Its average over the unitary group may be written as

$$\int\{\gamma_\lambda\Delta_\lambda\nabla_\lambda+\kappa\nabla\}dR\psi\Big/\int dR=0, \quad (2)$$

where dR is the element of volume of the four dimensional unitary group. If ψ is analytic we can write[1]

$$\nabla\psi=\prod_{\lambda=1}^{4}(\cos\omega r_{\lambda\rho}u_\rho)\psi, \quad (3)$$

where ω is the difference step, $r_{\lambda\rho}$ are the elements of R, and u_ρ are the differential operators $\partial/\partial x_\rho$. The first term of Eq. (2) can be written as[1]

$$\gamma_\lambda\Delta_\lambda\nabla_\lambda\psi=-\frac{1}{\omega^2}\sum_{\sigma=1}^{4}\gamma_\sigma\frac{\partial\nabla}{\partial u_\sigma}\psi. \quad (4)$$

Expanding $\cos\omega r_{\lambda\rho}u_\rho$ in a power series and integrating we get a result of the form

$$\int\prod_{\lambda=1}^{4}\cos\omega r_{\lambda\rho}u_\rho dR$$

$$=\sum_{k=0}^{\infty}\Big[\sum_{i_1+i_2+i_3+i_4=k}c_{i_1i_2i_3i_4}u_1^{i_1}u_2^{i_2}u_3^{i_3}u_4^{i_4}\Big]. \quad (5)$$

Inasmuch as the element of group volume is invariant under, say, right translations we have

$$\int f(Ru)dR=\int f(RTu)d(RT)=\int f(Ru')dR, \quad (6)$$

where T is unitary and $u'=Tu$. Hence

$$\sum_{k=0}^{\infty}\Big[\sum_{i_1+i_2+i_3+i_4=k}c_{i_1i_2i_3i_4}u_1^{i_1}u_2^{i_2}u_3^{i_3}u_4^{i_4}\Big]$$

$$=\sum_{k=0}^{\infty}\Big[\sum_{i_1+i_2+i_3+i_4=k}c_{i_1i_2i_3i_4}(u_1')^{i_1}(u_2')^{i_2}(u_3')^{i_3}(u_4')^{i_4}\Big]. \quad (7)$$

Equation (7) is an identity in u and implies that the homogeneous polynomials

$$\sum_{i_1+i_2+i_3+i_4=k}c_{i_1i_2i_3i_4}u_1^{i_1}u_2^{i_2}u_3^{i_3}u_4^{i_4} \quad (8)$$

are invariant under a unitary transformation. But the only invariant of a unitary transformation is the unitary scalar product (and functions of it). Hence the polynomials (8) cannot be invariant. It now follows from the validity of Eq. (7) that

$$c_{i_1i_2i_3i_4}=0 \quad (9)$$

except for $i_1=i_2=i_3=i_4=0$. Thus

$$\int\nabla dR\psi=\text{const }\psi \quad (10)$$

and consequently from Eq. (4) it follows that

$$\int\gamma_\lambda\Delta_\lambda\nabla_\lambda dR\psi=0. \quad (11)$$

Combining Eqs. (2), (10), and (11) we get finally

$$\kappa\ (\text{constant }\psi)=0 \quad (12)$$

or

$$\psi\equiv 0. \quad (13)$$

We may now conclude that the rotation group is the only compact group we can use to obtain relativistic invariance.

[1] B. T. Darling, Phys. Rev. 80, 460 (1950).
[2] F. D. Murnaghan, *The Theory of Group Representations* (Johns Hopkins Press, Baltimore, Maryland, 1938), p. 57.

Actual Path Length of Electrons in Foils

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THE angular and spatial distribution due to multiple scattering of a beam of charged particles after passing through a foil has been extensively studied by many authors. We wish to discuss in this note a related problem: the distribution with respect to the actual path length of the particles.

Let $t+\Delta$ be the actual path length of a particle after passing through a foil of thickness t.[1] Under the small angle approximation

$$\Delta=\Delta_y+\Delta_z,\quad \Delta y=\tfrac{1}{2}\Sigma_i t_i\theta_{yi}^2,\quad \Delta z=\tfrac{1}{2}\Sigma_i t_i\theta_{zi}^2, \quad (1)$$

where t_i, $t_i\tan\theta_{yi}$, and $t_i\tan_{zi}$ are the projections along the t, y, and z axes of the displacement of the particle between the $(i-1)$- and i-th scatterings. The approximation made in (1) in neglecting terms of order θ^4 amounts to a correction on Δ of less than 3 percent for θ as large as $\sim 20°$.

