By (G-I:7) and (G-I:65),

$$\begin{aligned}\int' r_l'\rho'\nabla_k'(1/r) &= -\delta_{kl}\int'\rho'/r-\int'(r_l'/r)\nabla_k'\rho' \\ &= -\delta_{kl}V+\int'(r_l'/4\pi r)\Delta'\nabla_k'V' \\ &= -\delta_{kl}V+2\int'[\nabla_k'V']\nabla_l'(1/4\pi r). \qquad \text{(B.7)}\end{aligned}$$

Thence

$$\int' V'\nabla_k'\nabla_l'(1/4\pi r) = -\tfrac{1}{2}\delta_{kl}V-\tfrac{1}{2}\int' r_l'\rho'\nabla_k'(1/r), \qquad \text{(B.8)}$$

so that, by (B.5) with (B.6) and (B.8),

$$\begin{aligned}\delta\mathfrak{A}_k = {} & \int'(\mathbf{b}\cdot\mathfrak{E}')\nabla_k'(1/4\pi r)-\tfrac{1}{2}b_kV \\ & +\tfrac{1}{2}\int'(\mathbf{b}\cdot\mathbf{r}')\rho'\nabla_k'(1/r). \qquad \text{(B.9)}\end{aligned}$$

Now adding (B.1a, b, c) to (B.4a, b, c) and making use of Eqs. (8), (1), (3), (B.9), and finally of Eqs. (7), (G-I:48), and (G-I:20), we find

$$\begin{aligned}\delta\, {}^{\mathrm{H}}T_{\mathrm{m}}^{00}+2\textstyle\sum_k b_k\, {}^{\mathrm{H}}T_{\mathrm{m}}^{0k} &= \tfrac{1}{2}e\int'(\mathbf{b}\cdot\mathbf{r}')[\boldsymbol{\nabla}'(1/r)]\cdot\mathfrak{R}\{\psi^\dagger[\mathbf{j}';\psi]\} \\ &= -\tfrac{1}{2}e\int'(\mathbf{b}\cdot\mathbf{r}')\,\delta(\mathbf{r}')\,\mathbf{j}\cdot\boldsymbol{\nabla}'(1/r)=0; \qquad \text{(B.10a)}\end{aligned}$$

$$\begin{aligned}\delta\, {}^{\mathrm{H}}T_{\mathrm{m}}^{0k}+b_k\, {}^{\mathrm{H}}T_{\mathrm{m}}^{00}+\textstyle\sum_l b_l\, {}^{\mathrm{H}}T_{\mathrm{m}}^{kl} &= \tfrac{1}{2}e\int'(\mathbf{b}\cdot\mathbf{r}')[\nabla_k'(1/r)]\mathfrak{R}\{\psi^\dagger[\rho';\psi]\} \\ &= -\tfrac{1}{2}e\int'(\mathbf{b}\cdot\mathbf{r}')\,\delta(\mathbf{r}')\,\rho\,\nabla_k'(1/r)=0; \qquad \text{(B.10b)}\end{aligned}$$

$$\begin{aligned}\delta\, {}^{\mathrm{H}}T_{\mathrm{m}}^{kl}+b_l\, {}^{\mathrm{H}}T_{\mathrm{m}}^{0k}+b_k\, {}^{\mathrm{H}}T_{\mathrm{m}}^{0l} &= [\![\tfrac{1}{4}e\int'(\mathbf{b}\cdot\mathbf{r}')[\nabla_k'(1/r)]\mathfrak{R}\{\psi^\dagger\alpha_l[\rho';\psi]\}]\!] \\ &\qquad +[\![k,\, l \text{ interchanged}]\!] \\ &= -\tfrac{1}{4}e\int'(\mathbf{b}\cdot\mathbf{r}')\,\delta(\mathbf{r}')\,\{j_l\nabla_k'+j_k\nabla_l'\}(1/r)=0. \qquad \text{(B.10c)}\end{aligned}$$

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Energy Distribution, Drift Velocity, and Temperature of Slow Electrons in Helium and Argon

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The energy distribution, and the drift velocity and electron temperature which are closely dependent on the distribution, were calculated for slow electrons in He and A under fields ranging from $X/p=1$ to 4 volts/cm mm. The calculations were based on the theory developed by Holstein. The diffusion cross sections of the gases for electrons which account for the effect of elastic collisions were computed from the scattering data of Ramsauer and Kollath and of Normand. The effect of excitation collisions was taken into account by comparing the results for three representative values of excitation cross section: $Q_{\mathrm{ex}}=0$ and $Q_{\mathrm{ex}}=\infty$ which form the limiting boundaries and $Q_{\mathrm{ex}}=$ a constant obtained from Maier-Leibnitz. Although a considerable percentage of the electrons were to be found in the excitation region for $Q_{\mathrm{ex}}=0$, the results for $Q_{\mathrm{ex}}=$ constant were nearly the same as those for $Q_{\mathrm{ex}}=\infty$. In the case of small fields, therefore, the electron energy distribution in the elastic region and related quantities may be calculated with Q_{ex} assumed infinite.

