separating ΔE_F into two terms, $\Delta E_F'$ and $\Delta E_F''$ in a manner originated by Feynman.[5] $\Delta E_F'$ contains explicit functions of the momenta and involves the potential at most linearly. Apart from renormalizations, which we ignore, it yields an order α correction to the hyperfine structure arising only from the anomalous electron moment. There are no order α^2 corrections. $\Delta E_F''$ contains the potential quadratically only and also contains S_F^e. Its evaluation to order α^2 is enormously simplified by neglecting the space parts of the momenta p_1 and p_2 associated with the state ψ_a everywhere except in $\psi_a(P_1)$ and $\bar{\psi}_a(P_2)$, and by replacing S_F^e by its zero field approximation.[6] The corrections to the hyperfine structure appear as cross terms between the coulomb and dipole potentials. The validity of the approximation used depends upon the reduced weight of low momentum transfers associated with the dipole field and with the Feynman separation.

The evaluation of $\Delta E_F''$ yields an energy $-[(13/4)-\ln 2]\alpha^2 z'\delta_{l0}$, again in units of the Fermi energy. The total correction from ΔE_P and ΔE_F is thus $-[(5/2)-\ln 2]\alpha^2 z \approx -1.81\alpha^2 z$,[7] which is more than twelve times as large as the Bethe-Longmire estimate. Inclusion of this correction reduces the value of α^{-1} obtained from the hyperfine structure from 137.043 to 137.036. Recent measurements of the $2P_{\frac{1}{2}}-2P_{\frac{3}{2}}$ separation in deuterium indicate that a reduction in α^{-1} is indeed required, and these measurements are not inconsistent with the corrected value given here.[8]

Inclusion of meson field effects may be expected to decrease further α^{-1}. In fact, an accurate determination of α^{-1} by a method independent of mesonic effects would be a useful means of experimentally determining their magnitude.

* Work supported in part by the Signal Corps and ONR.

[1] Kusch and Prodell, Phys. Rev. **79**, 1009 (1950).

[2] Dumond and Cohen, least-squares adjustment of the atomic constants as of December 1950, (report to the national research council), p. 30.

[3] Estimates of a part of these omitted effects have been given by Bethe and Longmire (Phys. Rev. **75**, 306 (1949)), where they are described as arising from a spatial extension of the anomalous moments.

[4] See Karplus and Kroll, Phys. Rev. **77**, (1950) for explicit definitions of D_F, S_F and $A_\mu^P(x)$. There A_μ^P is denoted by $(\alpha/2\pi)\bar{A}_\mu^{e2}$ (Eq. 14).

[5] Private communication. The separation is described in detail in the dissertation of M. Baranger, *Relativistic Corrections to the Lamb Shift* (Cornell University, New York, 1951).

[6] The authors are indebted to H. A. Bethe for pointing out to them the possibility of these approximations, and to M. Baranger for a copy of his thesis, in which their application to another problem is illustrated.

[7] A similar result, using a different method, has been obtained by R. Karplus and A. Klein. We wish to thank these authors for discussing their results with us.

[8] W. E. Lamb (private communication).

Continuous γ-Spectrum Accompanying Electron Capture

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It is well known that the emission of electrons from a radioactive nucleus is accompanied by a low intensity continuous spectrum of γ-radiation (bremsstrahlung). An analogous effect occurs when a nucleus captures an electron of the atomic shell. The theory of this radiative K-capture has been given by Morrison and Schiff.[1] An experimental evidence of this γ-radiation has been found in the study of the disintegration of Fe^{55}.[2] With the new technique using scintillations from NaI crystals for detecting γ-rays and measuring their energies, we were able to study the shape of the spectrum of the low intensity γ-radiation emitted by a Fe^{55} sample.

Figure 1 shows the pulse distribution of the scintillations excited by the Fe^{55} γ-radiation. To determine the upper limit of the γ-spectrum of Fe^{55} we proceed in the following way: From measurements of the pulse distribution for several monochromatic γ-rays of different energies, we are able to construct the pulse distribution which should result from the shape of the continuous γ-ray spectrum theoretically predicted by Morrison and Schiff, assuming different upper limits of the spectrum. As an example of the calibration, the pulse distribution for the nonconverted part of the isomeric 88-kev transition in Ag^{109} is shown in Fig. 2. (The $Ag-K\alpha$-line has been partly absorbed.) A good agreement between the experimental pulse distribution and the calculated one is obtained for an upper limit of the γ-spectrum at $E_{\max}(h\nu)=0.205$ Mev. The qualitative agreement of the curves supports the correctness of the assumed spectrum.