We want to calculate the probability $P(t;\,y,\,\theta_y,\,\Delta_y;\,z,\,\theta_z,\,\Delta_z)\times dyd\theta_yd\Delta_ydzd\theta zd\Delta_z$ that a particle at the thickness t has lateral displacements, direction of motion, and increment of path length in the specified ranges. Under the usual gaussian approximation (as in reference 1) we can write down the following diffusion equation for P:

$$\frac{\partial P}{\partial t}=-\theta_y\frac{\partial P}{\partial y}-\theta_z\frac{\partial P}{\partial z}+\frac{1}{w^2}\frac{\partial^2 P}{\partial\theta_y^2}+\frac{1}{w^2}\frac{\partial^2 P}{\partial\theta_z^2}-\frac{1}{2}\theta_y^2\frac{\partial P}{\partial\Delta_y}-\frac{1}{2}\theta_z^2\frac{\partial P}{\partial\Delta_z}. \quad (2)$$

This is separable into a product of distribution in $(t-y)$ and $(t-z)$ planes:

$$P=F(t;\,y,\,\theta_y,\,\Delta_y)F(t;\,z,\,\theta_z,\,\Delta_z), \quad (3)$$

where F satisfies

$$\frac{\partial F}{\partial t}=-\theta\frac{\partial F}{\partial y}+\frac{1}{w^2}\frac{\partial F}{\partial\theta}-\frac{1}{2}\theta^2\frac{\partial F}{\partial\Delta}. \quad (4)$$

The initial condition is

$$F(0;\,y,\,\theta,\,\Delta)=\delta(y)\delta(\theta)\delta(\Delta). \quad (5)$$

Equation (4) can be solved by first taking the fourier transform with respect to Δ and y:

$$F=\int\int_{-\infty}^{\infty}dpdq\psi(t;\,p,\,q,\,\theta)\exp(ipy+iq\Delta), \quad (6)$$

$$\partial\psi/\partial t=[(1/w^2)(\partial^2/\partial\theta^2)-\tfrac{1}{2}iq\theta^2-ip\theta]\psi. \quad (7)$$

This is easily reducible to a form identical with the Schrödinger equation for a harmonic oscillator. The only difference is that the "frequency" is here complex. Expanding ψ into normalized eigenfunctions ϕ of the harmonic oscillator one could solve for the t-dependence of the coefficients. One then uses the initial condition (5) to determine the constants of integration. The solution obtained this way is

$$\psi=\frac{1}{4\pi^2}\sum_{n=0}^{\infty}\phi_n\Big(\frac{wp}{q}\Big)\phi_n\Big(w\theta+\frac{wp}{q}\Big)\exp\Big\{t\Big(\frac{ip^2}{2q}-[2n+1]\omega\Big)\Big\}, \quad (8)$$

where

$$\phi_n(x)=(\pi^{\frac{1}{2}}2^n n!)^{-\frac{1}{2}}\omega^{\frac{1}{4}}H_n(\omega^{\frac{1}{2}}x)\exp(-\tfrac{1}{2}\omega x^2), \quad (9)$$

$$\omega^2=iq/2w^2, \quad (10)$$

and $H_n(x)$ are the hermite polynomials.[2] The real parts of ω, $\omega^{\frac{1}{2}}$, $\omega^{\frac{1}{4}}$ are chosen positive. The distribution function P can be computed from (8), (6), and (3).

