurity were counted with poor efficiency, the counts from a 5-percent impurity of argon not being distinguishable from the background. This discrimination against impurity counts is a property of the proportional counter used as the detector. The impurity-scattered particles have a higher energy than the particles scattered by helium or hydrogen. These higher energy particles produce smaller pulses in the counter than the real counts. The discriminator level that is correct for the desired counts, therefore, cuts out most of the impurity-scattering events.

The 0.5-percent impurity possible in the scattering gas according to the supplier's information can cause the pressure used to be incorrect. The quoted probable errors in N_n and the scattering cross sections include a ± 0.5-percent allowance for an error in the pressure due to the presence of impurities.

VII. ENERGY OF THE CYCLOTRON BEAM

The value of the energy of the cyclotron beam used in this work is based on several independent determinations.[5] Most of these determinations were based on the measurement of track lengths in photographic emulsions. It is thought that the quoted limits of error include all the seemingly reasonable measurements that have been brought to the attention of the author.

TABLE II. The results of the proton-alpha scattering work. θ is the scattering angle in the laboratory system, n is the number of counts observed, R is the ratio of the number of genuine counts to the number of background counts, N_n is the observed number of counts corrected for background and normalized to uniform conditions, and σ is the differential scattering cross section in the laboratory system expressed in barns.

θ	n	R	N_n	σ
		5.10±0.10-Mev protons incident		
30°	13,000	70	328.0 (±1.3%)	0.628 (±2.4%)
45°	7000	25	197.3 (±1.6%)	0.378 (±2.6%)
60°	3600	10	106.3 (±2.2%)	0.204 (±3.0%)
75°58′	2000	7	58.4 (±2.8%)	0.112 (±3.4%)
150°	1600	4	17.2 (±3.6%)	0.033 (±4.1%)
		4.80±0.10-Mev protons incident		
30°	11,000	15	333.2 (±1.5%)	0.638 (±2.5%)

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[5] Mather, see reference 4; Bondelid, Ph.D. thesis, Washington University, 1950; Shull and Keller, private communication.