A complete set of curves are given below illustrating some of the properties of the calculated quantities. The agreement between experiment and the curves obtained with the scattering data of Ramsauer and Kollath is good, while the curves obtained from the corresponding data of Normand indicate that his cross-section values are too low.

I. INTRODUCTION

BECAUSE of the lack of a complete theory and sufficient cross-section data, the energy distribution and related parameters for electrons in gases have heretofore been calculated under various simplifying assumptions. These approximations have resulted in considerable discrepancies between the theoretical results and experiment. In the discussion below, the energy distribution, drift velocity, and electron temperature of slow electrons in helium and argon are more exactly calculated for fields up to $X/p=4$ volts/cm mm. The following factors are considered, some or all of which have not been taken into account in previous calculations: (a) an electron loses a small fraction of its energy by recoil in elastic collisions with molecules of finite mass, (b) the effect of excitation collisions cannot be neglected, (c) the elastic cross section of gases for electrons is a function of the electron energy, (d) the type of elastic cross section that should be used in computing the energy distribution is not the Ramsauer cross section but, more exactly, the diffusion cross section. The latter cross section is also referred to as the cross section for momentum transfer.

The calculations below are based on the theoretical investigations of Holstein,[1] whose work represents the most inclusive theory to date on the energy distribution of electrons. Values for the diffusion cross section could not be found in the literature and were therefore calculated from existing angular scattering data. Energy distribution, drift velocity, and electron temperature curves were obtained for three values of excitation cross section: $Q_{\mathrm{ex}}=0$, $Q_{\mathrm{ex}}=\infty$ and $Q_{\mathrm{ex}}=$ a reasonable constant. For the small fields considered here, the calculated curves for the latter two cases were found to be nearly coincident. As a result, for the calculation of gas parameters depending on the electron energy distribution in the non-excitation region, Q_{ex} may be

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[1] T. Holstein, Phys. Rev. **70**, 367 (1946).

assumed infinite for fields up to $X/p=4$ volts/cm mm with little error.

II. DISCUSSION OF THEORY

The physical situation considered by Holstein was that of a gas discharge region between two parallel plane electrodes with an electric field X perpendicular to the electrodes. The steady-state, homogeneous, dc solutions for the isotropic portion of the energy distribution are given by the differential equations:

$$0=\frac{4a^2}{3}\frac{d}{du}\left(u\lambda_D\frac{df}{du}\right)+\frac{2m}{M}\frac{d}{du}\left(\frac{u^2f}{\lambda_D}\right)+\sum\nolimits_h(u+u_h)\lambda_h^{-1}(u+u_h)f(u+u_h) \quad (1a)$$

for $u<u_1$;

$$0=\frac{4a^2}{3}\frac{d}{du}\left(u\lambda_D\frac{df}{du}\right)+\frac{2m}{M}\frac{d}{du}\left(\frac{u^2f}{\lambda_D}\right)-\frac{uf}{\lambda_{\mathrm{ex}}} \quad (1b)$$

for $u>u_1$, where $a=eX/m$ represents the electron acceleration in a field, X; $u=v^2=$ electron velocity squared; $m/M=$ ratio of the electronic mass to the molecular mass; $\lambda_D(u)=$ the diffusion mean free path; $\lambda_h(u)=$ the mean free path between those excitation collisions, which excite a gas atom to the hth excited level; $\frac{1}{2}mu_h=$ the energy of the hth excited level; $\frac{1}{2}mu_1=$ the energy of the first excited level; and $\lambda_{\mathrm{ex}}=1/\sum_h\lambda_h^{-1}=$ the mean free path between excitation collisions.