The general solution given above can be used to compute the probability $A(\Delta)d\Delta$ (for any given geometry) that the charge particle has in a foil of thickness t a path length between $t+\Delta$ and $t+\Delta+d\Delta$. The final result for two common cases is given below:

Case I. All particles are detected irrespective of their position and angle of emergence.

$$\text{probability}=A(\Delta)d\Delta=B_I(v)dv, \quad (11)$$

$$v=2w^2\Delta/t^2, \quad (12)$$

$$B_I(v)=2\pi^{-\frac{1}{2}}v^{-\frac{3}{2}}(u-3u^9+5u^{25}-7u^{49}+\cdots), \quad (13)$$
$$u=\exp(-1/v).$$

The following asymptotic approximations of B_I are good to within 1 percent in the ranges indicated:

$$B_I(v)=2\pi^{-\frac{1}{2}}v^{-\frac{3}{2}}(e^{-1/v}-3e^{-9/v})\quad\text{for}\quad v\leqq 2.0,$$

$$B_I(v)=\tfrac{1}{4}\pi\exp(-\pi^2v/16)\quad\text{for}\quad v\geqq 2.0.$$

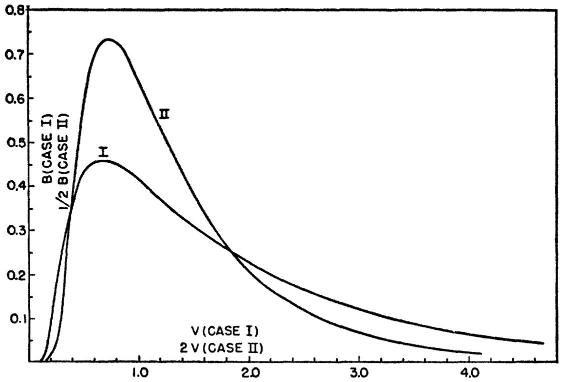

FIG. 1. Distribution function with respect to path length. v = (actual path length − thickness of foil) $\times 2w^2/t^2$ Bdv = probability.

Case II. Only those particles with $\theta_y=\theta_z=0$ at emergence are detected. The position of emergence is arbitrary. In this case

$$B_{II}(v)=4\pi^{-\frac{1}{2}}v^{-5/2}\left[(u+9u^9+25u^{25}+\cdots)-\frac{v}{2}(u+u^9+u^{25}+\cdots)\right].$$

The following asymptotic approximations of B_{II} are good to within 1 percent in the ranges indicated:

$$B_{II}(v)=4\pi^{-\frac{1}{2}}v^{-5/2}\left(1-\frac{v}{2}\right)e^{-1/v}\quad\text{for}\quad v\leqq 1.0,$$

$$B_{II}(v)=\tfrac{1}{2}\pi^2\exp(-\pi^2v/4)\qquad\text{for}\quad v\geqq 1.0.$$

The distribution function B_I and B_{II} are plotted in Fig. 1. The average value $\langle\Delta\rangle_{\text{Av}}$ of Δ for any case can be easily computed by the following method without first solving for the complete distribution with respect to Δ:

Equation (1) can be written

$$\Delta=\tfrac{1}{2}\int_0^t(\theta_y^2+\theta_z^2)dt'.$$

Hence

$$\langle\Delta\rangle_{\text{Av}}=\tfrac{1}{2}\int_0^t(\langle\theta_y^2+\theta_z^2\rangle_{\text{Av}})dt'. \tag{14}$$

For case I the probability for a particle to have the angle θ_y at thickness t' is given by $G(t',\theta_y)d\theta_y$ where G is defined in reference 1, Eq. (1.63). From this we find

$$\langle\theta_y^2\rangle_{\text{Av}}=\langle\theta_z^2\rangle_{\text{Av}}=2t'/w^2.$$

Hence by (14)

$$\langle\Delta\rangle_{\text{Av}}=t^2/w^2.$$

For case II suppose N is the number of incoming particles. The number of particles at thickness t' having angles $\cong\theta_y$ and θ_z is

$$NG(t',\theta_y')G(t',\theta_z')d\theta_y'd\theta_z'. \tag{15}$$

The probability for these particles to come out with angles of energence $d\theta_y\geqq\theta_y\geqq 0$, $d\theta_z\geqq\theta_z\geqq 0$ is

$$G(t-t',-\theta_y')d\theta_yG(t-t',-\theta_z')d\theta_z. \tag{16}$$

The product of (15) and (16) therefore gives the number of particles with the specified angle of emergence that had the angles $\cong\theta_y'$ and θ_z' at t'. Hence the probability for a detected particle to have the angles θ_y' and θ_z' at t' is

$$[G(t,0)]^{-2}G(t',\theta_y')G(t',\theta_z')G(t-t',-\theta_y')G(t-t',-\theta_z')d\theta_y'd\theta_z'.$$

