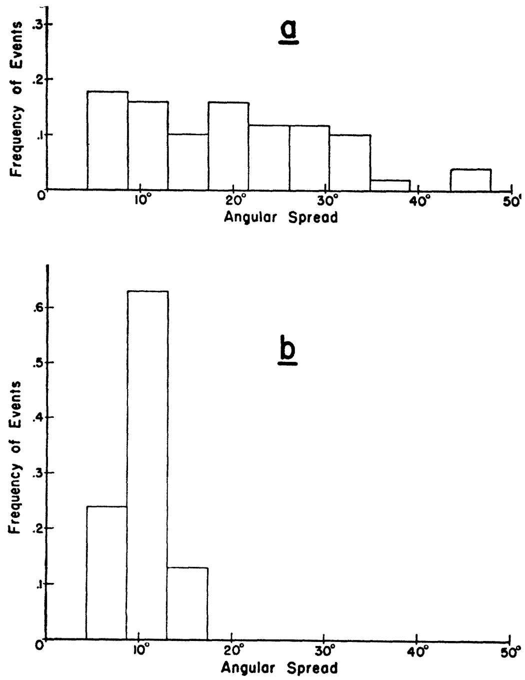

FIG. 2. Angular distribution of events of type 1a in which three or more counters were activated in tray T originating in: a, the walls of the Dewar flask; b, the liquid hydrogen.

which 2 and 3 counters were discharged in tray T is approximately equal (see Table I) indicates that the multiplicity of events observed to originate in liquid hydrogen is high. From geometrical considerations it follows that in case three or more counters were activated, the shower must have consisted of 6 or more particles. Taking this fact into account, due to the low flux of α-particles present in the cosmic radiation ($\sim$15 percent) and to their relatively low average energy per nucleon (compared to the corresponding energy for primary protons), it can be estimated that not more than 10 to 20 percent of the observed counting rate is due to the interaction of α-particles and heavier nuclei with the target protons. The remaining 80 percent to 90 percent has to be attributed to multiple meson production in proton-proton collisions.

Table II gives the flux of primary protons required to explain the observed counting rate corresponding to the following values of the collision cross section: (a) cross-sectional area of a single proton πr_0^2, where $r_0=1.4\times10^{-13}$ cm; (b) two times πr_0^2; and (c) four times πr_0^2. From the known flux of primary protons, the minimum proton energy corresponding to each value of the collision cross section was calculated. The results are listed in Table II.

It is concluded that primary protons of an energy of 50 to 100 Bev do interact in hydrogen with a cross section equal to 1 to 2 times the geometrical cross section of a single proton, producing on the average 4 or more charged mesons. These results are in satisfactory agreement with the predictions of Fermi's theory of high energy nucleon-nucleon interactions.[1] Since primary protons of energy lower than 10 Bev are about 10 times more numerous in the cosmic radiation above Chicago than those between 50 and 100 Bev, it is evident that multiple meson production in hydrogen must have a very low probability for nucleon energies $E<10$ Bev.

TABLE II. Proton flux which would yield the observed counting rate of collimated showers under hydrogen for various assumed collision cross sections. The third column gives the minimum proton energy necessary to produce such a shower as estimated from the flux and the known energy spectrum.

Collision cross sections (in units of πr_0^2 6.15×10^{-26} cm^2)	Proton flux (per cm^2 sterad sec)	Minimum proton energy (Bev)
1	0.01	50
2	0.0055	130
4	0.0028	250

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* Assisted by the joint program of the ONR and AEC.

[1] E. Fermi, Prog. Theor. Phys. 5, 570 (1950); Phys. Rev. 81, 683 (1951).

Radiative Corrections to the Hyperfine Structure and the Fine Structure Constant

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RECENT determinations of the fine structure constant, α, have rested almost entirely upon the precision measurement[1] of the hyperfine structure of hydrogen together with the corrected Fermi formula.[2] Omitted from the formula are all radiative corrections and meson field corrections other than those which can be described as anomalous magnetic moments.[3] We shall describe here a calculation of the second order radiative corrections. While these are fully described to order α by the anomalous moment as computed by Schwinger, it has long been known that a more precise evaluation of the second order energy would yield smaller corrections, the most important of which would be of order $\alpha^2 z$ rather than α. Our evaluation has been confined to that part of the energy which is of order $\alpha^2 z$ relative to the Fermi formula. In units of mc^2 then, the energy of interest is of order $\alpha^6 z^5 g_P(m/M)$, g_P being the nuclear magnetic moment in nuclear magnetons.

