data, however, agree better with the calculations for a uniform charge distribution than for the surface charge distribution.

VI. SUMMARY

The observed scattering of 15.7-Mev electrons by nuclei is compared with the calculations of Elton and of Acheson on the assumption that the radiation correction as calculated by Schwinger is valid. The observed deviations from coulomb scattering are consistent with the picture of a nuclear charge whose radius is given by the neutron experiments and whose density distribution is uniform or possibly slightly greater at the center of the nucleus.

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Measurement of Multiple Scattering of 15.7-Mev Electrons*

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The angular distribution of electrons scattered by thin Be and Au foils has been measured for angles where the multiple scattering is important. The $1/e$ widths of the distributions obtained with Au foils of 18.66 and 37.28 mg/cm^2 are 2.58° and 3.76°, respectively. These widths are about 10 percent narrower than those calculated from the theories of Williams or of Goudsmit and Saunderson but are in good agreement with the calculations of Molière.

The $1/e$ widths obtained with Be foils of 257 and 495 mg/cm^2 are 3.06° and 4.25°. These widths are about 5 percent smaller than those given by Molière's theory increased by $(1+1/Z)^{1/2}$ for the contribution of electron-electron collisions. The discrepancy may be qualitatively explained by the fact that the Thomas-Fermi screening used in the calculation is different from the effective screening in the Be metal.

The scattering from the two Au foils was measured for larger angles where the scattering can be considered as single scattering modified by the effect of multiple scattering. The ratio of the scattering from the thick to the thin foil can be represented by the relation $2+95/\theta^2$ from 9° to 30°, and is in fair agreement with theoretical expressions for this ratio.

THERE have been a number of experimental investigations of the scattering of electrons by multiple collisions in passing through various materials. The most complete series of experiments has been carried out by Kulchitsky *et al.*,[1] who investigated the scattering of 2.25-Mev electrons by elements from lithium to lead. They found that their results were in agreement with the theories of Williams[2] and of Goudsmit and Saunderson[3] for the light elements if the electron-electron scattering were accounted for by increasing the width calculated for nuclear collisions by a factor $(1+1/Z)^{1/2}$. Their results for the heavy nuclei, however, gave widths which were from 10 to 15 percent narrower than predicted by the theories mentioned.

The present work[4] with higher energy electrons confirms the above results. It will be shown that the experimental results to be presented here, as well as those of Kulchitsky *et al.*, are in better agreement with the theory of Molière,[5] who has investigated the basic single scattering law at small angles, which enters into the theory, in more detail than other authors.

A brief resumè of the various theories is given by Groetzinger, Berger, and Ribe,[6] in connection with the discussion of some cloud chamber observations.

EXPERIMENTAL METHOD

The experimental arrangement used was the same as that described in the work on electron-electron scattering[7] except for the insertion of a smaller aperture defining the scattered beam, and the use of a different detector.

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[1] L. A. Kulchitsky and G. D. Latyshev, Phys. Rev. **61**, 254 (1942); Andrievsky, Kulchitsky, and Latyshev, J. Phys. USSR **6**, 279 (1943).
[2] E. J. Williams, Phys. Rev. **58**, 306 (1940).
[3] S. Goudsmit and J. L. Saunderson, Phys. Rev. **58**, 36 (1940).
[4] Preliminary results were reported at the Chicago meeting of the Physical Society, Phys. Rev. **81**, 309 (1951).
[5] G. Molière, Z. Naturforsch **3a**, 78 (1948); **2a**, 133 (1947).
[6] Groetzinger, Berger, and Ribe, Phys. Rev. **77**, 584 (1950). See also Mott and Massey, *The Theory of Atomic Collisions* (Oxford University Press, London, 1949), second edition, p. 193.
[7] Scott, Hanson, and Lyman, Phys. Rev. **84**, 638 (1951).

The arrangement can be described briefly as follows: A monoenergetic 15.7-Mev electron beam was focused upon a scattering foil at the center of a scattering chamber 20 inches in diameter. The converging cone of electrons had a full angular width of about 1°, and was focused to a spot about 0.08 inch in diameter. The scattered electrons were selected by means of a $\frac{1}{32}$-inch hole in a $\frac{1}{8}$-inch thick lead plate, placed about 10 inches from the scattering foil. After passing through this aperture, the electrons were deflected by an analyzer magnet into a large ionization chamber. The exit port on the analyzer was enlarged so that all electrons whose energy was within 6 percent of the maximum energy entered the ionization chamber.

The incident current was measured by means of a faraday cage connected to a vacuum tube electrometer. This current was maintained at a constant value throughout the experiment. The scattered current was measured by a vibrating reed electrometer connected to the ionization chamber.

At small angles, it was necessary to remove the faraday cage from the forward direction, since it would have intercepted the scattered beam. This was done by remote control, so that a measure of the incident current could be obtained immediately before and after each measurement of the scattered current.

