FIG. 1. Decay schemes for Sn^{118} and Sn^{117} .

found to be 393.3 keV. Two weaker electron lines with energies of 227.2 and 373.0 keV are present and appear by their half-life to be associated with the same activity and hence represent gamma-transitions in indium with energies of 255.2 and 400.9 keV. No evidence could be found either spectrometrically or by absorption for a gamma-ray of energy around 90 keV, as reported, nor could any positrons be observed coming from the specimen. The half-life of the enriched specimen as observed for 8 months appears to be 118 ± 2 days, a value higher than those previously reported, which ranged from 70 to 112 days. A possible decay scheme is presented in Fig. 1(A).

Tin-117. An excited state of tin-117, whose half-life has been variously reported as from 13.0 to 14.5 days, was produced by neutron capture in tin-116. This specimen yielded 4 electron conversion lines at energies of 126.7, 129.3, 151.5, and 155.0 keV in addition to lines due to indium-113 and one other of Auger origin. This indicates the existence of two gamma-rays of energy 155.9 and 159.4 keV. The former is highly converted with a K/L ratio of about 7. Only the weak K line is observed for the latter. No evidence is found for the existence of a 174-keV gamma-ray as reported.² The half-life is noted to be 14.0 ± 0.5 days. A nuclear level scheme is proposed in Fig. 1(B).

Tin-123. Neutron capture in enriched tin-122 yields a beta-active emitter whose half-life has been reported³ as from 130 to 136 days. The maximum beta-energy has been given as 1.42 Mev with a converted gamma-ray of 394 keV. It now appears highly probable that this gamma-ray when observed in any isotopic enrichment is due to traces of tin-113, and is the highly converted indium gamma-radiation. No gamma-radiation appears to be associated with tin-123. The half-life observed in this investigation, following the decay of the enriched specimen for 18 months and allowing for the presence of some 2.5-year antimony-125 radiation, appears to be slightly less than that previously reported, namely 125 ± 3 days. No further information of a conflicting nature applying to the other known radioactivities in tin has so far been found.

Tellurium. Using isotopes of tellurium enriched in masses 120 (22.3 percent), 122 (86.2 percent), 126 (95.4 percent), and 128 (90 percent), some information has been obtained of a revisory nature to that previously reported.⁴ It is believed that the energies of the internally converted gamma-rays as now determined photographically are accurate to ± 0.2 percent. The radioactive decay of the various specimens has been followed for over 300 days leading to half-lives somewhat different than are in the literature.

All of the gamma-rays found in this investigation to be associated with the enriched isotopes represent isomeric transitions in Te with the probable exception that a 575-keV gamma occurs in Sb-121 following K capture from Te-121. The evidence for this

TABLE I. Summary of half-lives and gamma-energies in tellurium.

Mass	121	123	125	127	129
Half-life	140-day	121-day	58-day ^a	115-day	33.5-day
Gamma energy (kev)	81.8	88.7	109.6	88.7	106.0
	213.6	159.2	35.3		
	575(Z=51)				

^a See reference 5.

575-keV gamma is a single electron line at 544.7 keV in the enriched Te-121 specimen. No evidence could be found for a gamma-ray of energy 615 keV following K capture in Te-121 as reported, indicating the above interpretation of the 544.7-keV electron line. The revised values of the gamma-energies and the half-lives as here determined are shown collectively in Table I.⁵

* This investigation was aided by the joint support of the AEC and ONR.

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Electrodynamic Displacement of Atomic Energy Levels

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THE modifications in the fine structure and hyperfine structure of hydrogenic atoms, produced by electrodynamic effects, have thus far been computed essentially only to the order $\alpha = 1/137$. In view of the high accuracy of recent measurements we have extended the theory to include effects of order $Z\alpha^2$. In contrast with corrections of order α^2 , these terms arise from a more precise treatment of the nuclear coulomb field in the relativistic domain, rather than from higher order electrodynamic phenomena.

For the hyperfine structure of S levels, we find the following fractional correction of order $Z\alpha^2$,

$$-(5/2 - \log 2)Z\alpha^2. \quad (1)$$

In conjunction with the known α^2 contribution to the electron moment,¹ we have, as the total electrodynamic correction factor

$$1 + \alpha/2\pi - (2.97/\pi^2)\alpha^2 - (5/2 - \log 2)Z\alpha^2 \\ = 1.001145 - 0.963 \cdot 10^{-4}Z \\ = 1.001049, Z=1. \quad (2)$$

In a recent survey of atomic constants, Dumond and Cohen² have provided tables of constants based upon an adjustable correction factor, which was arbitrarily written

$$1.001147/(1+\Gamma). \quad (3)$$

Thus the reciprocal of the fine structure constant is stated as

$$1/\alpha = (137.0429 \pm 0.0009)(1 - 0.486\Gamma). \quad (4)$$

The electrodynamic correction embodied in (2) would, in itself, imply

$$\Gamma = 0.98 \cdot 10^{-4}, \quad (5)$$

whence

$$1/\alpha = 137.0364. \quad (6)$$

However, this ignores the contribution to Γ associated with the spatially distributed nature of the proton magnetic moment.