The complete distribution function is obtained by fitting together the solutions of Eqs. (1a) and (1b). Equation (1a) will yield the energy distribution in the elastic collision region while Eq. (1b) refers to electrons with energies above the first excitation level of the gas atoms. For the small electric fields considered here, the vast majority of the electrons will be found in the elastic collision region so that the $f(u)$ from Eq. (1a) will generally very much exceed and contribute more to the drift velocity and temperature than the $f(u)$ from Eq. (1b). Since very few electrons under small fields possess sufficient energy to cause ionization, the ionization terms in the differential equations have been neglected. The density of electrons in the energy interval defined by u and $u+du$ is proportional to $u^{\frac{1}{2}}f(u)$, and the normalized probability for finding an electron in this interval is given by

$$n(u)du=u^{\frac{1}{2}}f(u)\bigg/\int_0^\infty u^{\frac{1}{2}}f(u)du.$$

Equations (1a) and (1b) represent solutions for dc fields. It was shown by Holstein[1] that when $(\omega^2\lambda_D^2/u_1)\ll 1$, the energy distribution equations for ac fields are identical with those for the dc case given above. The energy distribution, drift velocity, and electron temperature calculated from the dc Eqs. (1a) and (1b) may therefore be applied to discharges in ac fields satisfying this condition.

A number of statistical properties of a gas are closely dependent on the electron energy distribution in that gas. Among these are the electron drift velocity and electron temperature which may be calculated once the energy distribution has been determined.

The drift velocity induced in electrons by an applied electric field X is given by

$$w=-\tfrac{2}{3}(e/m)X\int_0^\infty u\lambda_D(df/du)du\bigg/\int_0^\infty u^{\frac{1}{2}}fdu. \quad (2)$$

The drift velocity per unit applied field, w/X, represents the mobility of the electrons. w was calculated here from Eq. (2) for electrons in He and A up to $X/p=4$ volts/cm mm. The variation of $\lambda_D(u)$ with energy was taken into account. The drift velocity of electrons in Ne has been obtained by Druyvesteyn.[2] The Ne case is simplified by a rather flat $\lambda_D(u)$ *versus* energy curve. The drift velocity of electrons in the noble gases was measured by Townsend and Bailey[3] and R. A. Nielsen.[4] The calculated curves compare favorably with their data, and a more complete comparison will be given below.

The concept of electron temperature arose from the early erroneous assumptions that the electron energy distribution is Maxwellian. Although the concept has no place in the discussion of non-Maxwellian statistics, it finds many uses in gas discharges. Furthermore, it is capable of direct experimental determination.

In gas discharges, the electron temperature is defined as e/k times the ratio of the diffusion constant of the electrons to their mobility, where k is Boltzmann's constant. In Maxwellian statistics this ratio is equivalent to the temperature of the particles. When appropriate expressions are substituted for the diffusion constant and the mobility of the electrons, the electron temperature reduces to

$$T_e=\frac{eX}{k}\frac{D_e}{w}=-\frac{m}{2k}\int_0^\infty u\lambda_D f du\bigg/\int_0^\infty u\lambda_D\frac{df}{du}du. \quad (3)$$

T_e was calculated here for He and A up to $X/p=4$ volts/cm mm. The theoretical curves agree very well with Townsend's results.[3]

The quantity $\lambda_D(u)$ appears throughout the above equations. Since curves for this cross section are not given in the literature, they were calculated for helium, neon, and argon from the angular scattering data of Ramsauer and Kollath.[5]

Introducing $\lambda_D(u)$ by way of the better known Ramsauer cross section we may write

$$1/p\lambda_R(u)=Q_R(u)=2\pi N\int_0^\pi q(u,\theta)\sin\theta d\theta, \quad (4)$$

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[2] M. J. Druyvesteyn, Physica **3**, 65 (1936). Kruithoff and Druyvesteyn, Physica **4**, 450 (1937).

[3] Townsend and Bailey, Phil. Mag. **42**, 873 (1921); **43**, 594 (1922); **44**, 1033 (1922); **46**, 657 (1923).

[4] R. A. Nielsen, Phys. Rev. **50**, 950 (1936).

[5] Ramsauer and Kollath, Ann. Physik **12**, 529 (1932).

TABLE I. Comparison of the Ramsauer and diffusion cross sections. All cross sections are given in units of cm²/cm³ at 0°C and 1 mm Hg and were computed from the angular scattering data of Ramsauer and Kollath.

