the impulse approximation.[2] It will be shown that the predominantly $\cos^2\theta$ angular distribution can be understood without great difficulty, and that the reaction may be used to obtain information on the high momentum components of the $n-p$ and $p-p$ interaction.

The matrix element for the transition is to be written

$$(\psi_f, T\psi_i), \tag{1}$$

where ψ_i is the internal wave function of the deuteron in its ground state and ψ_f the continuum wave function of a diproton with relative momentum, k_f. The operator T must have the form,

$$t_1 \exp(\tfrac{1}{2}i\mathbf{q}\cdot\mathbf{r}) + t_2 \exp(-\tfrac{1}{2}i\mathbf{q}\cdot\mathbf{r}),$$

where $\mathbf{q}$ is the meson momentum, $\mathbf{r}$ is the relative nucleon coordinate and, if the meson is pseudoscalar, t_i must be some pseudoscalar operator. We assume that t_i is of the form,

$$t_i = \boldsymbol{\sigma}_i \cdot [a\nabla_r + b\mathbf{q}]\boldsymbol{\tau}_i^+, \tag{2}$$

where $\boldsymbol{\sigma}_i$ and $\boldsymbol{\tau}_i$ are the nucleon spin and isotopic spin operators and a and b may be arbitrary scalar functions of q. The restricted form of the dependence of (2) on nucleon variables is motivated by meson theory.[3] Otherwise the formulation here is completely phenomenological.

The experiment to be analyzed involves sufficiently small meson energies that one may replace $\exp(\frac{1}{2}i\mathbf{q}\cdot\mathbf{r})$ by unity. The error incurred thereby is of the order $q^2/k_f^2 \approx 0.1$. In this approximation the term in (2) proportional to a can lead only to final states of odd parity (triplet) and obviously yields an isotropic angular distribution.

The term proportional to b gives a contribution to the cross section which may be anisotropic if the deuteron is partly a D state. The final nucleon states here must be of even parity (singlet), and do not interfere with the $\boldsymbol{\sigma}\cdot\nabla$ terms. The contribution to the square of the matrix element, appropriately summed and averaged, is

$$\tfrac{1}{3}|b|^2\, q^2\{F_0^2 + 2^{\frac{1}{2}}\cos(\delta_0-\delta_2)\cdot\tfrac{1}{2}F_0F_2(3\cos^2\theta - 1) + \tfrac{1}{2}F_2^2(3\cos^2\theta+1)\}, \tag{3}$$

where

$$F_0 = \int_0^\infty u_f(r)u_i(r)dr, \quad F_2 = \int_0^\infty w_f(r)w_i(r)dr,$$

if u_i and w_i are the radial parts of the deuteron S and D functions as defined by Rarita and Schwinger,[4] and u_f and w_f are the S and D radial functions for the continuum diproton system, normalized asymptotically to $\sin(kr+\delta_0)$ and $\sin(kr+\delta_2-\pi)$, respectively. The angle between the incident meson and one of the outgoing protons in the center-of-mass system is θ.

At very low meson energies the triplet transitions induced by the $\boldsymbol{\sigma}\cdot\nabla$ term in (2) dominate, but the experimental evidence shows that between 20 and 50 Mev the singlet transitions are more important. In the first place, the very small cross section for the reaction, $p+p\rightarrow\pi^0+2p$,[5] compared to that for $p+p\rightarrow\pi^++d$,[6] is easily explained[7] only if the $\boldsymbol{\sigma}\cdot\mathbf{q}$ term is dominant. Confirmation is given by the observed constancy of the angular distribution in the meson energy.[1] This constancy is not possible if singlet and triplet transitions are of comparable probability, since they have different energy dependences. It is therefore inconsistent with the experimental accuracy attained up to now in the energy region above 20 Mev to include the terms proportional to a in the analysis. They are omitted in the following.