Measurements of the angular distribution were made on both sides of the incident beam for two gold and two beryllium foils from 0° to 6°, using the $\frac{1}{32}$-inch aperture. Measurements of the scattering from the gold foils were extended from 6° to 30° with the larger aperture used in the electron-electron scattering work (0.269×1.00-inch). The scattering by the beryllium foils at the larger angles was not measured, since the analyzer would have excluded the electrons scattered by atomic electrons, giving results which would not be directly comparable to measurements at small angles.

No corrections have been made for the angular width of the incident beam, as this width was small enough to have a negligible effect on the observed distributions.

RESULTS AND COMPARISON WITH THEORY

The observed distributions of electrons scattered by two gold foils are shown in Fig. 1. The measurements were made using the small aperture. The data were normalized so as to present the fractional scattering per unit solid angle in square degrees. The solid lines in the figure represent the distribution calculated from the theory of Molière. It can be seen that the agreement in the angular range given in the figure is very good. The broken lines indicate how the first term of Molière's formulation, the gaussian function $\exp(-\theta^2/B\chi_c^2)$, deviates from the more complete theory.

The distributions represented by the solid line can, however, be approximated quite accurately by a renormalized, slightly narrower gaussian which fits the experimental results very well up to angles (w) where the intensity is $1/e$ of the maximum. This $1/e$ width can be obtained from the theory and is given to a good approximation by

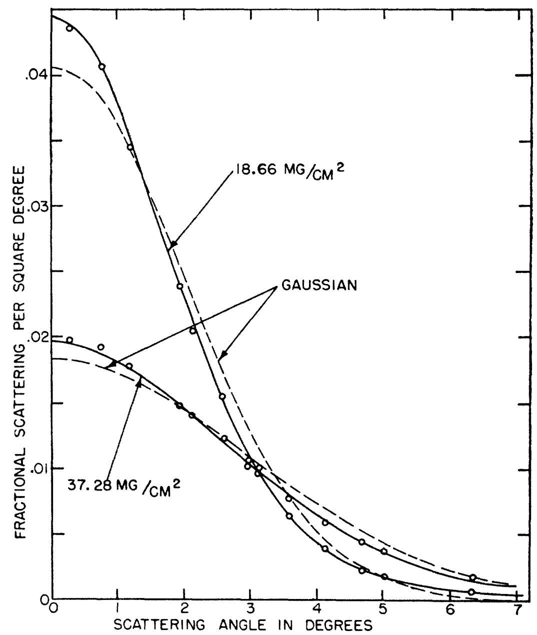

FIG. 1. Distribution of 15.7-Mev electrons passing through two thin gold foils in the region from 0° to 6°. The solid lines represent the theory of Molière. The dotted lines represent the first term of Molière, which is a gaussian having a width in agreement with the older theories.

$$w=\chi_c(B-1.2)^{\frac{1}{2}}\ \text{(spatial)}, \tag{1}$$

$$w=\chi_c(B-0.7)^{\frac{1}{2}}\ \text{(projected)}, \tag{2}$$

where

$$\chi_c^2=4\pi N(Z^2+Z)e^4/p^2v^2, \tag{3}$$

$$\chi_a^2=(\hbar Z^{\frac{1}{3}}/0.885pa_0)^2[1.13+3.76(Z/137\beta)^2], \tag{4}$$

$$\Omega_b=7e^B/6B=\chi_c^2/\chi_a^2, \tag{5}$$

where N is the number of atoms/cm², p and v the momentum and velocity of the incident electrons, and $a_0=\hbar^2/me^2$ is the Bohr radius of hydrogen. The effect of electron-electron collisions is taken into account by using Z^2+Z instead of Z^2 in Eq. (3). Ω_b can be seen to be proportional to the thickness and corresponds to the number of collisions in passing through the foil.

It might be pointed out that the projected distribution given by Molière is essentially the same as that calculated by Snyder and Scott[8] if the number of collisions is defined in the same way. Snyder and Scott's expression for the number of collisions is defined by the same equation if χ_a is determined in terms of the Born approximation solution to the scattering for exponential screening, namely

$$\chi_a=\hbar Z^{\frac{1}{3}}/pa_0. \tag{6}$$

For low Z, the Born approximation is valid and the difference between the two theories is slight. For high Z,

[8] H. Snyder and W. T. Scott, Phys. Rev. **76**, 220 (1949).

TABLE I.