The second order electromagnetic self-energy of a particle moving in a prescribed static external field can be written as

$\Delta E=\Delta E_P+\Delta E_F$, with

$$\Delta E_P=-(ie/cT)\int\bar{\psi}_a(x)\gamma_\mu A_\mu{}^{(P)}(x)\psi_a(x)d_4x$$

$$\Delta E_F=(i\hbar\alpha/cT)\int\bar{\psi}_a(x_2)\gamma_\mu S_F{}^e(x_2,x_1)\gamma_\mu\psi_a(x_1)D_F(x_2-x_1)d_4x_2d_4x_1.$$

Here $\psi_a(x)$ is a particular stationary state of a Dirac particle in the external field, $A_\mu{}^{(P)}(x)$ is the polarization potential induced in the vacuum, and $S_F{}^e(x_2,x_1)$ is, apart from a constant factor, a Greens function for the Dirac particle. $S_F{}^e$ reduces to $S_F(x_2-x_1)$ for a vanishing external potential. All integrals are from $-\infty$ to $+\infty$ with the understanding that the final timelike integration is really a time average over an extended interval; this being indicated by the factors $1/cT$. Henceforth, the potential will be taken to be that of a point magnetic dipole superposed upon a point charge.

Evaluation of the polarization energy, ΔE_P, is straightforward because of the availability of a convenient expression for $A_\mu{}^P(x)$.[4] As only the order α^2 corrections are desired, certain approximations can be made which facilitate the calculation. One contribution is obtained by taking the polarization potential of the nuclear dipole field with the wave functions of the coulomb field, while a second equal contribution arises from the polarization potential of the coulomb field in conjunction with the modification in the wave functions, ψ_a, produced by the nuclear dipole field. The total energy in units of the uncorrected hyperfine structure energy for the state ψ_a is $\frac{3}{4}\alpha^2 z\delta_{10}$.

The calculation of the fluctuation energy, ΔE_F, is complicated by the fact that it contains the infinite mass correction, and by the fact that a convenient expression for $S_F{}^e$ is not available. We proceed by first transforming to momentum space and then separating ΔE_F into two terms, $\Delta E_F'$ and $\Delta E_F''$ in a manner originated by Feynman.[5] $\Delta E_F'$ contains explicit functions of the momenta and involves the potential at most linearly. Apart from renormalizations, which we ignore, it yields an order α correction to the hyperfine structure arising only from the anomalous electron moment. There are no order α^2 corrections. $\Delta E_F''$ contains the potential quadratically only and also contains $S_F{}^e$. Its evaluation to order α^2 is enormously simplified by neglecting the space parts of the momenta p_1 and p_2 associated with the state ψ_a everywhere except in $\psi_a(P_1)$ and $\bar{\psi}_a(P_2)$, and by replacing $S_F{}^e$ by its zero field approximation.[6] The corrections to the hyperfine structure appear as cross terms between the coulomb and dipole potentials. The validity of the approximation used depends upon the reduced weight of low momentum transfers associated with the dipole field and with the Feynman separation.

The evaluation of $\Delta E_F''$ yields an energy $-[(13/4)-\ln 2]\alpha^2 z'\delta_{l0}$, again in units of the Fermi energy. The total correction from ΔE_P and ΔE_F is thus $-[(5/2)-\ln 2]\alpha^2 z \approx -1.81\alpha^2 z$,[7] which is more than twelve times as large as the Bethe-Longmire estimate. Inclusion of this correction reduces the value of α^{-1} obtained from the hyperfine structure from 137.043 to 137.036. Recent measurements of the $2P_{\frac{1}{2}}-2P_{\frac{3}{2}}$ separation in deuterium indicate that a reduction in α^{-1} is indeed required, and these measurements are not inconsistent with the corrected value given here.[8]

Inclusion of meson field effects may be expected to decrease further α^{-1}. In fact, an accurate determination of α^{-1} by a method independent of mesonic effects would be a useful means of experimentally determining their magnitude.

* Work supported in part by the Signal Corps and ONR.
[1] Kusch and Prodell, Phys. Rev. 79, 1009 (1950).
[2] Dumond and Cohen, least-squares adjustment of the atomic constants as of December 1950, (report to the national research council), p. 30.
[3] Estimates of a part of these omitted effects have been given by Bethe and Longmire (Phys. Rev. 75, 306 (1949)), where they are described as arising from a spatial extension of the anomalous moments.
[4] See Karplus and Kroll, Phys. Rev. 77, (1950) for explicit definitions of D_F, S_F and $A_\mu{}^P(x)$. There $A_\mu{}^P$ is denoted by $(\alpha/2\pi)\bar{A}_\mu{}^{e2}$ (Eq. 14).
[5] Private communication. The separation is described in detail in the dissertation of M. Baranger, *Relativistic Corrections to the Lamb Shift* (Cornell University, New York, 1951).
[6] The authors are indebted to H. A. Bethe for pointing out to them the possibility of these approximations, and to M. Baranger for a copy of his thesis, in which their application to another problem is illustrated.
[7] A similar result, using a different method, has been obtained by R. Karplus and A. Klein. We wish to thank these authors for discussing their results with us.
[8] W. E. Lamb (private communication).