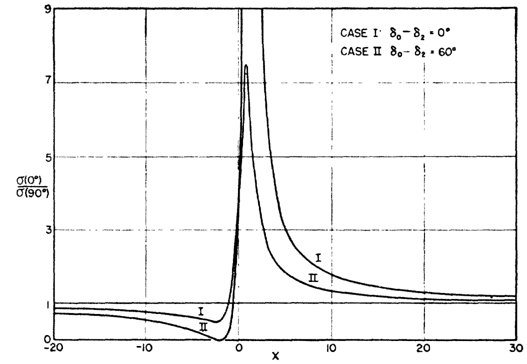

FIG. 1. Theoretical ratio of the cross section at 0° to that at 90° as a function of x.

Considering first the question of angular dependence, formula (3) may be written $\alpha+\beta\cos^2\theta$, where

$$\frac{\alpha+\beta}{\alpha} = \frac{\sigma(0°)}{\sigma(90°)} = \frac{x^2+4x\cos(\delta_0-\delta_2)+4}{x^2-2x\cos(\delta_0-\delta_2)+1}, \tag{4}$$

if $x=\sqrt{2}F_0/F_2$. The function (4) is plotted in Fig. 1 for two values of $\delta_0-\delta_2$. An attempted theoretical estimate of the constants F_0 and F_2, together with a discussion of terms of higher order in q^2/k_f^2, will be given later in a more complete report. It should be stated at once, however, that ignorance of the nuclear wave functions at short distances makes definite conclusions impossible. If the central part of the triplet $n-p$ force is equal to the singlet $p-p$ force, as suggested by the work of Pease and Feshbach,[8] then F_0 and x very nearly vanish. (The functions u_i and u_f are "almost" orthogonal.) Then $\sigma(0°)/\sigma(90°)=4$. On the other hand, nuclear force models designed only to fit scattering data have a sufficient number of free parameters that the sign and magnitude of x can be made almost anything. To fit the experimental value of $\sigma(0°)/\sigma(90°)=6$, x must be either 0 or 2-3, as seen from Fig. 1.

It will be shown in the more complete report that as the meson energy increases, the integrals F_0 and F_2 must decrease at least as fast as $1/k_f^2$ and probably as $1/k_f^4$. The observed increase in the absorption cross section then requires the parameter b in (2) to be an increasing function of meson energy. This is in definite disagreement with the weak coupling meson theory, indicating that the meson wave function is strongly perturbed in the region near the nucleon.

The authors wish to acknowledge great benefit from conversations with R. Serber. For comparison with other published phenomenological treatments of this problem, the following points of difference should be noted: Cheston,[9] Fujimoto and Yamaguchi,[10] and Brueckner[11] either omit or inadequately approximate both the diproton wave function and the deuteron D-state. Watson and Brueckner do not attempt to separate the meson-nucleon interaction from the interaction between the two nucleons and therefore their article is not an analysis in the sense of this work.

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Penetrating Showers Produced in Beryllium at Sea Level

W. Y. CHANG, G. DEL CASTILLO,* AND LEON GRODZINS
Physics Department, Purdue University, Lafayette, Indiana
(Received August 6, 1951)

THE cloud-chamber equipment, previously reported,[1] has been set up at Purdue to investigate penetrating showers produced in Be, C, and Pb. Figure 1 shows the arrangement. All the counters have a diameter of one inch. The lengths of trays A, B, C and D (fivefold coincidence) are respectively 6, 3.5, 12, and 14 inches. Tray C counters are connected for shower detection of at least two particles. The chamber expands, when a shower is produced (by a charged primary particle, presumably a high energy proton) in the 10-inch C block above (later replaced by

