Experimental Search for the Beta-Decay of the π^+ Meson*

HELEN L. FRIEDMAN AND JAMES RAINWATER
Columbia University, New York, New York

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The possibility that nuclear beta-decay is associated with the beta-decay of the meson, formed during an intermediate step, has long been of interest in the development of meson theories. To explain nuclear beta-decay, the beta-decay rate for the π-meson should be comparable to its μ-deay rate. The present investigation is an attempt to detect other than $\pi-\mu$ events for π^+ mesons which stop in G-5 400 and 600 micron emulsions. The result is one or zero $\pi-e$ event compared to 1419 $\pi-\mu$ events for mesons satisfying certain selection criteria.

A collimating exposure chamber was used in the fringing magnetic field inside the vacuum chamber of the Columbia 164-inch cyclotron to provide energy and direction selection of mesons at the photographic plate. The expected total energy spread for any point x along the plate was calculated to be about 1 Mev.

Only mesons were considered which entered the top surface of the emulsion, ended in the emulsion, and had directions and ranges within intervals including about 94 percent of the π-mesons. This procedure assured consideration of essentially all π-mesons but discriminated against μ-mesons produced by decays in flight.

I. INTRODUCTION

YUKAWA'S hypothesis[1] of the meson as a particle exchanged between nucleons in order to explain nuclear forces seemed strikingly confirmed by the discovery of particles of intermediate mass in cosmic rays.[2] The particles were observed to have a mass about 200 m_e which agrees, in order of magnitude, with the value predicted from the range of nuclear forces. Also they spontaneously decay into an electron, as first predicted by Bhabha.[3] These mesons are now called μ-mesons. However, it was gradually found that quantitative agreement between the nuclear force meson and the μ-meson of cosmic rays was lacking. Even if one adjusted the coupling constant with the nucleon field to account for nuclear forces and the coupling constant with the electron-neutrino field so as to obtain the correct results for nuclear beta-decay, then a mean lifetime of about 10^{-8} sec was predicted, whereas the observed lifetime is much larger (2×10^{-6} sec). Moreover, the experimental cross section for interaction of μ-mesons with nucleons is smaller than that required for nuclear forces by many orders of magnitude.

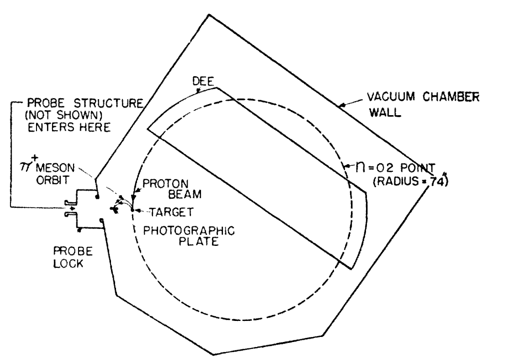

FIG. 1. Plan view of cyclotron showing π^+ meson orbits.

On the other hand the π-mesons, discovered by Powell *et al.*[4] are at least partly responsible for nuclear forces. They have been shown to interact with nucleons with about geometric cross section.[5] However, whether or not the creation and annihilation of virtual π-mesons can be used to explain nuclear beta-decay, according to the scheme: $p^+ \underset{G}{\rightarrow} \pi^+ + n \underset{G'}{\rightarrow} e^+ + \nu + n$ is quite another question. The theoretical investigations of this subject are summarized in the Appendix. Experimentally, one wishes to decide whether the π-meson does undergo beta-decay

$$\pi^{\pm} \xrightarrow{?} e^{\pm} + \nu$$

and if so, to determine the partial lifetime τ_e for beta-decay in terms of the known lifetime[6] $\tau \approx 2.5\times10^{-8}$ sec for total decay (which is mainly, if not entirely, due to μ-decay),

$$1/\tau = (1/\tau_\mu) + (1/\tau_e).$$

(Since π^- mesons stopped in matter are trapped in atomic orbits and absorbed by the nucleus in a time short compared to 10^{-8} sec, only π^+ mesons are suitable for such an investigation.)

When photographic plates were first used to detect π^+ mesons at Berkeley[7] there was some indication that a few percent of the π^+ mesons which stopped in the C-2 emulsions used did not give rise to μ^+. The difficulties inherent in any systematic investigation of the τ_e/τ_μ ratio are twofold: (1) One must, of course, use electron

* Assisted by the ONR and AEC.