Continuous γ-Spectrum Accompanying Electron Capture

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It is well known that the emission of electrons from a radioactive nucleus is accompanied by a low intensity continuous spectrum of γ-radiation (bremsstrahlung). An analogous effect occurs when a nucleus captures an electron of the atomic shell. The theory of this radiative K-capture has been given by Morrison and Schiff.[1] An experimental evidence of this γ-radiation has been found in the study of the disintegration of Fe^{55}.[2] With the new technique using scintillations from NaI crystals for detecting γ-rays and measuring their energies, we were able to study the shape of the spectrum of the low intensity γ-radiation emitted by a Fe^{55} sample.

Figure 1 shows the pulse distribution of the scintillations excited by the Fe^{55} γ-radiation. To determine the upper limit of the γ-spectrum of Fe^{55} we proceed in the following way: From measurements of the pulse distribution for several monochromatic γ-rays of different energies, we are able to construct the pulse distribution which should result from the shape of the continuous γ-ray spectrum theoretically predicted by Morrison and Schiff, assuming different upper limits of the spectrum. As an example of

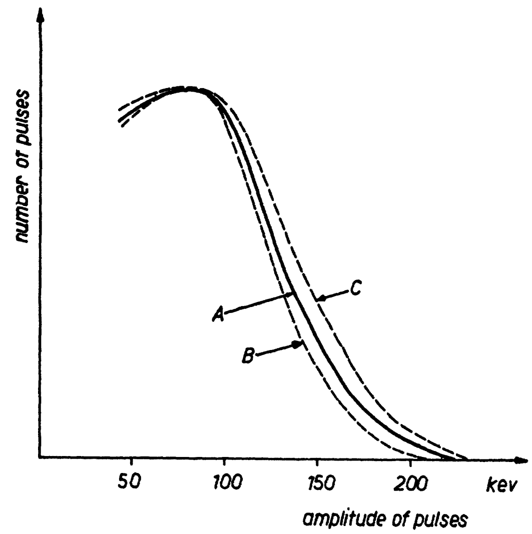

FIG. 1. Pulse distribution of the γ-radiation from Fe^{55}: A = observed curve; B = constructed curve with $E_{max}(h\nu)$ = 0.200 Mev; C = constructed curve with $E_{max}(h\nu)$ = 0.210 Mev.

the calibration, the pulse distribution for the nonconverted part of the isomeric 88-kev transition in Ag^{109} is shown in Fig. 2. (The Ag−$K\alpha$-line has been partly absorbed.) A good agreement between the experimental pulse distribution and the calculated one is obtained for an upper limit of the γ-spectrum at $E_{max}(h\nu) = 0.205$ Mev. The qualitative agreement of the curves supports the correctness of the assumed spectrum.

In the electron capture process, the whole energy available most frequently goes off with the neutrino, but may be shared between the neutrino and a γ-quantum. The upper limit of the γ-spectrum corresponds to the case when the whole disintegration energy is taken away by the γ-quantum. From the upper limit the mass difference of the parent and the daughter atoms can therefore be obtained directly; in our case it is:

$$M(Fe^{55}) - M(Mn^{55}) = E_{max}(h\nu) + E_K = (0.212 \pm 0.010) \text{ Mev}.$$

This value is in good agreement with the recent determination from the threshold of the $Mn^{55}(p, n)Fe^{55}$ reaction, for which a

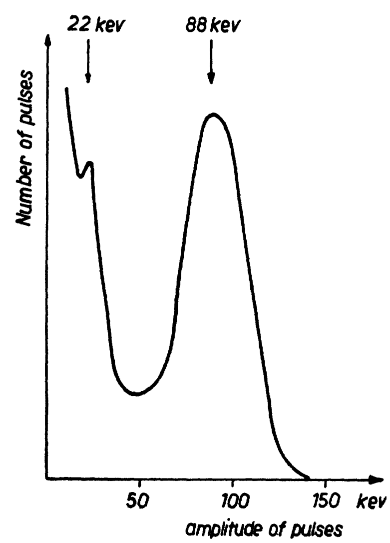

FIG. 2. Pulse distribution of the 88-kev γ-line from Ag^{109}.