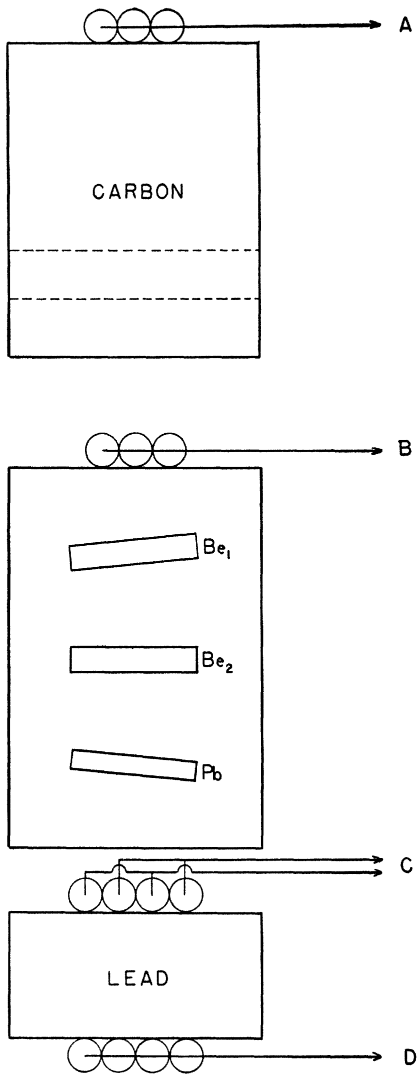

FIG. 1. Arrangement for shower equipments.

same amount of Pb, i.e., 43 g/cm²), or in the Be_1 or Be_2 plate (4.8 g/cm²) inside, and has at least two particles to discharge two neighboring tray *C* counters, one of them being energetic enough to pass through the Pb block (110 g/cm²) below and then discharge a tray *D* counter. The Pb plate (17 g/cm²) inside the chamber serves to infer the nature of the particles passing through it. The illuminated depth of the chamber was greater on both sides than the depth (4 inches) of the Be or Pb plate, so that particles which pass in front of the plates can easily be seen. Only those showers have been selected, which have their axes near to the central portion of the plates and appear to have all of their particles falling within the surface of the plates. Most of the small and medium showers have entered the chamber in this manner due to counter disposition. In this way we shall be more certain about the multiplicity and penetrating power of the shower particles. In all the experiments stereoscopic pictures have been taken.

The results for beryllium will be reported here, while those for carbon and lead will be discussed in another note, because the experimental conditions for C and Pb are the same. The total experimental period for Be was about 11 months; for the first 6 months carbon was above the chamber, for the remainder, lead.

A particle is defined as penetrating if it passes through the bottom Pb plate (17 g/cm²) inside the chamber without multiplication or multiple scattering greater than 20 degrees.[2] In the case of Be, a penetrating shower is defined as follows: Two or more tracks, as seen in the gas space immediately below their origin, must appear from the same point within a Be plate. (1) When only two particles are seen, both of them must be penetrating particles as defined above; for, if only one of them is penetrating, the stopped one may be just a knock-on electron. We have observed 36 cases of this latter type, which were therefore discarded, though some may be real consequences of nucleon-nucleon collisions. (2) When 3 or more are seen in the space below the origin, at least one of them must be penetrating. The chance is considerably smaller for two knock-on electrons to be ejected from about the same point by a single penetrating particle. Only 3 such cases (three or more particles seen and only one penetrating) have been found and are included in our statistics. A single penetrating particle can discharge two neighboring tray *C* counters if it has one knock-on electron in this direction, and discharge one tray *D* counter if it is energetic enough to penetrate the 110 g/cm² Pb block. Therefore, we are concerned here with showers, each of which has at least one particle having a range in Pb of at least about 130 g/cm².

TABLE I. Multiplicity distribution for beryllium. Working period—11 months. *N* is the number of charged particles per shower.

Absorber	Gas space \ *N*	1	2	3	4	5	6
	2nd		27	9	1		1
Be_1	3rd		30	7		1	
4.8 g/cm²	4th	3	32	2		1	
Be_2	3rd		18	3	2	1	1
4.8 g/cm²	4th		20	2	2	1	