Helium				Neon				Argon			
Electron energy in ev	Ramsauer cross section	$1-\langle\cos\theta\rangle_{\text{Av}}$	Diffusion cross section	Electron energy in ev	Ramsauer cross section	$1-\langle\cos\theta\rangle_{\text{Av}}$	Diffusion cross section	Electron energy in ev	Ramsauer cross section	$1-\langle\cos\theta\rangle_{\text{Av}}$	Diffusion cross section
1.8	20.8	1.102	22.9	0.99	5.06	1.023	5.18	1.1_5	5.92	0.983	5.82
2.9	22.0	1.070	23.6	1.17	5.77	1.025	5.92	1.5	8.37	0.855	7.15
4.2	20.8	1.081	22.5	1.35	6.01	1.016	6.11	2.0	12.2	0.875	10.7
5.3_5	19.4	1.075	20.8	1.67	6.23	0.972	6.05	2.3	14.0	0.856	12.0
6.7_5	18.1	1.065	19.3	1.70	6.30	0.973	6.13	2.4_5	15.1	0.848	12.8
8.2_5	16.7	1.008	16.9	1.80	6.41	0.938	6.02	2.8	17.5	0.835	14.6
10.7_5	15.1	0.992	15.0	2.2	6.94	0.924	6.41	3.2_2	20.3	0.835	16.9
13.8	14.2	0.949	13.5	2.6	7.31	0.883	6.46	3.3	20.7	0.807	16.7
15.8	12.3	0.928	11.4	2.8	7.42	0.899	6.67	3.6	22.6	0.842	19.0
19.2	11.3	0.893	10.0	3.6	8.13	0.825	6.70	4.0	24.2	0.831	20.1
				4.9	8.91	0.798	7.11	5.0	32.6	0.857	27.9
				6.4	9.50	0.767	7.29	5.4	34.5	0.848	29.3
				7.9	10.1	0.766	7.76	6.7	45.2	0.879	39.8
				10.4	10.9	0.768	8.36	8.0	53.5	0.809	43.2
				13.1	11.5	0.782	8.99	9.0	62.5	0.833	52.1
				15.9	12.0	0.790	9.47	10.3	71.6	0.735	52.6
								12.5	78.6	0.739	58.0

where $Q_R(u)$ is the Ramsauer cross section of 1 cc of gas (usually given at 1 mm Hg and 0°C) for electrons of incident energy u, and N is the gas density in molecules per cc. The quantity $q(u,\theta)$ represents the angular scattering function for the deflection by a molecule of an electron of energy u through an angle θ. Data on $q(u,\theta)$ in the rare gases were obtained by Ramsauer and Kollath[5] for electron energies up to the first excitation level of the gases.

The diffusion cross section of 1 cc of gas is given by the somewhat similar equation

$$1/p\lambda_D(u)=Q_D(u)$$

$$=2\pi N\int_0^\pi q(u,\theta)\sin\theta(1-\cos\theta)d\theta, \tag{5}$$

whence

$$Q_D(u)/Q_R(u)=1-\langle\cos\theta\rangle_{\text{Av}}, \tag{6}$$

where

$$\langle\cos\theta\rangle_{\text{Av}}=\int_0^\pi q(u,\theta)\sin\theta\cos\theta d\theta\Big/\int_0^\pi q(u,\theta)\sin\theta d\theta.$$

Values of $Q_R(u)$, $1-\langle\cos\theta\rangle_{\text{Av}}$, and $Q_D(u)$ for He, Ne, and A were computed from the data on $q(u,\theta)$ given by Ramsauer and Kollath and are shown in Table I. $Q_R(u)$ was also measured directly by Ramsauer and Kollath. The values calculated from Eq. (4) agree almost perfectly with their direct measurements. It can be seen from the $1-\langle\cos\theta\rangle_{\text{Av}}$ column that the deviation between the two types of cross sections can be as high as 25 percent.

Measurements of the total Ramsauer cross section, $Q_R(u)$, but not of $q(u,\theta)$, were also conducted by Normand.[6] His curves of $Q_R(u)$ display the same form as those of Ramsauer and Kollath but are displaced downward by about 20 percent, indicating a consistent source of error in one of the two experiments. For the sake of completeness, values of $Q_D(u)$ were also computed by Eq. (6) from Normand's data on $Q_R(u)$ and the $\langle\cos\theta\rangle_{\text{Av}}$ calculated from the angular scattering data of Ramsauer and Kollath. The drift velocity and electron temperature curves calculated from Normand's cross sections do not agree with experiment.