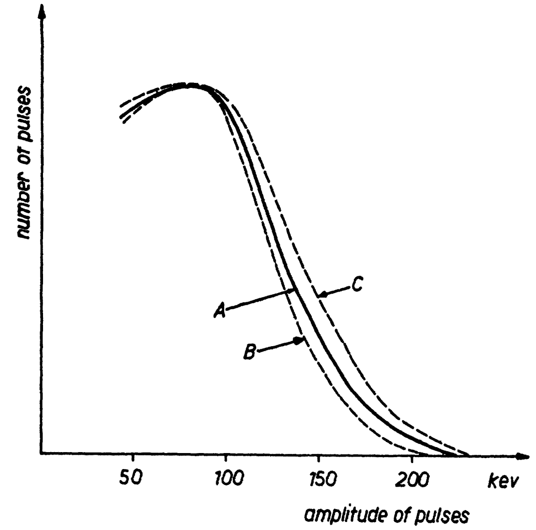

Fig. 1. Pulse distribution of the γ-radiation from Fe^{55}: A = observed curve; B = constructed curve with $E_{\max}(h\nu)=0.200$ Mev; C = constructed curve with $E_{\max}(h\nu)=0.210$ Mev.

In the electron capture process, the whole energy available most frequently goes off with the neutrino, but may be shared between the neutrino and a γ-quantum. The upper limit of the γ-spectrum corresponds to the case when the whole disintegration energy is taken away by the γ-quantum. From the upper limit the mass difference of the parent and the daughter atoms can therefore be obtained directly; in our case it is:

$$M(Fe^{55})-M(Mn^{55})=E_{\max}(h\nu)+E_K=(0.212\pm 0.010)\text{ Mev}.$$

This value is in good agreement with the recent determination from the threshold of the $Mn^{55}(p,n)Fe^{55}$ reaction, for which a

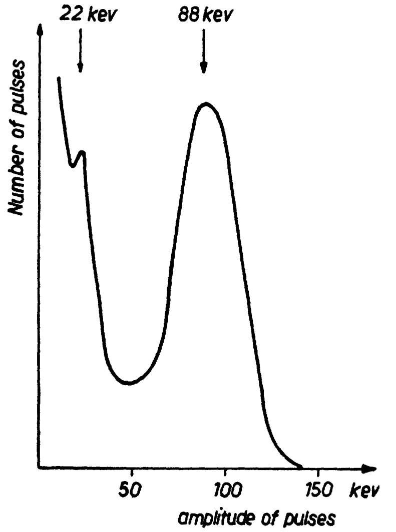

Fig. 2. Pulse distribution of the 88-kev γ-line from Ag^{109}.

Q-value of $-(1.006\pm0.010)$ Mev has been given[3] corresponding to a mass difference of (0.22 ± 0.01) Mev.

The new method of determining the energy-release in electron capture processes may be applied even in some cases where the disintegration goes over an excited level of the daughter nucleus with the emission of a nuclear γ-ray. A more detailed account will be published in *Helvetica Physica Acta*. We thank Professor P. Scherrer for his interest in this work.

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Angular Correlation of the Continuous Radiation Accompanying Beta-Decay*

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THE theory of the continuous gamma-radiation accompanying beta-decay was developed by Knipp and Uhlenbeck[1] and by Bloch.[2] An extension to forbidden transitions was made by Chang and Falkoff,[3] especially for the scalar interaction. This radiation has been found by several investigators,[4] and recently it has been possible to measure the shape of the gamma-ray continuum at low energies (50 kev–300 kev).[5] Theoretical predictions show that for low energies the shapes of the gamma-continua are almost identical for allowed and first-forbidden transitions for any interaction. This point has been verified experimentally (to be published).

We thought it might be of interest to calculate the angular correlation between the beta-particle and its associated gamma-quantum and to see if one could distinguish between allowed and forbidden transitions. We calculated the correlation for allowed and first-forbidden transitions, making use of the tensor interaction and neglecting charge dependence. We obtained the following expressions for the allowed and first-forbidden correlation functions $W(\theta)$. The first-forbidden formula is for the special selection rule ($\Delta J=2$; yes). Allowed correlation function

$$W_A(\theta)=\frac{[W_0-(W+k)]^2}{k}\sin\theta p\frac{(W+k)}{q^2}\left(\frac{(W+k)^2+W^2}{W+k}q-q^2-1\right),$$

first-forbidden correlation function, tensor interaction ($\Delta J=2$; yes)

$$W_F(\theta)=CW_A(\theta)\{[(W_0-(W+k)]^2+(W+k)^2-1-2kq\},$$

where W = final energy of beta-ray, $q=W-p\cos\theta$, $p=(W^2-1)^{\frac{1}{2}}$, k = photon energy in relativistic units, and C = a constant for equalization at 90° (see below).

