Electron-Electron Scattering at 15.7 Mev*

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The differential scattering cross section for electron-electron scattering has been measured at relativistic energies. A focused beam of 15.7-Mev monokinetic electrons from the 22-Mev betatron was scattered by 1- and 2-mil Nylon foils in a 20-inch scattering chamber. The incident current was collected by a faraday cage after it had passed through the scattering foil and was measured by a vibrating reed electrometer. The scattered beam was defined by gold-lined slits at a known angle to the incident beam. The number of scattered electrons in a given momentum interval chosen by a magnetic analyzer was counted by a Geiger counter.

Energy analysis of the scattered electrons made it possible to separate the nuclear scattering events from the electron scattering events. The energy of the scattered electrons was measured as a function of the scattering angle and was found to be in agreement with the laws of relativistic mechanics within an experimental accuracy of 0.4 percent.

For the electron-electron collisions the relative scattering cross sections agree with the Møller formula. For the electron-nuclear collisions they agree with the relativistic Mott theory (Born approximation) corrected for the radiative losses as calculated by Schwinger (3 to 8 percent). The experimental consistency of the cross sections was about 4 percent.

The absolute differential scattering cross sections for the electron-nuclear scattering were measured at angles between 10° and 43°. The average of the absolute values was 2 percent lower than theory. This is probably due to systematic errors in the solid angle, and in the collection and counting efficiencies.

The absolute differential scattering cross sections for electron-electron scattering were measured at the same set of angles, which correspond to the angular interval of 70° to 150° in the center-of-mass system. The average of these absolute cross sections was 7 percent lower than theory predicts. The bulk of the 5 percent additional departure from theory may be due to a systematic error in the measurement of the momentum interval defined by the counter.

I. INTRODUCTION

THE problem of the single scattering of electrons by electrons has been treated from a theoretical point of view by several investigators.[1–3] The theory now generally accepted is due to Møller,[2] who gives the differential scattering cross section for the scattering of free electrons in the center-of-mass system as:

$$\sigma(\theta^*)d\Omega^*=\frac{\gamma+1}{2\gamma^2}\left(\frac{e^2}{mv^2}\right)^2\left\{\csc^4\frac{\theta^*}{2}+\sec^4\frac{\theta^*}{2}-\csc^2\frac{\theta^*}{2}\sec^2\frac{\theta^*}{2}+\left(\frac{\gamma-1}{\gamma}\right)^2\left[1+\csc^2\frac{\theta^*}{2}\sec^2\frac{\theta^*}{2}\right]\right\},\quad(1)$$

where θ^* is the angle of scattering in the center-of-mass system, Ω^* is the solid angle in the center-of-mass system, γ is the total energy of the incident electron in units of mc^2 in the laboratory system, v is the velocity of the electron in the laboratory system, m is the rest mass of an electron, and e is the charge of an electron.

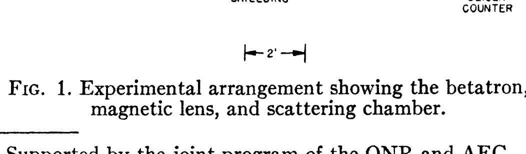

FIG. 1. Experimental arrangement showing the betatron, magnetic lens, and scattering chamber.

The first two angular terms in Eq. (1) represent the coulomb scattering and the corresponding recoil electrons and define a "classical" theory of scattering in which quantum-mechanical interference is not taken into account. The third angular term is the interference between the first two terms arising from the exclusion principle and the identity of the particles. The sum of the first three terms represents the angular dependence of the Mott[1] theory based on the nonrelativistic Schroedinger wave equation. The fourth term is a relativistic correction introduced by Møller arising from the use of the Dirac wave equation. In the nonrelativistic limit ($\gamma\rightarrow1$) the Møller expression (1) reduces to the Mott formula. In the relativistic limit the Møller formula becomes approximately classical due to the near cancellation of the interference term by the relativistic correction.