Curves for the diffusion cross section were calculated for energies up to the first excitation level (u_1) of the gas atoms. Above u_1, $Q_D(u)$ was assumed constant and equal to $Q_D(u_1)$. The differential Eqs. (1a) and (1b) for the energy distribution were solved for $Q_{\text{ex}}=0$, $Q_{\text{ex}}=\infty$ and $Q_{\text{ex}}=$ some reasonable constant. The constant chosen was 0.01 $Q_D(u_1)$ which was obtained from the curves of Maier-Leibnitz.[7]

Assuming $1/p\lambda_{\text{ex}}=Q_{\text{ex}}=0$, both Eqs. (1a) and (1b) reduce to

$$0=\frac{4a^2}{3}\frac{d}{du}\left(u\lambda_D\frac{df}{du}\right)+\frac{2m}{M}\frac{d}{du}\left(\frac{u^2f}{\lambda_D}\right) \tag{7}$$

whose solution is

$$f_1(u)=C\exp\left[-\frac{3}{4a^2}\frac{2m}{M}\int_0^u\frac{u}{\lambda_D{}^2}du\right]$$

$$=C\exp\left[-\frac{6}{(X/p)^2}\frac{m}{M}\int_0^V Q_D{}^2(V)VdV\right] \tag{8}$$

where $a=eX/300m$ with X in volts/cm; $\frac{1}{2}mu=eV/300$ with V in volts; and $(p\lambda_D)^{-1}=Q_D$ where p is the pressure in mm Hg. The energy distribution for this case is proportional to $u^{\frac{1}{2}}f_1(u)$.

Equation (8) is identical with that derived by Morse, Allis, and Lamar[8] with excitation collisions neglected.

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[6] C. E. Normand, Phys. Rev. **35**, 1217 (1930).

[7] H. Maier-Leibnitz, Physik **95**, 499 (1935).

[8] Morse, Allis, and Lamar, Phys. Rev. **48**, 412 (1935).

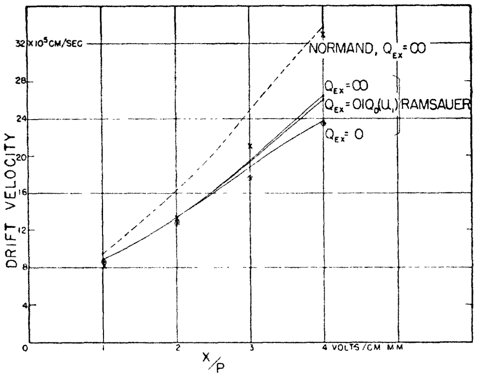

FIG. 1. Drift velocity of electrons in helium. See reference 11. Crosses are from Nielsen, and circles from Townsend and Bailey.

It was also obtained previously by Druyvesteyn[9] who assumed Q_D independent of energy.

For $Q_{\text{ex}}=\infty$, Eq. (1b) becomes meaningless and Eq. (1a) reduces to

$$\frac{4a^2}{3}u\lambda_D\frac{df}{du}+\frac{2m}{M}\frac{u^2f}{\lambda_D}=-\phi(u), \tag{9}$$

where

$$\phi(u)=\int_0^u \sum\nolimits_h[(u+u_h)f(u+u_h)\lambda_h^{-1}(u+u_h)]du.$$

By standard methods one obtains the solution

$$f_2(u)=f_1(u)\left[\frac{3}{4a^2}\int_u^{u_1}\frac{\phi du}{\lambda_D f_1}\right], \tag{10}$$

where $f_1(u)$ is given by Eq. (8) and represents the solution to the homogeneous portion of Eq. (9).

The quantity $\phi(u)$ represents the contribution of electrons to the energy element around u by excitation collisions. To simplify matters it was assumed that electrons lose all their energy per excitation collision. Actually, only u_h units are lost when an atom is excited to its hth level, where $u_1 \leq u_h \leq$ all the energy of the electron. A few representative calculations were performed with the assumption that the electrons lose u_1 units of energy per excitation collision. It was found that the difference in results between the two extremes, i.e., a loss of u_1 units and a complete loss, was less than 2 percent in the worst case ($X/p=4$). With the complete loss assumption, ϕ becomes independent of u and may be readily obtained from Holstein's equations.[10]

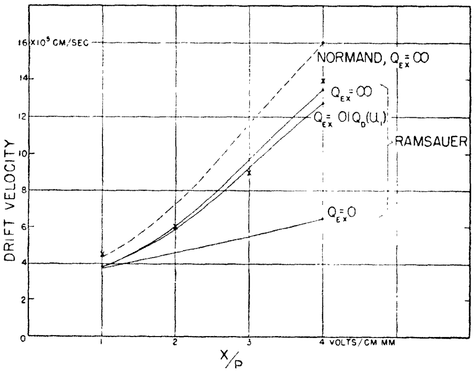

FIG. 2. Drift velocity of electrons in argon. See reference 11. Crosses are from Nielsen.