From this we can easily calculate the average $\langle\theta_y^2\rangle_{\text{Av}}$ at t'. Substitution of the result into (14) gives finally

$$\langle\Delta\rangle_{\text{Av}}=t^2/3w^2.$$

It is evident that this method of calculation of $\langle\Delta\rangle_{\text{Av}}$ can be applied whenever we know the angular and spatial distribution functions. One could, for example, dispense with the gaussian approximation and use the exact numerical solution of Snyder and Scott[3] to compute $\langle\theta_y^2\rangle_{\text{Av}}$ and $\langle\Delta\rangle_{\text{Av}}$.

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1 B. Rossi and K. Greisen, Revs. Modern Phys. 13, 240 (1941). We use these authors' notation in their Sec. 23.
2 See, e.g., L. I. Schiff, *Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1949), Sec. 13.
3 H. Snyder and W. T. Scott, Phys. Rev. 76, 220 (1949).

Alpha-Alpha Correlations in the Photodisintegration of C^{12} and the Resonant Absorption of Electromagnetic Radiation of Non-E.D. Character

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THE photodisintegration of C^{12} leading, in the final state, to three alpha-particles, has been the subject of a number of recent investigations.[1–5] These established the following facts:

(a) The cross section for the reaction goes through a pronounced resonance, with a peak value of the order of 10^{-28} cm^2 at approximately $E_\gamma=18$ Mev and a half-width of roughly 3.5 Mev, and increases again after a minimum around $E_\gamma=21$ Mev.

(b) In the region of the first resonance, the reaction proceeds predominantly via the well-known 3-Mev level of Be^8, while some other mechanism(s) prevail in the region of re-increase.

(c) A small fraction of the disintegrations, increasing with E_γ, involves the ground state of Be^8.

A better understanding of these facts as well as some information on the spins of Be^{8*} and Be^8 can be gained by analyzing more deeply some anomalies encountered in the investigation of this reaction with the resonant gamma-radiation from Li^7+p.

Diagram A in Fig. 1 shows a histogram of the energy distribution of the alpha-particles from 483 stars produced by the 17.6-Mev line in nuclear emulsions. To permit an analysis after Bethe[6] the distributions to be expected for the first alpha-particle (W_1) and the alpha-particles from the break-up of $Be^{8*}(W_{23})$ in the 3-Mev state have been drawn in. For the calculation of W_{23} a level width Γ of 1.1 Mev and absence of correlation between the direction of flight of Be^{8*} and the velocities, in the C. G. system of Be^{8*}, of the alpha-particles resulting from the break-up have been assumed. The gamma-ray momentum has been neglected throughout.

While the general agreement of the theoretical distribution $W=W_1+W_{23}$ warrants statement (b), the deviations for $E\lesssim 3.5$ Mev seem to be outside statistical fluctuations. A somewhat more satisfactory agreement can be obtained by assuming[2] that about 16 percent of the disintegrations proceed via an 8-Mev level of $\Gamma=0.75$ Mev in Be^8. This mechanism would explain the appearance of an extra maximum around $E\simeq 1.5$ Mev, but fails to produce the very noticeable minimum between 2.0 and 4.0 Mev. It is also discredited by the observation that at higher photon energies this level does not participate more strongly if at all.[5] Further, Nabholz *et al.*[7] have emphasized that similar discrepancies occur for $E_\gamma=14.6$ Mev, which is insufficient to excite the 8-Mev level.

We wish to show that the appearance of this minimum can be explained quite naturally on the assumption that the 3.0-Mev level in Be^8 has $J=2$. It might be noted that this is in contradiction with Wheeler's[8] conclusions. On the basis of the assignment $J=2$, the observed distribution may arise as a result of angular correlation effects. Electric quadrupole (E.Q.) absorption would not give correlation, because the alpha-particle emitted from the 2^+ state of carbon would be expected to be mainly an s wave. Magnetic dipole (M.D.) absorption, on the other hand, would lead to a 1^+ state in carbon (j_1), decaying by emission of an alpha-particle of two units of angular momentum (l_1) to the 3-Mev state of Be^8 ($j=2^+$) which breaks up with a d wave (l_2) with respect to its