[1] H. Yukawa, Proc. Phys.-Math. Soc. Japan **17**, 48 (1935); H. Yukawa and S. Sakata, Math Soc. Japan **19**, 1084 (1937).

[2] C. D. Anderson and S. H. Neddermeyer, Phys. Rev. **51**, 884 (1937); J. C. Street and E. C. Stevenson, Phys. Rev. **51**, 1005(A) (1937); Nishina, Takeuchi, and Ichimiya, Phys. Rev. **52**, 1193 (1937).

[3] H. J. Bhabha, Nature **141**, 117 (1938).

[4] Lattes, Muirhead, Occhialini, and Powell, Nature **159**, 694 (1947); Lattes, Occhialini, and Powell, Nature **160**, 453, 486 (1947).

[5] Bernardini, Booth, Lederman, and Tinlot, Phys. Rev. **80**, 924 (1950); H. Bradner and B. Rankin, Phys. Rev. **80**, 916 (1950).

[6] Jacobsen, Schulz, and Steinberger, Phys. Rev. **81**, 894 (1951). References to earlier work are given here.

[7] Burfening, Gardner, and Lattes, Phys. Rev. **75**, 382 (1949).

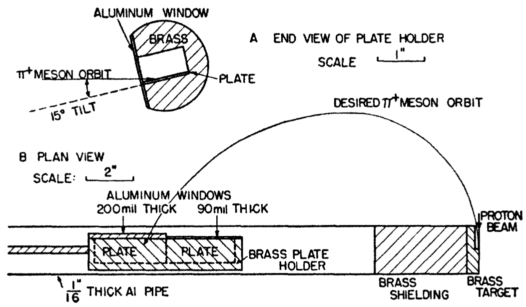

FIG. 2. Preliminary apparatus.

sensitive (G-5) emulsions to be able to detect the minimum ionization track produced by the 70-Mev electron one expects from a π^+ beta-decay. However, even so, a $\pi-\mu$ event is much more striking than a meson-electron event (for a photograph of a $\pi-\mu-e$ decay, see reference 8). In fact, it has been customary to detect positive mesons by their characteristic $\pi-\mu$ junction. Thus, unless one is careful to devise proper scanning techniques, there is danger of overlooking a greater proportion of meson-electron events than of $\pi-\mu$ events, or even of missing the meson-electron events entirely (assuming that they exist). (2) The μ meson does undergo beta-decay.

$$\mu^{\pm}\rightarrow e^{\pm}+2\nu.$$

Thus, one must contrive means of distinguishing between the $\mu-e$ events, which one expects to be present in considerable numbers, and the $\pi-e$ events being investigated. Now a μ meson arising from a $\pi-\mu$ decay occurring when the π-meson is at rest always has 4.1-Mev kinetic energy and a residual range in G-5 emulsion of about 595μ. Thus, by the simple expedient of working with mesons of residual range considerably greater than 600μ, one can eliminate from consideration any μ-electron events arising from $\pi-\mu$ decays at rest. What remains is to discriminate between true $\pi-e$ events and $\mu-e$ events where the μ-meson is formed in a $\pi-\mu$ decay in flight. The collimating system designed to provide this discrimination and the scanning methods devised to avoid missing meson-electron events will be described in the following sections.

II. APPARATUS

The π-mesons were produced by bombarding a copper target with the 385-Mev proton beam of the Nevis 164-inch cyclotron. Positive π's emitted in the backward direction were approximately refocused in the fringing magnetic field onto an Ilford G-5 plate placed at 180° with respect to the target (see Fig. 1).

Preliminary exposures were made with the simple arrangement shown in Fig. 2. The results were unsatisfactory in two respects. (1) The background was annoyingly high so that scanning was laborious and not wholly reliable. There was the usual collection of extraneous tracks, random in orientation and randomly distributed in energy, caused by neutron and γ-ray stars, knock-on protons, etc., which one expects to find on plates exposed inside the vacuum chamber. In addition, there were numerous high energy proton tracks, all roughly parallel to each other and to the direction of the meson tracks. This particularly irritating background is caused by protons which are scattered in traversing the target and then follow trajectories such as (1) in Fig. 3. The particles spiral outward onto the plate because of the rapid decrease of magnetic field with increasing radius beyond the $n=0.2$ point. (2) The π-mesons observed at a given position x along the plate, corresponding to a given radial distance from the target in the cyclotron magnetic field, possessed no characteristic energy or entrance angle. Therefore, one could not hope to be able to distinguish between π-mesons and μ-mesons from $\pi-\mu$ decays in flight. Offhand, at a given coordinate x on the plate, one would expect, despite the smear in energies arising from the inhomogeneity of the magnetic field, that there would be a peak in the meson energy distribution at that energy which satisfied the (modified) 180° focusing condition, and also a correlation between meson energy and entrance angle. However, any such energy peaking or angular correlation is in practice obscured by the possibility of such π-meson trajectories as (2) in Fig. 3, where the meson makes a turn of 360° etc. before reaching the plate.