Under the above conditions the showers produced in the two Be plates have been selected and tabulated in Table I, which shows the multiplicity distribution of the charged particles as seen in the different gas spaces (in descending order) inside the chamber. The particles counted in the third or fourth gas space are those which appear to be a continuation of the tracks in the preceding space. Secondaries are not included. In this way we may learn more about the behavior of the shower particles with respect to their penetrating power in the Be and Pb plates. Most of the stopped particles are simply absorbed by the plates without producing any observable effect, some produce secondaries, and a few are scattered at angles greater than 20 degrees. From the total numbers of the shower particles in the different gas spaces, we have estimated the mean free paths of the particles respectively in Be_2 and Pb plates, using the relation $n=n_0e^{-x/L}$. For Be_1 showers, $L_{Be_2}=85$ g/cm² and $L_{Pb}=180$ g/cm². This may indicate that initially the showers might have more low energy particles which are easily absorbed by the Be_2 plate, while after the Be_2 plate relatively more penetrating particles remained which would then have a larger mean free path in Pb. Also, the effect of atomic number tends to decrease the mean free path in Be. The above figures are only approximate because of low statistics and difficulty in tracing the tracks, and so the conclusions are merely tentative. For Be_2 showers $L_{Pb}=200$ g/cm², comparable with L_{Pb} for Be_1 showers.

The average multiplicity per shower in the space immediately below the origin of production is about 2.5 from the data of both Be_1 and Be_2. This figure is smaller than that for C and Pb showers, both having 2.8 in the first gas space. This difference must not be

TABLE II. Angular distribution for beryllium. N_1 is the number of particles in the space just below the origin making the projected angle θ with the primary particles.

θ, deg	N_1	θ, deg	N_1	θ, deg	N_1
0–2	95	6.1–8	4	20–22	3
2.1–4	18	8.1–10	2	30–32	1
4.1–6	11	12 –14	3	34–36	1

over-emphasized because of poor statistics. Besides, exact comparison is not justified, for the experimental conditions and conditions for selecting showers are somewhat different for Be and for C and Pb. However, in general, one may say that Be, C, and Pb have similar multiplicity and angular distributions, which may indicate a mechanism of multiple production. This point will be discussed in more details in the note for C and Pb. It is to be noted that our multiplicity for Be seems to be smaller than that obtained recently by Fretter[3] from 13 cases for Li above a chamber at sea level. The projected angles with the primary were measured of the shower particles in the space just below the origin. The distribution is shown in Table II, indicating that the shower particles are emitted with an extremely strong preference for the direction of the primary.

We are thankful to Princeton University for the apparatus used in this work, which one of us (W.Y.C.) and his co-workers built at Princeton, and to our Colleagues at Purdue for their interest and support in these experiments.

* Supported in part by Purdue Research Foundation and Mexican Research Commission C.I.C.I.C.

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Penetrating Showers Produced in Carbon and Lead at Sea Level

W. Y. Chang and G. del Castillo*
Physics Department, Purdue University, Lafayette, Indiana
(Received August 6, 1951)

THE experimental arrangement and general conditions for recording the showers have been briefly described in the preceding note for the Be showers.[1] The first 6 months were spent studying the showers produced in the 10-inch C block above the chamber, while the next 5 months were for the showers from a Pb block of the same thickness, 43 g/cm², which replaced the C block (counter disposition kept the same); the Be plates remained inside the chamber for all the 11 months. Since the C and Pb blocks used had the same amount of material in g/cm², the absorption of shower particles with a given energy distribution would be approximately the same in the two elements. The Pb block was placed about 1.5 inches below the middle of the replaced 10-inch C block, so that the scattered (more in Pb) shower particles would have a larger chance to enter the chamber. The Be and Pb plates, respectively 4.8 and 17 g/cm², inside the chamber serve to study the particles of the C and Pb showers. There are altogether 63, 84, and 73 showers selected, respectively, from Be, C, and Pb. The statistics are compatible with the three different working periods and amounts in g/cm² of the three elements used.