For $Q_{\text{ex}}=0.01\ Q_D(u_1)$, the solution of Eq. (1a) is given by Holstein as

$$f_3(u)=f_1(u)\left[\frac{f_3'(u_1)}{f_1(u_1)}+\frac{3}{4a^2}\int_u^{u_1}\frac{\phi}{\lambda_D f_1}du\right]. \tag{11a}$$

In solving Eq. (1b) for the excitation region, it is assumed that the excitation losses very much exceed the recoil losses. The m/M term is therefore negligible, and the solution of Eq. (1b) becomes

$$f_3'(u)=Cu^{-\frac{1}{2}}e^{-\gamma u}, \tag{11b}$$

where $\gamma=(1/2a)[3/\lambda_{\text{ex}}\lambda_D]^{\frac{1}{2}}$. For the excitation region $1/p\lambda_{\text{ex}}=0.01\ Q_D(u_1)$ and $1/p\lambda_D=Q_D(u_1)$. The constants in Eq. (11a) have been arranged so that at $u=u_1$, $f_3(u_1)=f_3'(u_1)$.

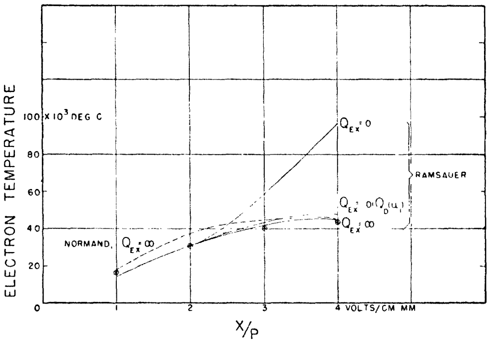

FIG. 3. Temperature of electrons in helium. See reference 11. Circles are from Townsend and Bailey.

III. DISCUSSION OF RESULTS

Graphs for the energy distribution, drift velocity, and temperature of electrons in helium and argon are shown in Figs. 1 through 9.[11] Figures 1 and 2 show the drift velocity, and Figs. 3 and 4 give the electron temperature, both quantities as a function of X/p from $X/p=1$ to 4 volts/cm mm. The solid curves in Figs. 1 through 4 were computed from the elastic cross-section data of Ramsauer and Kollath, while the broken curves were obtained from the corresponding data of Normand. Figures 5 and 6 show the electron energy distribution,

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[9] M. J. Druyvesteyn, Physica 10, 69 (1930).

[10] See Sec. VII of reference 1.

[11] In Figs. 1 through 3, the solid line curves were calculated from the elastic scattering data of Ramsauer and Kollath while the broken line curves were obtained from the corresponding data of Normand. The curves in Figs. 4 through 9 were obtained from the data of Ramsauer and Kollath.

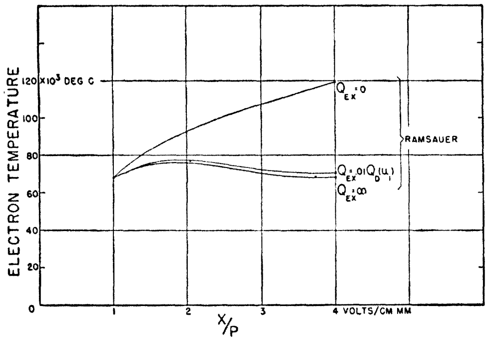

FIG. 4. Temperature of electrons in argon.

$u^{\frac{1}{2}}f(u)$, with X/p as a parameter. Figures 7 and 8 illustrate the effect of the excitation cross section on the shape of the energy distribution by comparing curves obtained for $Q_{\mathrm{ex}}=0$, ∞ and $0.01\ Q_D(u_1)$ at $X/p=4$ volts/cm mm. Figure 9 demonstrates the effect of a variable elastic cross section on the shape of the energy distribution curve by comparing the distribution in helium where $Q_D(u)$ decreases with energy with the distribution in argon where $Q_D(u)$ increases with energy.