Thus, the "meson exposure chamber," Fig. 4, was designed with the twofold purpose in mind of cutting down background and of collimating the π-mesons. The size was limited by the requirement that the apparatus would have to fit into the probe vacuum lock and pass through the opening between the vacuum lock and the vacuum chamber. This limited the possible length and diameter to less than 14 in. and 12 in. respectively.

The copper target, which was $\frac{1}{2}$-in. thick in the beam direction and $\frac{1}{16}$-in. thick radially, was situated at $r=74$ in. and 1 in. above the median plane. This is at the $n=0.2$ point where the beam "blows up" vertically[9] due to coupling of the radial and vertical oscillations.

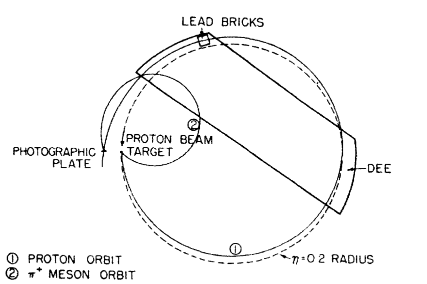

FIG. 3. Possible proton and π^+ meson orbits in the fringing magnetic field.

[8] Brown, Camerini, Fowler, Muirhead, and Powell, Nature **163**, 47 (1949).

[9] Henrich, Sewell, and Vale, Rev. Sci. Instr. **20**, 887 (1949).

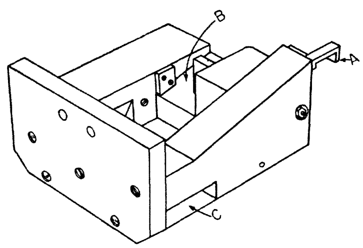

FIG. 4. Meson exposure chamber showing target A, channel B, and slot for plate holder C.

Experiment showed that the beam intensity at the target position was practically undiminished with respect to the median plane intensity. The activity of the target was found to be concentrated in a small region ($\approx\frac{1}{16}$ in.) near the bottom of the $\frac{1}{16}$-in. wide target so it acted, nearly, as a $\frac{1}{2}$-in. long line source of mesons.

Referring to Fig. 4, which shows a perspective view looking down at the exposure chamber, the metal plate on the end, to the left in the figure, was bolted onto the 4-in. probe pipe. The two bolt holes were about 1 in. above the median plane. The lower tip of the target, A, served as the source of mesons. The mesons then spiralled downwards in a roughly semicircular path through the slotted channel, B. The plate holder was inserted in the slot C. The dimensions can best be judged from Fig. 5, which shows the top view of the chamber.

For the meson orbits the important dimensions were as follows: the emulsion extended from 83-in. to 86-in. cyclotron radius and was 2.7 in. below the median plane. (The emulsions used were 1 in. by 3 in. and either 400 or 600μ thick, mounted on a glass backing of about 1.25 mm thickness.) Thus the mesons which reached the plate had to spiral down from 1 in. above the median plane to 2.7 in. below, which corresponded to an angle of dip of about 12°. This dip angle was chosen as a compromise between two opposing factors: one would like to have the plate subtend as large an effective solid angle as possible at the target and yet one must give the π-mesons, which have several mm of residual range, a reasonable chance of ending in the emulsion.