As with Be, a penetrating particle is defined as one passing through the Pb plate (17 g/cm²) inside the chamber without multiplication or scattering larger than 20 degrees. A penetrating shower is here defined as one which has at least two tracks seen in the first gas space and at least one of them a penetrating particle (the stopped one must show no appreciable scattering). They must, of course, appear to come from the same origin within the C or Pb block as examined in both the direct and stereoscopic pictures. Single isolated penetrating particles are discarded, for most of them are presumably mu-mesons. A few of them may be penetrating showers with only one penetrating particle entering the chamber and the rest absorbed by the material above the chamber. Here we are concerned with showers, each of which has at least one penetrating particle having a minimum range lying between 140 g/cm² Pb and 140 g/cm² Pb plus 43 g/cm² Pb or C (see Be note). In both experiments for C and Pb, quite a few large mixed showers with total number of particles around 10 or larger have been observed. They have not been included in our statistics because of uncertainty in their origins because of some particles being badly scattered in random directions.

TABLE I. Multiplicity distribution for carbon and lead. Working period —6 months for carbon and 5 months for Pb. N is the number of charged particles per shower.

Absorber	Gas space \ N	1	2	3	4	5	6	7	8	9	10	15
	1st	...	52	21	3	5	1	1	...	...	...	1
	2nd	1	55	18	3	4	2	...	...	...	...	1
Carbon	3rd	4	60	11	4	2	2	...	...	...	...	1
43 g/cm²	4th	40	33	8	1	1	1	...	...	...	...	...
	1st	...	47	15	3	2	2	1	1	1	1	...
	2nd	1	49	13	3	2	3	...	1	1	...	...
Lead	3rd	5	51	9	3	2	2	...	...	1	...	...
43 g/cm²	4th	40	27	5	1	...	...	...	...	...	...	...

Under the aforementioned conditions, the data for C and Pb showers have been obtained and are presented in Table I, according to the multiplicity distributions of the charged particles observed in the four gas spaces. As in the case of Be showers, the particles in the various spaces are those which appear to be a continuation of the tracks in the preceding space (see Be note for manner of recording showers), and secondary particles have not been included. One hopes to learn more in this way about the penetrating power of the shower particles. As before, most of the stopped particles do not produce any observable effect, some have secondaries, and a few are scattered at angles larger than 20 degrees. The mean free paths have been estimated, assuming as before that the shower particles are absorbed exponentially according to $n=n_0e^{-x/L}$. The values are shown in Table II. It is to be noted that here we are concerned mostly with the latter sections of the range of the shower particles, while in the case of Be we are in the early parts of their range. When a C or Pb shower is produced in the upper portion of the material, its particles have to traverse the lower portion before entering the chamber. The mean free path in Be appears to decrease in the downward direction and in Pb becomes still smaller. This is conceivable from the energy consideration. The effect of atomic number must be remembered in comparing L_{Be} and L_{Pb}. Values for C showers are consistently larger than the corresponding values for Pb showers. However, as before, these figures must not be emphasized too much, because of low statistics and difficulty in tracing a track in the different spaces.

TABLE II. Mean free paths in g/cm² at successive points of the range.

	C showers	Pb showers	Be_1 showers	Be_2 showers
L_{Be1}	120	85	...	...
L_{Be2}	70	50	85	...
L_{Pb}	45	40	180	200

The average multiplicity per shower has been estimated for the charged particles as observed in the first gas space. Both C and Pb showers have the same value of 2.8. The average multiplicity at the origin may be higher because some low energy shower particles may have been absorbed by the material between the origin and the chamber. Reference to the Be results seems to indicate that the initial absorption is not too serious and hence the increase in multiplicity may not be high. The multiplicity of all ionizing particles as obtained recently by Fretter[2] and Green[3] for Pb and C,

TABLE III. Angular distribution—carbon and lead. N_1 is the number of particles in the first gas space making the projected angle θ with the shower axis.[a]

θ, deg	N_1(C)	N_1(Pb)	θ, deg	N_1(C)	N_1(Pb)
0–2	140	98	14–16	3	3
2.1–4	41	36	16.1–18	...	4
4.1–6	22	27	18.1–20	4	...
6.1–8	9	17	23–25	1	2
8.1–10	8	12	...	...	...
10.1–12	...	6	42–44	...	1

[a] See reference 5.