Material	Be	Be	Au	Au
Target thickness mg/cm²	257.0	491.3	18.66	37.28
Average energy in the foil (Mev)	15.47	15.24	15.69	15.67
	Theoretical constants			
Molière χ_c(degrees)	1.08	1.52	1.10	1.55
Molière B	9.56	10.29	6.49	7.30
Number of collilisions Eq. (4)	1397	2667	118	236
Number of collisions Eq. (6)	2043	3907	366	732
	Central intensities			
I_0 per degree²	0.0317	0.0153	0.0441	0.0200
	$1/e$ widths in degrees			
w_{expt} (direct)	3.06	4.25	2.58	3.76
$w_{\text{expt}}(1/\sqrt{I_0})$	3.01	4.33	2.55	3.78
$(w_{\text{th}})_{\text{M}}$ Eq. 4	3.13	4.57	2.52	3.83
$(w_{\text{th}})_{\text{M}}$ Eq. 6	3.16	4.63	2.81	4.21
$(B^{\frac{1}{2}}\chi_c)$	3.34	4.94	2.81	4.18
$(w_{\text{th}})_{\text{G and S}}$	3.23	4.65	2.92	4.23

Molière gives a number of collisions which is 2 to 3 times smaller than that of Snyder and Scott and corresponds to a width which is about 10 percent narrower.

The experimental $1/e$ widths (w) and the corresponding calculated widths are given in Table I. Two values for the experimental widths are given. One is the observed angle at which the intensity is reduced to I_0/e; the other is based on the magnitude of the central intensities, which should be inversely proportional to the square of the $1/e$ widths. The latter widths are normalized to the two gold foils and yield values for Be which agree with the direct measurement to about 2 percent. The theoretical values of the $1/e$ width included are those from the tables of Goudsmit and Saunderson, the basic gaussian used by Molière, and the $1/e$ value according to Molière. Theoretical widths using the values for χ_a from Eq. (6) are also given. It can be seen that the agreement between the experiment and Molière's theory is good for gold, but that the experimental $1/e$ widths for beryllium are 3 to 7 percent narrower.

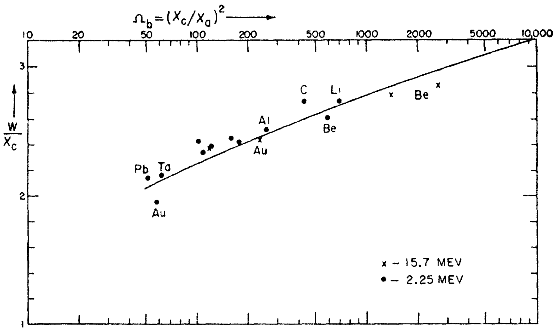

FIG. 2. Experimental values of the $1/e$ widths in units of w/χ_c, as a function of the number of collisions Ω_b. The line is that given by Molière's theory [Eq. (1)].

The results of Kulchitsky *et al.*, at 2.25 Mev, and the present results are shown as a function of the number of collisions, Ω_b, in Fig. 2. The solid line represents the theory of Molière, modified for the effect of electron-electron collisions by using the factor Z^2+Z as in Eq. (3). In such a plot, the light elements which yield a given width correspond to points at large Ω_b. It can be seen that the individual points scatter considerably, but that the over-all agreement is fairly good.

The discrepancy between the observed and calculated widths for Be can probably be explained by the fact that the effective screening of the nuclear charge in Be metal is considerably different than that given by the Thomas-Fermi screening used by Molière. Shinohara[9] has presented a treatment of multiple scattering based on the scattering from microcrystalline units in a solid. This appears to offer an optional way of calculating the scattering from solid materials and may ultimately describe the scattering somewhat better.

SINGLE SCATTERING REGION

The scattering for Au was observed for larger angles, by the use of the larger aperture. A correction (which was small except for the smallest angles) was made for the effect of the aperture size. The data extend from 6°, where they agree fairly well with the small angle data, to 30°, where they are normalized to agree with the observed results of the nuclear single scattering experiment. This single scattering at 30° was about 36 percent larger than that given by the relativistic Mott formula (Born approximation).[10] It was, for the purposes of this work, represented by the relation

$$f(\theta)=2\chi_c^2(1+0.012\theta)/\theta^4.$$

The observed points are shown on a logarithmic scale in Fig. 3, which also gives the complete Molière theory (solid line) over the entire angular range investigated.

The dotted line at smaller angles represents the tail of the gaussian in Fig. 1. The dotted line at larger angles represents the single scattering modified for the above-mentioned departure from the Born approximation.

The ratio of the theoretical scattering to single scattering as given by Molière at large angles is

$$\frac{I}{I_s}=1+\frac{4\chi_c^2}{\theta^2}\left[\ln\left(\Omega_b\frac{\theta^2}{\chi_c^2}\right)-2\right]. \qquad (7)$$

The experimental points follow the theory reasonably well at very small and very large angles but are somewhat higher than theory in the region from 6 to 20 degrees.

At large angles, the effect of multiple scattering on the observed scattering may be presented in a manner

[9] Shinohara, Inoue, and Hirohata, J. Phys. Soc. (Japan) 4, 227 (1949).