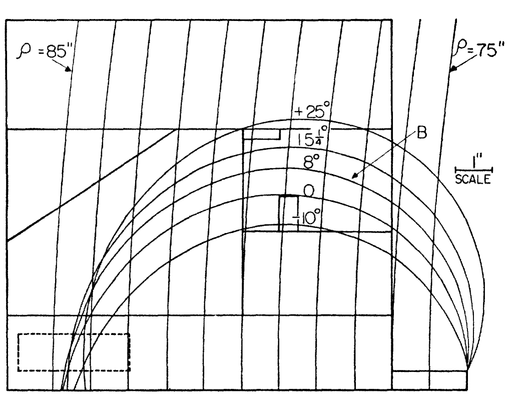

FIG. 5. Allowed and rejected orbits for 12-Mev π^+ mesons. Mesons emitted at $+25°$ and $-10°$ with respect to the proton beam direction fail to pass through the opening of channel B.

The plate holder had a $\frac{1}{16}$-in. thick aluminum window through which the mesons entered approximately normally. This was screwed to the main block and a rubber gasket kept the inside sealed so the plate was not exposed to the cyclotron vacuum. The aluminum window completely stops heavier charged particles (protons) of the same momentum as the mesons which would otherwise give a heavy blackening of the surface of the emulsion.

As can be seen from Fig. 4, there was 7 in. of copper shielding the plates from any direct radiation from the target-neutrons, γ-rays, or charged particles. (385-Mev protons have a range in copper of about 5 in.) Since only 4 to 6 in. of copper was available to shield the plates from the full energy elastically scattered protons of Fig. 3, a 6-in. thickness of lead bricks was placed inside the vacuum chamber in the position shown in Fig. 3 in order to provide additional shielding. The bricks covered a 6-in. radial distance starting just beyond the $n=0.2$ radius and a 4-in. vertical distance extending from $2\frac{1}{2}$ in. below the median plane to $1\frac{1}{2}$ in. above. Also acting to reduce background was the vertical selection imposed by the channel B in Fig. 5. Of the particles leaving the target with the correct dip angle to reach the plate, those which have longer spiral paths outside the collimating system than corresponds to a turn of about 180° are cut out, since by the time they reach the channel, they are too far below the median plane to pass through.

In order to be able to collimate the π-mesons to give optimum energy definition at the plates, accurate knowledge must be obtained of their trajectories in the fringing magnetic field of the cyclotron. To this end, trajectories of π-mesons of a given energy leaving the target at 74-in. radius, at different angles relative to the proton beam direction, were determined graphically. In Fig. 5, some of these orbits have been reproduced for a 12-Mev π-meson. Superimposed is a drawing of the collimating system, so that one can see how the edges of the collimator were located to give optimum sharpness of focus at the plate and still give a reasonable solid angle of aperture relative to the source. (A 12-Mev π has a range in copper of about 0.05 in., so the baffles were very thick compared to the meson range.)

The arc lengths of allowed meson orbits were measured, and this data as well as the known vertical drop of 3.7 in. from target to emulsion was used to adjust the height of channel B in Fig. 5, so as to pass all allowed orbits. Those mesons, on the other hand, which follow trajectories such as (2) in Fig. 3 are cut out, since they are too far below the median plane when they reach the channel to be passed.

The expected energy spread of the π-mesons reaching a given x coordinate of the plate was estimated from a consideration of the meson trajectories together with the size of the meson source. Over the region of the plate used, a 1-Mev change in meson energy corresponds to a change in x of about 0.6 in. For the allowed orbits shown in Fig. 5, the mesons of a given energy are seen to be concentrated in a region of x of about $\frac{3}{8}$ in. (Variation in x coordinate due to both variation in entrance angle and to y coordinate are included in this estimate.) It will be remembered that the source width is $\frac{1}{16}$ in. Thus, in all, a total energy spread of about 1 Mev is expected. The expected spread in entrance angle is less than 15°.

The basis of the method is then as follows. Only mesons which enter the emulsion at about the correct direction and which have the proper residual range are considered. Most μ-mesons formed from $\pi-\mu$ decays in flight will accordingly be rejected, as discussed below. In practice, angle criteria were found useful only in the actual scanning technique finally adoped, which is discussed in Sec. III. Range criteria, however, did serve to exclude most μ-mesons. The range limits for a given x were chosen to be as narrow as possible while still including greater than 90 percent of the total π-mesons. The limits are indicated in Fig. 6 and are discussed at greater length in Sec. IV. (Note that the mesons considered in Fig. 6 have energies reduced by passage through the $\frac{1}{16}$-inch aluminum window of the plate holder.)