An indication of the magnitude of this effect can be obtained from the ratio of the hyperfine structures in hydrogen and deuterium. If the residual discrepancy in this quantity³ is ascribed to

the spatial distribution of the nucleon moments, we should have

$$\Gamma_{\text{nuc}} = 0.26 \cdot 10^{-4}, \quad (7)$$

which, incorporated in Γ , gives

$$1/\alpha = 137.0346. \quad (8)$$

The effects under consideration produce the following contribution to the displacement of an nS level,

$$\frac{8Z^4 \alpha^3}{n^3} \frac{1}{3\pi} \left[3\pi Z\alpha \left(1 + \frac{11}{128} - \frac{1}{2} \log 2 + \frac{5}{192} \right) \right] \text{Ry}. \quad (9)$$

For the $n=2$ level of hydrogen this amounts to 7.08 Mc/sec.⁴ If the previous computation⁵ of the $2S-2P_{1/2}$ displacement is corrected slightly for the new value of α , and 0.94 Mc/sec subtracted to account for the α^2 contribution to the magnetic moment, the addition of (9) yields

$$1057.75 \text{ Mc/sec} \quad (10)$$

for the improved theoretical value of the "Lamb shift" in hydrogen. This may be compared with the published experimental value⁶ of 1062 ± 5 Mc/sec. Still not included in the theoretical value are α^2 effects other than in the magnetic moment, and the possible influence of nucleon structure and mass.⁷

We are indebted to N. M. Kroll for numerous enlightening discussions.

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⁷ Apart from the simple mass correction employed in reference 5. Note that if the reduced mass is not substituted for the electron mass in the logarithm, the "Lamb shift" (10) becomes 1057.82 Mc/sec.

Influence of Initial Velocities on Electron Transit Times in Diodes

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IN a recent letter¹ and a more detailed article² Barut has given a method to calculate the transit time of electrons with initial velocities in diodes with partial space charge, assuming a homogeneous initial velocity distribution. In practical cases calculations for a nonhomogeneous velocity distribution are of more interest. The author has undertaken such calculations by means of a first-order perturbation method.

Assume a parallel plane diode with a homogeneous flow of electrons. The electric field, potential, etc., may be calculated from published data on partial space charge.²⁻⁵ Consider, then, a small number of electrons with a velocity that differs from the main velocity by the amount Δ . If the number is small enough its influence on the space charge conditions may be neglected, corresponding to a first-order approximation.

The transit time for these electrons is

$$\tau = \int_0^d dx/v.$$

By substituting the values

$$v = [(2e/m)(V \pm \Delta)]^{1/2}$$

$$dx = \frac{dV}{[(16/9)E^2\eta(V/V_a)^{1/2} + E_0^2]^{1/2}}$$

where E is the electric field strength without space charge, E_0 is the electric field strength at the cathode, $\eta = I/I_s$ is the relative current compared to space charge saturated current, and V_a is the anode voltage we obtain the elliptic integral

$$\tau = \left(\frac{m}{2e}\right)^{1/2} \int_0^{V_a} \frac{dV}{[(16/9)E^2\eta(V/V_a)^{1/2} + E_0^2]^{1/2}(V \pm \Delta)^{1/2}}$$

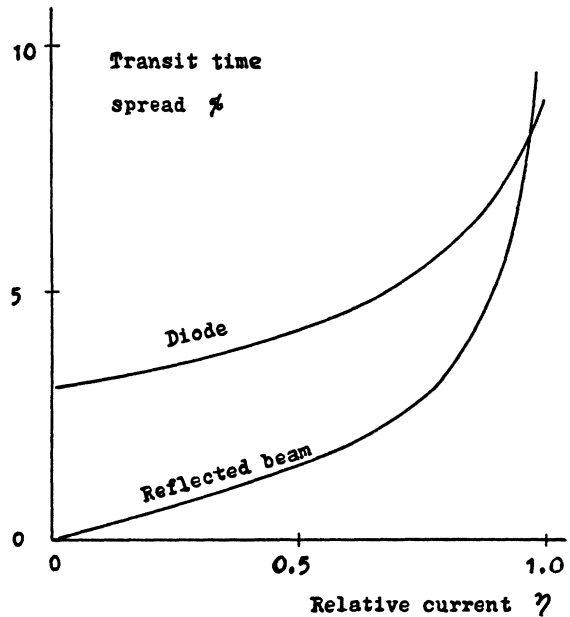


FIG. 1.

The integral may be solved by standard methods. The results will be shown applied to two practical cases, first on ordinary diode with an anode voltage of 100 volts and an anode-cathode distance of 5 mm. Figure 1 shows the spread in transit time as a function of current density in the diode for a difference in initial velocity of 0.1 volt (roughly corresponding to conditions for an oxide cathode at 1100°K).

Figure 1 also shows the spread in transit time for a reflected beam under the same circumstances. In that case the difference in transit time is much reduced as a result of the influence of different path lengths.

A complete report is in preparation.

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Group Uniqueness in the Irreducible Volume Character of Events

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THE derivation of the mass quantization condition on the basis of the irreducible volume character of events¹ involved, for purposes of relativistic invariance, an averaging of the difference equation (corresponding to Dirac's equation) over the four-dimensional rotation group. This led of necessity to complex values for the space-time variables. In view of this complex character of space one should consider averaging over more general bounded subgroups of the linear group than just the rotation group. (The Lorentz group cannot be used because it is not compact.) Every bounded subgroup of the full linear group is, however, equivalent to a unitary group.² It will now be shown that the integration (averaging) of the difference equation over the complete four-dimensional unitary group annihilates the dynamical equation.

The difference equation corresponding to Dirac's equation is

$$\{\gamma_\lambda \Delta_\lambda \nabla_\lambda + \kappa \nabla\} \psi = 0. \quad (1)$$

(For the definition of the operators Δ_λ , ∇_λ , and ∇ see reference 1).