Experimental study of the scattering problem has been carried out by several investigators. Williams[4] found good agreement with the nonrelativistic Mott theory for electrons of 20 kev, by counting forked tracks in a cloud chamber. After the theory of Møller indicated that deviations from the Mott formula might be expected for relativistic collisions, efforts were made to measure the scattering of fast beta-particles. Champion[5] found good agreement with the Møller theory for 250 collisions of radium E beta-particles in nitrogen found in cloud chamber pictures. More recently, Groetzinger

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[4] E. J. Williams, Proc. Roy. Soc. (London) **A128**, 459 (1930).
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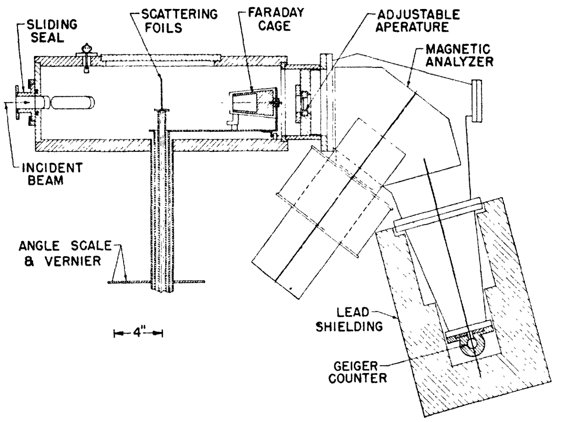

FIG. 2. Scattering chamber.

et al.,[6] have analyzed 98 electron-electron collisions using P^{32} beta-rays in a gas-filled cloud chamber obtaining agreement with Møller. However, the energy of the electrons was too low and the statistics too poor to distinguish unambiguously between the Møller theory, the relativistic Mott theory, or the theory of Kar and Basu.[3] Page and Woodward[7] have also reported good agreement with Møller for the scattering cross section. Champion[8] was the first to confirm the kinematic relations for relativistic collisions of electrons.

The present work was carried out to measure the differential scattering cross section and energy of relativistic electrons scattered by atomic electrons. The principal improvements in this experiment are the use of monokinetic electrons of high energy (15.7 Mev), which permitted the testing of the Møller relativistic correction, and the use of a counter as a detector of the scattered electrons to improve the statistics of the data.

II. EXPERIMENTAL ARRANGEMENT

The source of electrons for this experiment was the 22-Mev betatron with a special donut equipped with a magnetic shunt to extract the electron beam[9] (Fig. 1). The divergent beam from the betatron was collimated and focused by means of a magnetic lens to a spot about 2 mm in diameter on the scattering foil at the center of the scattering chamber ten feet from the betatron. The convergent cone of rays had a spread of about $\frac{1}{2}°$.

Electrons were admitted to the brass scattering chamber (Fig. 2) 6 inches high and 20 inches in diameter through a series of slots in the cylindrical wall. The slots were covered by a polished brass cylindrical cover plate. A sliding vacuum seal between the wall and plate was maintained by a single *O*-ring imbedded in the chamber wall and enclosing all of the slots. The beam entered the chamber through a tube attached to the cover plate. The scattering angle was altered by rotating the chamber and detector system, while the cover plate was held stationary. All scattering angles up to 53° were accessible, and most of the angular range was accessible on both sides of center.

A remotely controlled target holder in the center of the chamber had the following targets mounted on it: a fluorescent screen for locating the beam visually, a vertical 20-mil tungsten wire for measuring the shape and position of the beam by the scattering from the wire, a one-mil nylon scattering foil (2.287 mg/cm^2), a two-mil Nylon foil (4.468 mg/cm^2), and several blank spaces. The Nylon was $C_{12}H_{22}O_2N_2$, furnished by the DuPont Company. The target holder could be moved so as to bring any of the positions on it into the path of the beam without turning off the betatron.

After passing through the scattering foil, the unscattered portion of the beam was collected by an aluminum faraday chamber having a body depth of 1.6″ and subtending 12°. The charge collected by the chamber was measured by the voltage change across a condenser attached to a model 30 vibrating reed electrometer[10] which could be switched by remote control.

The scattered beam was defined by adjustable gold-edged slits (Fig. 3) located in the cylindrical wall of the scattering chamber. The slits were formed of two interlocking brass pieces $\frac{3}{4}$ inch thick, which could slide relative to each other, thereby changing the width of the slit up to a maximum of $\frac{3}{4}$ inch. The slit opening was one inch high and completely edged with a gold strip $\frac{1}{16}$ inch wide and $\frac{1}{8}$ inch deep inset into the brass. The brass behind the gold was relieved by 3° so that the electrons passing through the slit could be scattered only by the gold edges.