It is evident from the graphs that in all cases the results for $Q_{\mathrm{ex}}=\infty$ and $Q_{\mathrm{ex}}=0.01\ Q_D(u_1)$ are extremely close. Figures 7 and 8 indicate that although a considerable percentage of the electrons are to be found in the excitation region for $Q_{\mathrm{ex}}=0$ at $X/p=4$, an excitation cross section of $0.01\ Q_D(u_1)$ is sufficient to drive almost all of the electrons from the excitation region producing a distribution curve for $Q_{\mathrm{ex}}=0.01\ Q_D(u_1)$ that approaches the curve for $Q_{\mathrm{ex}}=\infty$. Physically, it can be reasoned that when the steady state is attained in a gas discharge the energy losses sustained by electrons through the medium of collisions are replaced by gains from the electric field. However, the energy losses from excitation collisions are large so that most of the electrons involved in such collisions are thrown back close to the zero energy level with very little chance, in the case of small fields, of regaining the loss. Apparently, the small values of excitation cross section chosen in these calculations to approximate the actual values in helium and argon were sufficient to create a distribution close to that of $Q_{\mathrm{ex}}=\infty$. The discrepancy between the curves for $Q_{\mathrm{ex}}=0.01\ Q_D(u_1)$ and $Q_{\mathrm{ex}}=\infty$ increases with X/p. However, the difference is small up to $X/p=4$.

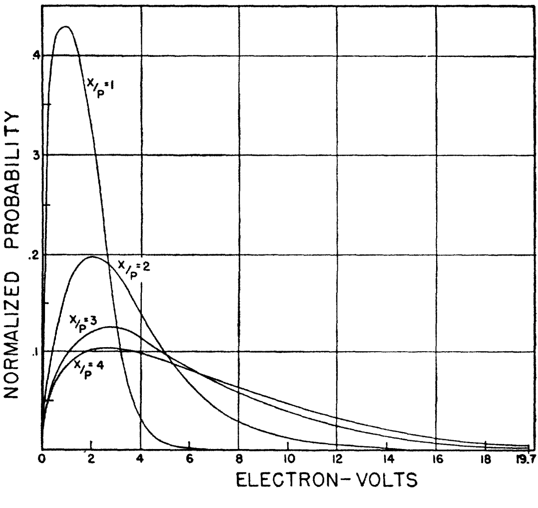

FIG. 5. Energy distribution of electrons in helium for $Q_{ex}=0.01\ Q_D(u_1)$.

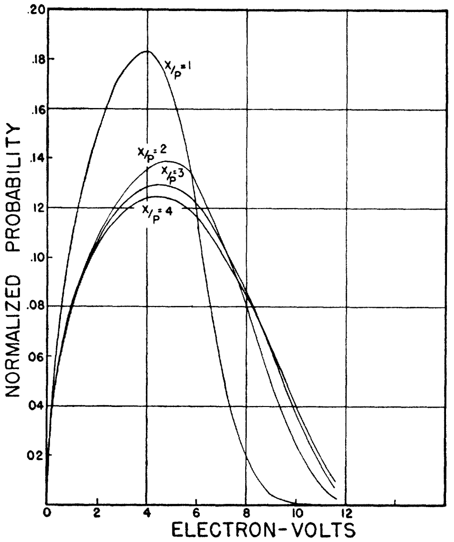

FIG. 6. Energy distribution of electrons in argon for $Q_{ex}=0.01\ Q_D(u_1)$.

Also shown in Figs. 1 through 4 are the drift velocity measurements of Townsend and Bailey[3] and Nielsen,[4] and the electron temperature measurements of Townsend and Bailey.[3] The agreement between the curves computed from the data of Ramsauer and Kollath and experiment is good while Normand's curves are much too high.[12] In the case of argon, Townsend's data is too

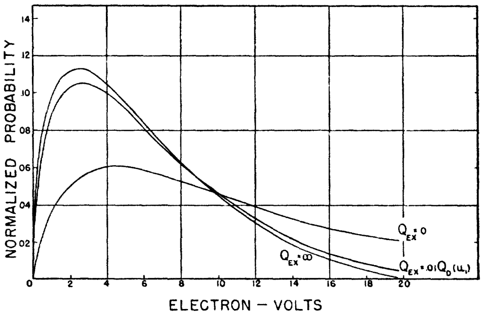

FIG. 7. Effect of the excitation cross section on the energy distribution of electrons in helium for $X/p=4$ volts/cm mm.

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[12] It is interesting to note that all the quantities discussed above are a function of the product $X\lambda$. Thus, since λ (Normand) is approximately equal to 1.2λ (Ramsauer and Kollath), a result calculated from Normand's data for X should equal the one calculated from the data of Ramsauer and Kollath for $1.2X$. This fact is illustrated in Figs. 1, 2, and 3.

