the spatial distribution of the nucleon moments, we should have

$$\Gamma_{\text{nucl}}=0.26\cdot 10^{-4}, \quad (7)$$

which, incorporated in Γ, gives

$$1/\alpha=137.0346. \quad (8)$$

The effects under consideration produce the following contribution to the displacement of an nS level,

$$\frac{8Z^4}{n^3}\frac{\alpha^3}{3\pi}\left[3\pi Z\alpha\left(1+\frac{11}{128}-\frac{1}{2}\log 2+\frac{5}{192}\right)\right]\text{Ry}. \quad (9)$$

For the $n=2$ level of hydrogen this amounts to 7.08 Mc/sec.[4] If the previous computation[5] of the $2S-2P_{\frac{1}{2}}$ displacement is corrected slightly for the new value of α, and 0.94 Mc/sec subtracted to account for the α^2 contribution to the magnetic moment, the addition of (9) yields

$$1057.75 \text{ Mc/sec} \quad (10)$$

for the improved theoretical value of the "Lamb shift" in hydrogen. This may be compared with the published experimental value[6] of 1062±5 Mc/sec. Still not included in the theoretical value are α^2 effects other than in the magnetic moment, and the possible influence of nucleon structure and mass.[7]

We are indebted to N. M. Kroll for numerous enlightening discussions.

[1] R. Karplus and N. M. Kroll, Phys. Rev. 77, 536 (1950).
[2] J. W. M. Dumond and E. R. Cohen, National Research Council Report, December, 1950, Phys. Rev. 82, 555 (1951).
[3] F. E. Low and E. E. Salpeter, Phys. Rev. 83, 478 (1951).
[4] Essentially the same result has also been obtained by M. Baranger, Ph.D. thesis, Cornell (1951). We are grateful to H. A. Bethe for informing us of this calculation, and to Dr. Baranger for supplying us with a copy of his thesis.
[5] Bethe, Brown, and Stehn, Phys. Rev. 77, 370 (1950).
[6] R. C. Retherford and W. E. Lamb, Phys. Rev. 75, 1325 (1949).
[7] Apart from the simple mass correction employed in reference 5. Note that if the reduced mass is not substituted for the electron mass in the logarithm, the "Lamb shift" (10) becomes 1057.82 Mc/sec.

Influence of Initial Velocities on Electron Transit Times in Diodes

J. T. Wallmark
Royal Institute of Technology, Stockholm, Sweden
(Received August 27, 1951)

In a recent letter[1] and a more detailed article[2] Barut has given a method to calculate the transit time of electrons with initial velocities in diodes with partial space charge, assuming a homogeneous initial velocity distribution. In practical cases calculations for a nonhomogeneous velocity distribution are of more interest. The author has undertaken such calculations by means of a first-order perturbation method.

Assume a parallel plane diode with a homogeneous flow of electrons. The electric field, potential, etc., may be calculated from published data on partial space charge.[2–5] Consider, then, a small number of electrons with a velocity that differs from the main velocity by the amount Δ. If the number is small enough its influence on the space charge conditions may be neglected, corresponding to a first-order approximation.

The transit time for these electrons is

$$\tau=\int_0^d dx/v.$$

By substituting the values

$$v=[(2e/m)(V\pm\Delta)]^{\frac{1}{2}}$$

$$dx=\frac{dV}{[(16/9)E^2\eta(V/V_a)^{\frac{1}{2}}+E_0^2]^{\frac{1}{2}}},$$

where E is the electric field strength without space charge, E_0 is the electric field strength at the cathode, $\eta=I/I_s$ is the relative current compared to space charge saturated current, and V_a is the anode voltage we obtain the elliptic integral

$$\tau=\left(\frac{m}{2e}\right)^{\frac{1}{2}}\int_0^{V_a}\frac{dV}{[\{(16/9)E^2\eta(V/V_a)^{\frac{1}{2}}+E_0^2\}(V\pm D)]^{\frac{1}{2}}}.$$

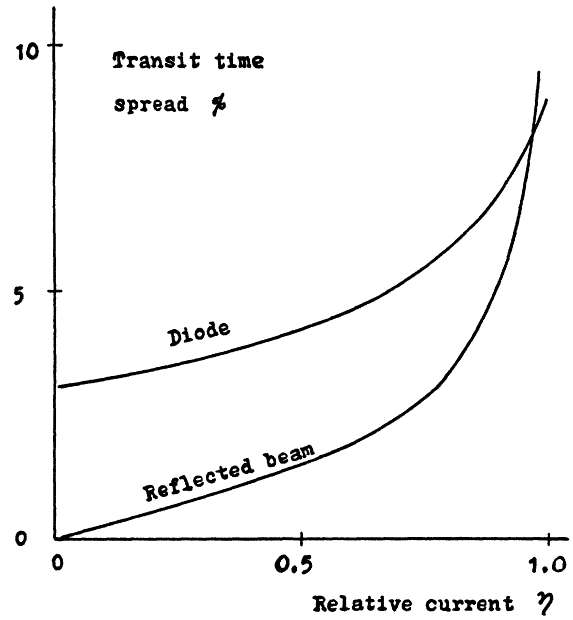

FIG. 1.

The integral may be solved by standard methods. The results will be shown applied to two practical cases, first on ordinary diode with an anode voltage of 100 volts and an anode–cathode distance of 5 mm. Figure 1 shows the spread in transit time as a function of current density in the diode for a difference in initial velocity of 0.1 volt (roughly corresponding to conditions for an oxide cathode at 1100°K).

Figure 1 also shows the spread in transit time for a reflected beam under the same circumstances. In that case the difference in transit time is much reduced as a result of the influence of different path lengths.

A complete report is in preparation.

[1] A. O. Barut, Phys. Rev. 81, 274 (1951); 82, 554 (1951).
[2] A. O. Barut, Z. angew. Math. Phys. II: 1, 35 (1951).
[3] J. T. Wallmark, Proc. Inst. Radio Engrs., to be published.
[4] H. F. Ivey, Phys. Rev. 76, 554 (1949).
[5] R. Cockburn, Proc. Phys. Soc. (London) 47, 810 (1935).

Group Uniqueness in the Irreducible Volume Character of Events

B. T. Darling and M. Leichter
Department of Physics and Astronomy, Ohio State University, Columbus, Ohio
(Received May 16, 1951)

The derivation of the mass quantization condition on the basis of the irreducible volume character of events[1] involved, for purposes of relativistic invariance, an averaging of the difference equation (corresponding to Dirac's equation) over the four-dimensional rotation group. This led of necessity to complex values for the space-time variables. In view of this complex character of space one should consider averaging over more general bounded subgroups of the linear group than just the rotation group. (The Lorentz group cannot be used because it is not compact.) Every bounded subgroup of the full linear group is, however, equivalent to a unitary group.[2] It will now be shown that the integration (averaging) of the difference equation over the complete four-dimensional unitary group annihilates the dynamical equation.

The difference equation corresponding to Dirac's equation is

$$\{\gamma_\lambda\Delta_\lambda\nabla_\lambda+\kappa\nabla\}\psi=0. \quad (1)$$

(For the definition of the operators Δ_λ, ∇_λ, and ∇ see reference 1).