Electrons admitted by the defining slit passed through a 75° magnetic analyzer and into a thin-walled glass Geiger counter detector surrounded by lucite and lead shielding. With a 0.375-inch slit above the Geiger counter detector, the energy spectrum admitted to the counter was 1 percent wide at the peak. Except for the wall of the Geiger counter, the only scattering material in the path of the beam was the the scattering foil itself.

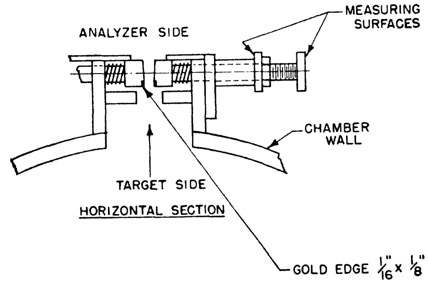

FIG. 3. Detail of defining aperture.

[6] Groetzinger, Leder, Ribe, and Berger, Phys. Rev. **79**, 454 (1950).
[7] L. A. Page and W. M. Woodward, Phys. Rev. **79**, 228 (1950).
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[9] Lyman, Hanson, and Scott, Phys. Rev. **84**, 626 (1951).

[10] Applied Physics Corporation, Pasadena, California.

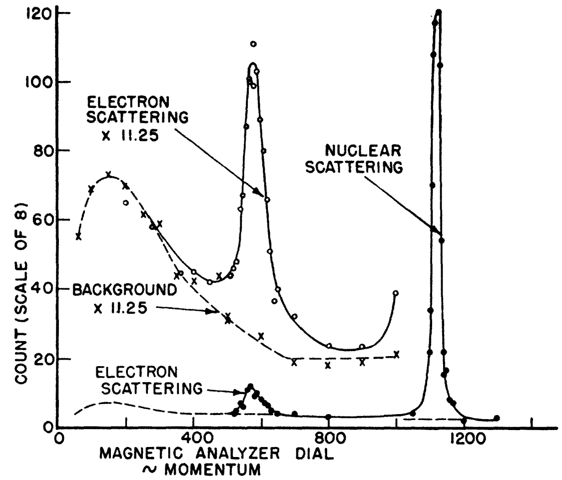

FIG. 4. Momentum spectrum of electrons scattered through 14 degrees. Laboratory angle = 14°. Center-of-mass angle = 90°.

III. EXPERIMENTAL PROCEDURE

The procedure for lining up the electron beam was to determine the path of the beam through the vacuum system as the system was built up piece by piece, either by observing the beam with x-ray film or by using a fluorescent screen and mirror-telescope arrangement. The final alignment was checked by passing the tungsten wire across the center of the chamber and observing the scattering from the wire to determine the size and position of the focused spot relative to the center of the chamber.

The data consisted of the observed number of scattered electrons per unit incident charge as a function of the analyzer magnetic field (i.e., the momentum of the scattered electrons). Counts were taken at intervals of 30 to 70 kev across the peak due to electron-electron scattering, at several points off this peak and across the elastic scattering peak for each foil, with foils absent, and at each of 9 angles. To eliminate systematic errors in measurement of the scattering angles, data were obtained for angles on each side of center in some cases. At one of the angles (14 degrees) measurements were taken on one side of the center at the start of the experiment and repeated on the other side of the center at the end of the experiment as a check on the consistency of the data.

At various times during the course of the experiment the shape and position of the focused spot at the center of the chamber were observed with the tungsten wire. The width at half-maximum of the spot was 1.5 to 2.0 mm, and the center of the spot moved about 1 mm during the experiment.

Transmission ratios for the thick foil were obtained by finding the charging time of the vibrating reed electrometer condenser for constant incident current, with and without the scattering foil in position. The transmission ratio of the thick foil was found to be 0.992 and the ratio for the thin foil was estimated to be 0.996.

In the case of one angle (14°) data were obtained for five different slit widths. The results indicated that the observed count was linear with the slit width up to at least 0.37 inch. At all the other angles the data were obtained with a slit width of 0.269 inch.