[10] Lyman, Hanson, and Scott, Phys. Rev. 84, 626 (1951).

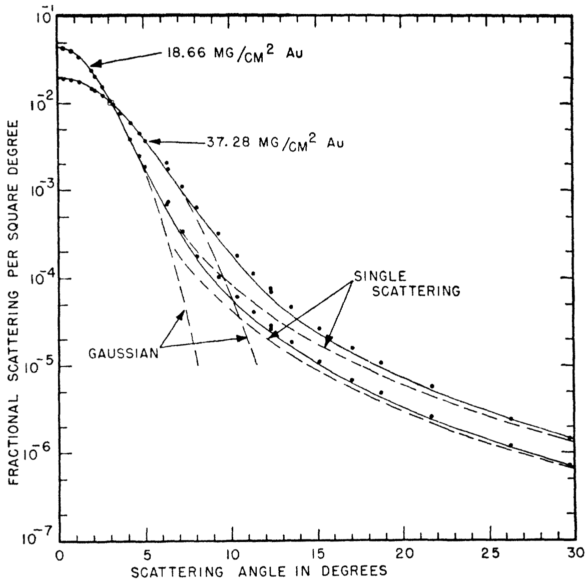

FIG. 3. Angular distribution of electrons from thick and thin gold foils from 0° to 30°. The solid line represents the theory of Molière extrapolated through the region where his small and large angle approximations give different values. The dotted lines at small angles represent the continuation of the gaussians of Fig. 1. At larger angles, the dotted line represents the single scattering contribution.

which is less dependent on the single scattering law. This is done by taking the ratio of the observed scattering intensities for the two gold foils. These ratios are shown in Fig. 4. It is found that these are represented fairly well by the expression $2+95/\theta^2$ in the region from 9° to 30°. The solid lines in the figure are values obtained from Molière's theory. The representations at small and large angles do not quite agree in the region in which they overlap. (This disagreement was ignored in the condensed scale used in Fig. 3.)

The ratio of scattering by two foils can also be obtained from the theory of Butler.[11] Good agreement is obtained with experiment if the gaussian used in Butler's theory is reduced by 9 percent. The deviation from the single scattering law at even larger angles has been discussed more generally by Chase and Cox.[12] At angles up to 30°, where the single scattering law can be represented by $1/\theta^4$, both Chase and Cox and Butler find that the increase in scattering can be represented to a first approximation by the factor $1+(4w^2/\theta^2)$. This factor is considerably smaller than Molière's at large angles. Butler's expression, including the higher order terms, represents the experimental results in this angular range somewhat better.

[11] S. T. Butler, Proc. Phys. Soc. (London) **63A**, 599 (1950).
[12] C. T. Chase and R. T. Cox, Phys. Rev. **58**, 246 (1940).

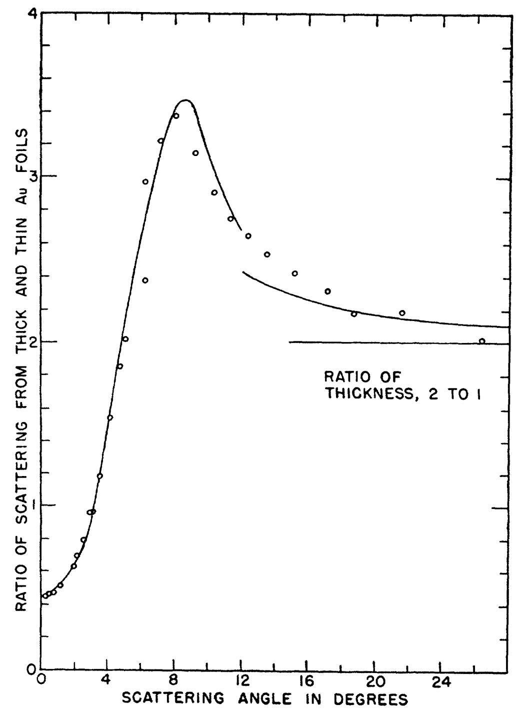

FIG. 4. Ratio of the scattering from thin and thick gold foils. The solid lines represent the values predicted by Molière's small and large angle approximations.

SUMMARY

The observed multiple scattering distributions of fast electrons after passing through thin gold foils is in good agreement with Molière's theory within the experimental accuracy of 2 to 3 percent. The widths are about 10 percent narrower than those given by the theories of Williams or Goudsmit and Saunderson.

The widths of the multiple scattering distributions for beryllium are slightly narrower than those given by Molière's theory. This result may be explained by the fact that the screening used in the theory is different than the effective screening in Be metal.

In the single scattering region, the departure from single scattering toward smaller angles is more rapid than that given by Molière's approximate formula.

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