Once the mesons have been selected in this way, it becomes possible to distinguish between π-mesons and most μ-mesons arising from $\pi-\mu$ decays in flight. Consider a π-meson of 15 Mev decaying into a μ-meson. In the rest system of the π-meson, the μ-meson is emitted isotropically with 4.1-Mev kinetic energy. As observed in the laboratory system, however, the μ-meson is emitted at an angle to the original direction of the π-meson which can vary only from 0° up to a maximum of about 37°. The energy of the μ-meson varies from about 3 Mev for backward emission in the π-rest system, through about 15 Mev for the maximum laboratory angle case, to about 29 Mev for forward emission in the π-rest system. Thus, one expects that μ-mesons having roughly the same energy as the original π will make large angles with the original direction of the π and therefore either not get through the collimating system at all or at least enter the emulsion at an angle recognizably different from those of the accepted π-mesons. Those μ-mesons emitted in roughly the original direction of the π should have recognizably different energies and therefore residual ranges than the original π-either greater or less, corresponding to forward or backward emission in the π-rest system. Of course one cannot expect perfection. For example, a 15-Mev π headed in a direction which will not permit its passage through the collimating system, might decay in the target region to a roughly 15-Mev μ-meson which would then have the correct direction to have an allowed orbit. However, such instances will be rare and one expects on the whole good discrimination between π's and μ's from in flight $\pi-\mu$ decays. That this aim has been accomplished is shown by the results of the experiment.

III. SCANNING AND MEASUREMENTS

The scanning techniques finally used were gradually evolved over a period of months. What was required was a method of searching for events which would not be biased in favor of $\pi-\mu$ events over the less noticeable meson-electron events, and yet would not be unreasonably time consuming.

The plates obtained with the preliminary arrangement of Fig. 2 were scanned by conventional methods whereby each field of view was carefully examined throughout its depth for meson endings. Each meson found was recorded in a notebook by locating its entrance position and end with respect to cartesian coordinates and roughly drawing its trajectory. The μ-meson or electron originating from the meson ending was also drawn. Gradually, by dint of much practice, it was found possible to identify mesons well before

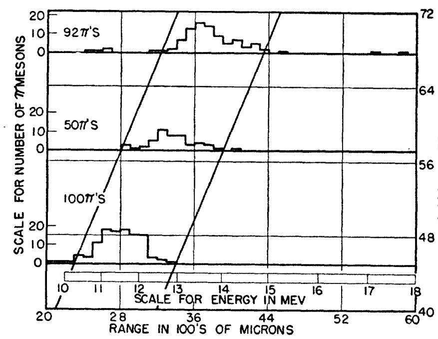

FIG. 6. Histogram of π-meson ranges and graph of accepted range limits *vs* x coordinate.

their endings by their characteristic small angle scattering. By the time scanning was begun on the plates obtained with the meson exposure chamber of Fig. 4, confidence had been built up in one's ability to recognize a meson as such and the conventional method of searching for meson endings was dropped as being prejudiced in favor of the more striking $\pi-\mu$ event.

The first new method, used on the 400 μ-plates, consisted of searching each field of view in a given column for meson trajectories. Every time a meson was found it was followed and mapped as before. Each meson was first identified when it entered the emulsion through the air surface, and then in several fields of view in each column, and also in 2 or 3 different columns, depending upon the entrance angle. This method of scanning is very thorough since each meson is identified as such several times without any reference to a $\pi-\mu$ or meson-electron event. However, the counting rate of about 10 mesons a day was too low for the method to be of practical use.

A counting rate of between 25 and 50 mesons a day was obtained with a scanning technique whereby only

TABLE I. Meson-electron events (ranges given in microns).

	x coordinate	Range	Accepted range limits
[a]1	65.0	2280	3120–4240
2	57.1	3090	2800–3900
[b]3	55.8	4180	2750–3860
4	46.0	4490	2350–3450
5	54.1	4950	2690–3790
[a]6	48.9	5770	2470–3570
[b]7	58.9	5890	2880–3990
8	48.4	6000	2450–3550
9	47.0	6350	2390–3490
10	46.7	6550	2380–3480
11	52.0	7000	2600–3700
12	51.0	7730	2560–3660
13	56.3	8880	2770–3880
14	63.3	9000	3050–4170
15	46.7	9280	2380–3480
16	62.6	9750	3030–4140
17	55.8	10,900	2750–3860

[a] No electron was observed here. However, out of 15μ-mesons arising from $\pi-\mu$ decays at rest whose endings were carefully examined for electrons, in two cases the electron was not visible. Thus, it seems reasonable that, in both cases, electrons were emitted, but at an angle unfavorable for observation.
[b] Found on 400μ-plates.

the top surface of emulsion is examined for entering mesons. The method was carried out as follows on 600μ-plates. Two notebooks were kept. In one, a plot was made of the surface fields of view. All tracks entering the emulsion in a given field at roughly the proper angle were recorded before any were followed. Each track was then followed far enough to determine whether it was a proton (C), a meson leaving the emulsion (B), or a meson ending in the emulsion (A). Those mesons which ended in the emulsion and thus comprised the desired data, were mapped in the second notebook as always.