IV. TREATMENT OF DATA

In order to obtain the true number of counts corresponding to an observed number, it was necessary first to reduce the observed number by subtracting the natural background of the counter and second, to adjust for the fractional loss of counts due to the high peak intensities used. The fractional loss was taken to be the counting rate observed divided by twice the repetition frequency of the betatron (180 cps). This latter correction never exceeded 10 percent.

In order to obtain the true number of counts due only to electrons scattered by the target foil and to eliminate those due to electrons and x-rays scattered

TABLE I. Electron-electron scattering results.

Laboratory scattering angle (degrees)	Target thickness (mils)	Differential scattering cross section (lab system) (barns/sterad) experiment	Exp't / Møller	Experimental cross section in center-of-mass system (barns/sterad)	Scattering angle in center-of-mass system (degrees)
9.98	2	0.435	0.95	0.0144	70.9
	1	0.442	0.96	0.0146	
13.77	2	0.201	0.99	0.0111	89.5
	1	0.195	0.96	0.0107	
14.37	2	0.183	0.97	0.0109	92.0
16.98	2	0.124	0.83	0.0106	102.0
	1	0.136	0.91	0.0116	
19.33	2	0.125	0.94	0.0146	109.4
	1	0.118	0.88	0.0138	
23.57	2	0.113	0.91	0.0225	120.9
	1	0.109	0.87	0.0217	
27.84	2	0.124	0.95	0.0406	129.8
	1	0.120	0.92	0.0393	
33.83	2	0.133	0.88	0.081	139.5
	1	0.133	0.88	0.081	
42.65	2	0.212	0.99	0.286	150.0
	1	0.213	1.00	0.288	

about the room, the background count with no foil present was obtained and subtracted for each point of the energy spectrum.

Figure 4 is an example of the data obtained in this experiment. There are two peaks, one due to nuclear scattering at the incident energy, and one due to electron scattering at a lower energy. The energy lost in the latter case is carried away by the other electron participating in the collision. The upper curve is an expansion of the low energy end of the lower curve to show the relative size of the peak and background. The example shown is for the angle of symmetry in the center-of-mass system, for which the cross section is small and hence the background is large relative to the scattering. The electron peak has a considerable energy width due to the angular spread associated with the finite aperture of the detector and the multiple scattering in the foil.

The total number of electrons associated with the electron-electron scattering peak was determined from the area under this peak and the fractional resolution of the analyzer. The fractional momentum resolution of the analyzer (r) was obtained from the width at half-maximum, of the nuclear scattering peak which is nearly monoenergetic.

If we let $C_{\text{obs}}(p)$ be the observed count at an analyzer setting which admits to the detector electrons with an average momentum p in the small momentum interval rp, then the total number of electrons associated with the electron-electron peak is given by

$$C_t=\int_{\text{peak}} C_{\text{obs}}(p)/rp\,dp\cong(1/r\bar{p})\int_{\text{peak}} C_{\text{obs}}(p)dp,$$

where $\bar{p}$ is the momentum at the peak and the integral is the area under the experimental curves of the net count plotted as a function of the momentum.

V. EXPERIMENTAL RESULTS

The experimental results are given in Tables I, II, and III. Table I gives the differential scattering cross section for electron-electron scattering. The scattering angle of column 1 has been averaged over the defining slit in the laboratory system. The corresponding angle in the center-of-mass system is given in column 7. The target thicknesses in column 2 are approximate and are used only to identify the foil. The experimental scattering cross section in barns per unit solid angle per electron is given in column 3. The ratio of the experimental cross section to the theoretical cross section is given in column 4. The cross section in the center-of-mass system corresponding to the experimental cross section in the laboratory system (column 3) is given in column 5. Figure 5 shows the experimental cross sections from Table I, compared with the Møller, relativistic Mott, Kar, and classical theories. We can see that the experimental points follow the Møller theory and depart as much as 100 percent from the Mott theory for the 90° scattering. The points average about 7 percent low, which may be due to a systematic error in the measurement of the momentum interval defined by the counter.