We have plotted a correlation for the following experimental situation. The beta-transition is taken as first-forbidden tensor for the special case ($\Delta J=2$; yes). The upper energy of the beta-spectrum is $W_0=4mc^2$. We assume that we have a gamma-counter which accepts all gammas above 50 kev. The angular correlations for allowed cases are also plotted and are equalized to the forbidden curves at 90°. The results are plotted for various final energies of the beta-particle.

The curves demonstrate that the ratio of the number of coincidences at the forward peak to that at 90° is greater for the allowed than for the first forbidden. The ratio, of course, depends on the value of the range of final beta-energy one accepts. This ratio becomes quite appreciable if one accepts only high energy gamma-rays in the counter ($h\nu>2mc^2$). This occurs when one beta-particle is created with a high energy in the intermediate state and then emits a high energy photon. An experiment based on this condition is not feasible, however, since for an upper energy of $W_0=4mc^2$, only 10^{-5}-gammas per beta are emitted in the range above 1 Mev.

The calculations also show that it becomes easier to distinguish between allowed and forbidden as the upper energy of the beta-spectrum is increased. From the calculation we can conclude the following results:

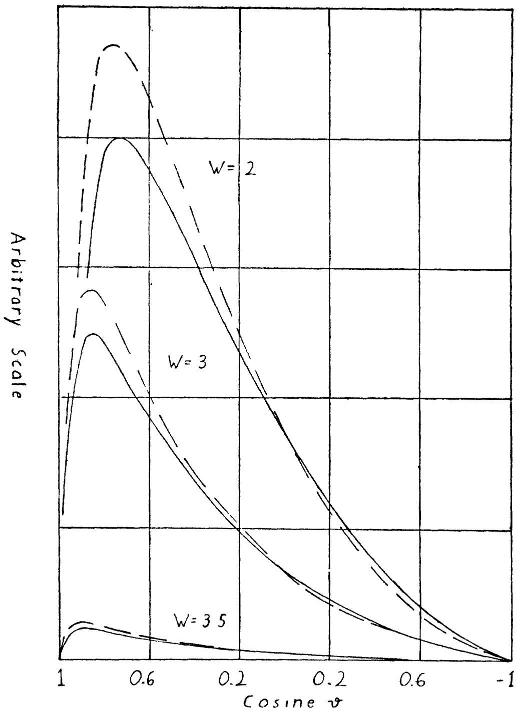

FIG. 1. The angular correlation $W(\theta)$ as a function of $\cos\theta$ for final beta-energies of 2, 3, and 3.5 mc^2. Dotted curves are for allowed transitions, solid curves for first-forbidden transitions—tensor interaction ($\Delta J=2$, yes). The curves for equalized at 90° for each final beta-energy.

1° The correlation for allowed transitions is independent of the form of the interaction;

2° The nuclear matrix elements for first-forbidden transitions are identical with the ones obtained in the ordinary beta-decay. Hence the shape of the beta-ray spectrum uniquely determines the angular correlation.

Although the internal bremsstrahlung does not provide any new information on the form of beta-decay interaction, it does provide a secondary means of studying beta-decay.

* This work was supported in part by the AEC.
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Additional Data on the Radioactive Isotopes of Tin and Tellurium*

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THE existence of many stable isotopes in tin together with the small cross section for neutron capture displayed by most of them, has made difficult the correct assignment of the observed radioactivities. Spectrometric studies of specimens enriched in masses 112 (72.5 percent), 116 (89.6 percent), 122 (45.8 percent), and 124 (83.1 percent) and irradiated in the pile for two months, yield results differing from those previously reported[1,2] for certain of the isotopes.

Tin-113. This isotope is produced by neutron capture in Sn-112 with the largest cross section of any of the isotopes, and gives in the spectrometer several electron lines. Three low energy lines are interpretable as of Auger origin accompanying indium x-rays. Three strong lines are the K, L, and M electron groups for the known transition in indium-113 (112 min) whose energy is here