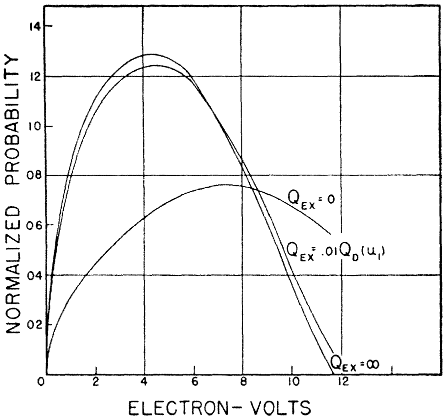

FIG. 8. Effect of the excitation cross section on the energy distribution of electrons in argon for $X/p=4$ volts/cm mm.

high to be shown. In his day, argon was the most difficult of the rare gases to purify and because of its Ramsauer effect the most sensitive to impurities.

In Fig. 9 the energy distributions in helium and argon are compared, where in helium $Q_D(u)$ decreases [$\lambda_D(u)$ increases] with energy and in argon $Q_D(u)$ increases [$\lambda_D(u)$ decreases] with energy. The peak of the distribution curve, which indicates the most probable energy of the electrons, falls at a higher energy level in argon than in helium. The distribution curve in argon, however, decreases rapidly for energies above the most probable value while the downward slope in helium is considerably more gradual. Thus, we would expect a greater relative percentage of electrons at the higher energies in helium than in argon. For energies below the most probable energy peak, the distribution decreases more rapidly in helium. These effects have been observed experimentally and are very well explained in Loeb's book.[13]

A study of the effect of excitation collisions and of a variable elastic cross section on the form of the distribution law in the rare gases was made by Allen.[14] The distribution was evaluated for variable elastic free paths from the Morse, Allis, and Lamar equation which in neglecting inelastic collisions is identical with Eq. (2) derived for $Q_{ex}=0$. The effect of excitation impacts was then taken into account by cutting off the distribution curve and having it cross the zero axis at some energy value (E_1) between the first excitation potential and the ionization potential of the gases. This assumption is equivalent to setting $Q_{ex}=\infty$ for all energies beyond E_1. The exact value of E_1 was chosen by comparing the calculated results with the measured drift velocities or with the measurements of the average electron energy by Townsend and Bailey.[3] For helium, agreement was obtained for $E_1=19.7$ volts, the first excitation potential of the gas. For argon, agreement regarding the drift velocity was obtained at $E_1=11.57$ volts, the first excitation potential of the gas, but for agreement in the case of the average electron energy it was necessary to set E_1 at 15.6 volts, the ionization potential of the gas. Since Townsend's measurements in argon were often inaccurate, the former value of E_1 should be considered as the more reliable point. Thus, for both argon and helium, optimum agreement with experiment was obtained when the distribution curves were cut off at E_1, the first excitation level of the gas. The manner in which Allen accounted for the excitation collisions was, therefore, to set $Q_{ex}=\infty$. This fact is confirmed by the closeness of Allen's results to those of this paper for $Q_{ex}=\infty$.

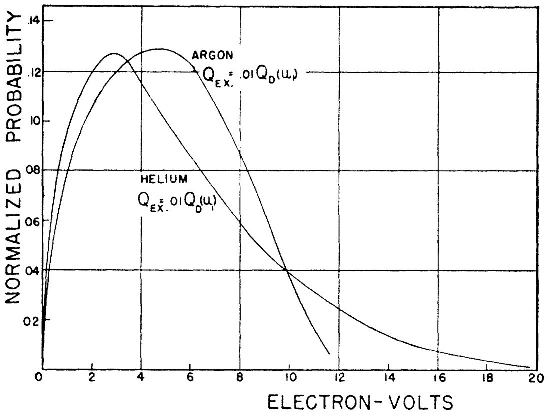

FIG. 9. A comparison of the shape of the energy distribution curves in helium and argon illustrating the effect of a variable elastic cross section, for $X/p=3$ volts/cm mm. The diffusion cross section decreases with energy in helium and increases with energy in argon.

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[13] L. B. Loeb, *Fundamental Processes of Electrical Discharge in Gases* (John Wiley and Sons, Inc., New York, 1939), p. 214.

[14] H. W. Allen, Phys. Rev. **52**, 707 (1937).