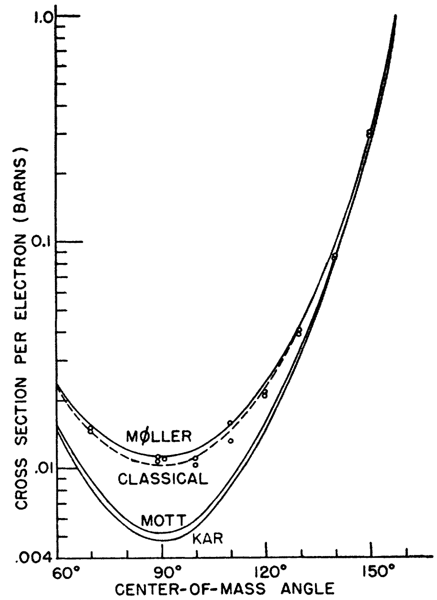

FIG. 5. Differential scattering cross section for electron-electron scattering as a function of the angle in the center-of-mass system. The curves represent the theoretical values according to the Møller, Kar, and Mott formulations. The curve indicated as classical represents the first two terms of the Møller formula.

Table II gives the data on the kinetic energy of the scattered electrons as a function of the scattering angle. The calculated kinetic energy is based on the formulas:

$$T=T_0\cos^2\tfrac{1}{2}\theta^*;\quad (\tfrac{1}{2}T_0+1)^{\frac{1}{2}}\tan\theta=\tan\tfrac{1}{2}\theta^*,$$

where θ^* is the angle in the center-of-mass system, θ is the angle in the laboratory system, T is the kinetic energy in units of mc^2, and T_0 is the incident kinetic energy (15.7 Mev) in units of mc^2. These formulas are based on the assumption that during the collision energy and momentum are conserved and that the velocities combine according to the relativistic law of addition of velocities.

An experimental check on the absolute energy of the

TABLE II. Energy of scattered electrons *vs* laboratory scattering angle.

Average laboratory angle degrees	Observed kinetic energy Mev	Calculated kinetic energy Mev
0	15.672	15.700
9.98	10.413	10.420
13.77	7.924	7.918
14.37	7.567	7.570
16.98	6.211	6.216
19.33	5.212	5.210
23.57	3.829	3.816
27.84	2.808	2.822
33.83	1.893	1.880
42.65	1.037	1.055

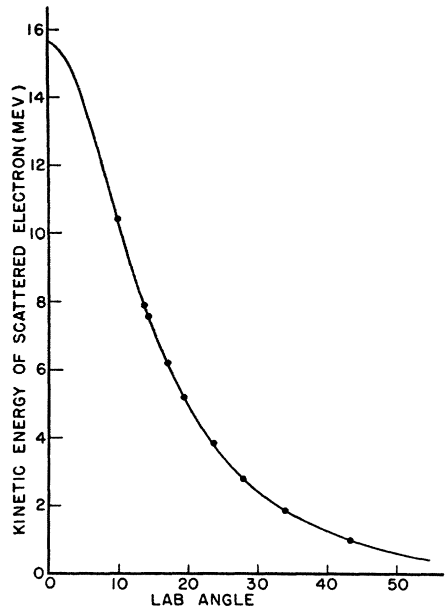

FIG. 6. Energy of scattered electrons at the maximum of the electron-electron peak as a function of the angle. The solid line represents the theoretical variation of energy with the angle. The incident energy is 15.7 Mev.

incident electrons was obtained by comparing the observed variation of momentum of the scattered electrons as a function of the angle with that expected from the above kinematic relations. The absolute values listed in column 2 of Table II are based on a value of the incident energy determined in this way. The relative values in the column are determined from the observed magnetic field corresponding to the peak of the electron-electron distribution at the indicated angle. The calculated energies given in column 3 of the table are based on an incident energy of 15.7 Mev which is determined by using the value of 10.9±0.2 Mev for the threshold of the $Cu^{63}(\gamma, n)$ reaction. It can be observed that the energy scale obtained from the observed variation of the energy with angle in an electron-electron collision is in agreement with the energy scale usually used to better than 0.5 percent. The data of Table II are shown in Fig. 6, where the line is the theoretical calculation and the points are the experimental data.