As a check on our scanning techniques, several rows in each of the 4 plates used were scanned for $\pi-\mu$ junctions. Of the 463 π-mesons so found, only two had been missed by the top surface scanning method. Keeping in mind that any missing of events by the latter method is completely unprejudiced in favor of $\pi-\mu$ events *versus* meson-electron events, such a small fraction of events missed was considered quite satisfactory.

It was found that of the tracks entering the emulsion at approximately the correct angle, about 40 percent are mesons, of which about 40 percent end in the emulsion. The ratio $A/(C+B)$ was studied as a function of x coordinate along the plate and this study served as basis of selecting for use only the upper energy third of the plate. The area actually used extended from $x=3.5$ mm to $x=29.5$ mm (from the upper energy edge) and from $y=8.0$ mm to $y=23.0$ mm. The border was left out as being undesirable due to edge effects.

Three types of measurements were made on a sampling of the mesons which ended in the emulsion: projection on xy plane of angle of entrance, projected length of track, and vertical distance of the ending from the air or glass surface of the emulsion. The accuracy of measurement of the projected meson track lengths was about 1 percent. An attempt was made to find the true track lengths of a sample of 10 mesons by also measuring the change in z coordinate for each field of view. However, the corrections so obtained were less than 3 percent and consequently negligible for our purposes.

IV. ANALYSIS AND RESULTS

As discussed in Sec. II, it was expected that, due to focusing of the π-mesons in the fringing magnetic field of the cyclotron, each coordinate x along the plate would correspond to a reasonably definite meson energy. Originally it was thought that for a given x, finer resolution could be obtained by correlating energy with y coordinate and angle of entrance. However, straggling in range for each x was sufficiently great that no systematic variation of range with y or θ was observed, so that in practice no y or θ corrections were actually used.

The total number of $\pi-\mu$ events found was 1559, of which 264 were found on 400μ-plates, and the rest on 660μ-plates. Two corrections reducing this number were applied.

The first correction consisted in eliminating from consideration those $\pi-\mu$ events where the π-meson ended in a layer of the emulsion within 5μ[10] from the air or glass surfaces of the processed plate. The reason for applying this correction is the following one. For meson-electron events where the meson ended in the rejected layers, since the electron is easily overlooked in the process of scanning, the meson might be mistakenly classified as one leaving the emulsion. The 5μ-criterion was determined experimentally on the basis that a meson ending in the emulsion at a distance greater than 5μ from the glass or air surfaces would during the scanning be recognized as ending, with complete certainty, independently of whether or not an electron was noticed. Of a sampling of 587 $\pi-\mu$ events in the 600μ-plates, it was found that 17 mesons ended within the rejected layers. Therefore a 2.9 percent correction was applied to the 1295 $\pi-\mu$ events found in the 600μ-plates, and a $\frac{2}{3}\times 2.9$ percent correction to the 264 $\pi-\mu$ events found in the 400μ-plates, leaving 1510 mesons in all.

The second correction was concerned with range. In Fig. 6 is shown a histogram of the π-meson ranges for x coordinates corresponding to the edges of the region used, and to the center. The range spread for a given x is greater than that predicted from orbit considerations. This is due to straggling. For comparison, the variation of meson ranges for 31 mono-energetic μ's arising from the decay of a π-meson at rest has been given by Powell.[11] The straggling coefficient in percent $\delta/100=[(\Sigma(R-\bar{R})^2)/n]^{\frac{1}{2}}/\bar{R}$, where $\bar{R}$ is the mean range for n particles, is computed to be 5.1 percent for Powell's

[10] Actual thickness of rejected layer $=5\mu$ increased by a factor equal to the index of refraction of the emulsion.
[11] Lattes, Occhialini, and Powell, Proc. Phys. Soc. (London) **61**, 173 (1948).

measurements on μ-mesons, whereas the theoretical[12] value is estimated to be about 6 percent. In the case of the π-mesons for x coordinate 56.1–58.1, the theoretical value is estimated to be 5.2 percent and the computed value for the 49 accepted π's is 6.8 percent. If, however, one subtracts from the computed value of $(R-\bar{R})_{\text{Av}}^2$ an amount corresponding to a reasonable estimate for the expected energy spread at each x due to varying θ and due to varying y, one obtains a corrected straggling coefficient of 5 to 6 percent which agrees well with the theoretical value. This shows that the collimator worked according to theory.