Table III gives the nuclear scattering results. The scattering angle in column 1 is the angle to the center of the defining slit. In column 3 are the experimental differential cross sections in barns/sterad corrected for the finite size of the defining slit and reduced to $Z=1$. The corresponding theoretical cross section is given by Mott as:

$$\sigma(\theta)=\left(\frac{e^2}{2mc^2}\right)^2\frac{1-\beta^2}{\beta^4}\,\frac{1-\beta^2\sin^2\frac{1}{2}\theta}{\sin^4\frac{1}{2}\theta}. \qquad (2)$$

Column 5 is the ratio of the experimental to the theoretical cross section corrected for the loss of counts from a detector of 1 percent energy resolution due to radiation straggling and emission and reabsorption of virtual quanta as given by Schwinger.[11] This effect varies from 3 to 8 percent. The experimental data and the Schwinger correction are given in Fig. 7 as a function of the scat-

TABLE III. Experimental results—electron-nuclear scattering.

Scattering angle at center of defining slit degrees	Foil thickness mils	Experimental differential scattering cross section corrected to center of slit barns/sterad ($Z=1$)	Theoretical cross section (Mott)	Exp't/theory corrected for radiation (Schwinger)
9.88	2	0.353	0.357	1.023
	1	0.362		1.049
13.70	2	0.0869	0.0963	0.948
14.30	2	0.0742	0.0811	0.963
	1	0.0748		0.970
16.92	2	0.0358	0.0413	0.918
	1	0.0386		0.990
19.28	2	0.0224	0.0244	0.971
	1	0.0226		0.984
23.53	2	0.0106	0.0110	1.027
	1	0.0104		1.018
27.80	2	0.00492	0.00559	0.945
	1	0.00473		0.911
33.80	2	0.00226	0.00253	0.968
	1	0.00232		0.996
42.63	2	0.000918	0.000980	1.023
	1	0.000893		0.996

[11] J. Schwinger, Phys. Rev. **76**, 790 (1949).

tering angle, normalized to Mott scattering (2). The observed points follow somewhat better the line of the Mott theory with the Schwinger correction. The points average about 2 percent low, which is probably due to systematic errors in the solid angle and in the collection and counting efficiencies.

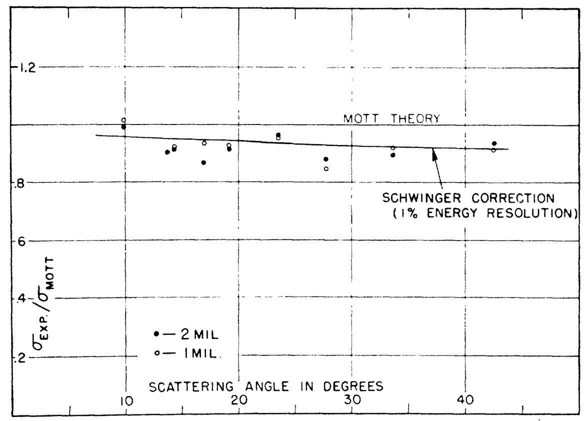

FIG. 7. Ratio of the experimental cross sections for the nuclear scattering from Nylon to those calculated according to the Mott formula. The curve takes into account the expected reduction in the cross section because of radiative energy losses.

SUMMARY

The observed relative cross section for electron-electron scattering agrees with that given by the Møller formula. The relative cross section for nuclear scattering by light nuclei agrees fairly well with that given by the Mott formula [Eq. (2)] after correcting for the loss of counts due to the radiative losses by the Schwinger formula.

The individual values of the observed cross sections seem to be consistent to about 4 percent. The absolute cross sections are 7 percent low in the case of electron-electron scattering and 2 percent low for nuclear scattering. The departure of the absolute values from theory is not significant and may be due to systematic errors common to each group of measurements. The energy of the electrons scattered by electrons was investigated as a function of the scattering angle and was found to be in agreement with relativistic kinematics.

We wish to express our appreciation to many of the members of the laboratory for their interest and cooperation in this work. In particular, we are indebted to D. E. Riesen for the maintenance and operation of the betatron throughout this work, and to A. J. Petersen, J. E. Leiss, and G. E. Mader for valuable assistance in developing the equipment and in obtaining the data.