Also given in Fig. 6 is a graph of accepted range limits *versus* x coordinate. Since 14 π-mesons out of a sample of 242 did not fall within the accepted limits, the total number of mesons, 1559, was reduced by a 5.8 percent correction factor. As a result of applying both corrections, the reduced number of $\pi-\mu$ events is 1419.

The accepted range limits of Fig. 6 were used to exclude all but one of the meson-electron events, as shown in Table I. Events 2 is the only meson-electron event falling within the accepted range limits, and therefore the only possible $\pi-e$ event.

As discussed in Sec. II, it is to be expected that occasionally a μ-meson of the proper range, arising from a $\pi-\mu$ decay in flight, will successfully get through the collimating system. Thus it is quite possible that event 2 is a $\mu-e$ event. It is rather gratifying, in fact, that more such events were not observed out of so large a number of total meson events. Consequently our experimental result is either zero or one $\pi\rightarrow e+\nu$ disintegration out of 1419 $\pi\rightarrow\mu+\nu$ decays.

APPENDIX. THEORETICAL ASPECTS OF THE $\pi-e$ DECAY

In what follows, it should be borne in mind that recent experimental evidence favors a pseudoscalar π-meson.[13]

Of the four possible coupling schemes recently discussed by Yukawa,[14] and illustrated in Fig. 7, in only one does nuclear beta-decay occur, according to the original Yukawa model, through the intermediate creation of a virtual π-meson, and its subsequent decay to an electron and a neutrino. Let us call this coupling scheme Model I. To obtain the correct magnitude of nuclear forces, $G^2/\hbar c\approx 0.1$. The mean lifetime τ_μ of the π-meson for μ-decay and the mean lifetime for capture of μ-mesons by nuclei are both in approximate agreement with a single choice of the coupling constant $G''[G''^2/\hbar c\approx 10^{-14}]$, as shown by Tiomno and Wheeler.[15] The coupling constant G_F' must be chosen[15] about equal

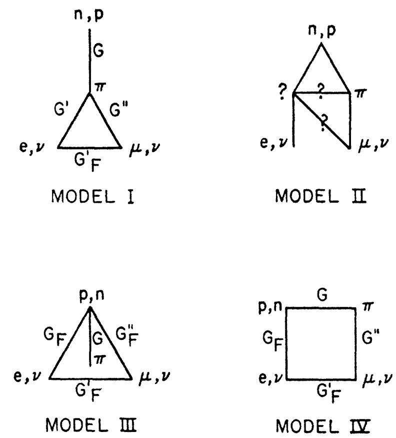

FIG. 7. Proposed coupling schemes.

to G_F in all cases except that of pseudoscalar coupling, if one is to obtain the correct lifetime for μ-meson decay. (G_F is the coupling constant for direct interaction of nucleons with the electron-neutrino system, according to the Fermi theory of nuclear beta-decay. The value of G_F is obtained, for example, from the beta-decay of the neutron ($\tau=12.5\pm 2$ min)[16].)

The only difficulty with Model I is one directly involving the $\pi-e$ decay. Thus, if one chooses G' according to the relation $G_F=GG'/k^2$ [17] where $k=m_\pi c/\hbar$ in order to account for nuclear beta-decay, one finds $G'\approx G''$, and

$$\tau_e\approx(\tau_\mu/6)(G''/G')^2 \tag{1}$$

so that the lifetime for $\pi-e$ decay is in general even shorter than that observed for $\pi-\mu$ decay, in contradiction with experiment. The one promising possibility seemed to be that of the pseudoscalar meson with pseudovector coupling. In this case one finds[18]

$$\tau_e\approx(\tau_\mu/6)(F''/F')^2(m_\mu/m_e)^2, \tag{2}$$

where the F's are pseudovector coupling constants. If one now assumes with d'Espagnat[18] that $F'\approx F''$ one obtains $\tau_e\approx 10^4\tau_\mu$ which might agree with experiment. A similar conclusion was inferred in reference 14 from the earlier calculations of Sakata.[19]

However, this peculiarity of the case of the pseudoscalar meson with pseudovector coupling does not remedy the situation.[19,20] For a self-consistent scheme, one must choose, not $F'\approx F''$, but $(m_e/m_\pi)F'=G'\approx G''=(m_\mu/m_\pi)F''$ which again yields the general result (1). In fact, if one carries through in detail the calculation of the probability $1/\tau_\beta$ for the beta-decay of a free neutron according to Model I, in the case where the intermediate

[12] M. L. Livingston and H. Bethe, Revs. Modern Phys. **9**, 284 (1937).
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meson is pseudoscalar, one obtains an additional factor $(m_e/m_\pi)^2$ not present in the general case, as first shown by Nelson.[20] As a result, it is found that, instead of removing the difficulty inherent in Model I, the pseudoscalar meson predicts a value $\tau_e \approx 10^{-15}$ sec,[21] which is much less than the value predicted in general.

Thus if one wishes to retain the Yukawa scheme for nuclear beta-decay, one is forced to consider some such coupling scheme as Model II, where a virtual hypothetical meson serves as intermediate state between Fermions. Such proposals have been made by Sasaki[22] and by Caianiello.[21] Of course no difficulties are encountered here since one can assign properties to the hypothetical meson in arbitrary fashion.

If one confines one's attention to coupling schemes involving known particles, another possibility is Model III, the symmetric coupling scheme, which was proposed by Tiomno and Wheeler.[14] Here, there is direct coupling between nucleon and electron-neutrino, or between any other pair of Fermions, and the coupling constants are of the same order of magnitude in all three cases. There is no longer any direct coupling between π-meson and either electron-neutrino fields, or μ-meson-neutrino fields. However, $\pi-e$ decay and $\pi-\mu$ decay will still take place via the creation and annihilation of virtual nucleon pairs. The difficulty encountered with this model is that the probabilities for these decays are divergent with respect to the momentum of the nucleon pairs virtually created in the intermediate state.[23] On the other hand, if one makes the decay probabilities finite by some cut-off procedure, one again expects, contrary to experiment, that the lifetimes for both decay possibilities will in general have the same order of magnitude, since the coupling constants involved are of the same order of magnitude.

Detailed calculations according to the symmetric coupling scheme (Model III) have been carried out for the $\pi-e$ and $\pi-\mu$ decay rates by Steinberger[24] and by Ruderman and Finkelstein[25] using different cut-off procedures. Since the results are sensitive to the choice of cutoff employed, no conclusions can be drawn from the absolute lifetimes so calculated. However, as the same divergent integral appears in both lifetime expressions, if the integral can somehow be made finite, the ratio of lifetimes should be independent of the method used. The results are that in some cases the decay of the π-meson by either mode is forbidden, in others that the ratio of lifetimes is about unity. Only if the π-meson is pseudoscalar, with either pseudoscalar or pseudovector coupling to nucleons, and if the nuclear beta decay coupling contains a pseudovector term do we obtain a result not in contradiction with experiment: $\tau_e/\tau_\mu \approx 10^4$. (The nuclear beta-decay coupling may also contain arbitrary amounts of scalar, vector, or tensor terms, since for these in the case of a pseudoscalar meson, the decay of the π is forbidden.) Whether or not Model III is the true coupling scheme depends on whether or not a correct calculation of τ_μ and τ_e would yield absolute results in agreement with experiment for the case of the pseudoscalar π-meson. These questions cannot be answered with existing meson theory.

An alternate possibility is Model IV, which differs from the symmetric coupling scheme in that the μ-meson-neutrino fields are now coupled to the nucleon fields only indirectly via the π-meson. Here, the $\pi-\mu$ decay takes place directly, while the $\pi-e$ decay again proceeds through intermediate virtual nucleon pairs, which again leads to divergence difficulties. It is quite conceivable that this model would yield acceptable values of τ_μ and τ_e, but again, existing meson theory is incapable of deciding.

In any event, it is quite apparent that the experimental value of the lifetime of the π-meson for beta-decay will be of vital importance in making the choice among the various possible coupling schemes.

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