Letters to the Editor

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The Absorption of Pions by Deuterons*

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THE study of the absorption of positive pions by deuterons, which, together with its inverse, has been used to determine the spin of the meson,[1,2] has been extended. We present here results on the energy dependence of the angular distribution and of the total cross section, in the hope that they will prove useful in the attempt to understand the pion-nucleon interaction.

The reaction is $\pi^+ + d \rightarrow p + p$. The reaction rate is determined by the rate of recoil proton pairs in coincidence with incoming mesons, incident successively on water and heavy water targets. The experimental technique has already been described.[1] The water cells are 1″ thick along the beam direction. The energy dispersion due to target thickness and meson beam inhomogeneity is approximately ± 7 Mev. The angular dispersion is approximately $\pm 14°$. Despite this poor resolution the coincidence rate of recoil protons is only ~ 1/minute in the Nevis meson beam of ~ 500/sec. The uncertainties in the result are almost entirely statistical; the geometrical factors have been calculated with greater accuracy, and the combined uncertainties in counting efficiency and beam composition are less than 10 percent.

The results are given in Table I. All parameters as well as differential cross sections are in the center of mass system. The recoil angles listed are averaged over the angular dispersion. This affects only the angles near 90°, since, because of the symmetry, 90° is the extreme angle, and larger angles are recorded as smaller angles. When the proton counter axes are set at 90° relative to the meson beam, the average detection angle is 83°.

It is possible to represent the angular dependence as $\alpha + \beta \cos^2\theta$, since meson angular momenta with respect to the deuteron of more than two Planck units should not contribute appreciably at these energies. The best fits to such a distribution are given in Table II, together with absolute cross sections.

In Table II we have also included the results of Cartwright, Richman, Wilcox, and Whitehead.[3] The angular distribution found at Berkeley and our results are only in fair agreement.

There is no large change in the angular distribution in the energy range 25–53 Mev for the incident meson in the center of mass system. The total cross section increases by a factor 2.25 ± 0.32.

To see the implication of this result on the meson nucleon interaction, it is necessary to separate the effects of the nuclear binding in initial and final states as well as the kinematical factors. The effects of the binding on the angular distribution are complicated, and are discussed in the following note.[4] The kinematical factors are

TABLE I. Differential cross sections for the absorption of pions by deuterons in the center of mass system. Rms statistical errors are given.

$E_m = 25$ Mev		$E_m = 40$ Mev		$E_m = 53$ Mev	
θ_{cm}	$d\sigma/d\Omega$ cm²/sterad	θ_{cm}	$d\sigma/d\Omega$	θ_{cm}	$d\sigma/d\Omega$
29°	$8.5 \pm 1.1 \times 10^{-28}$	30°	$17.4 \pm 2.6 \times 10^{-28}$	30°	$20.1 \pm 4 \times 10^{-28}$
35°	7.3 ± 1.6	50°	9.2 ± 1.5	60°	5.7 ± 1.8
45°	7.5 ± 2.2	70°	8.4 ± 2.4	83°	5.5 ± 2.1
50°	7.1 ± 0.9	83°	4.0 ± 1.7	...	...
82°	1.7 ± 0.6	...	...	...	...

TABLE II. Total cross sections and best fit angular distributions for the data in Table I.

E_{cm}	Angular distribution	Total cross section
21.5 Mev[a]	$10.7 (\cos^2\theta + 0.07) \times 10^{-28}$ sterad/cm²	$2.8 \pm 0.8 \times 10^{-27}$ cm²
25 Mev	$9 (\cos^2\theta + 0.22) \times 10^{-28}$ sterad/cm²	$3.1 \pm 0.3 \times 10^{-27}$ cm²
40 Mev	$18 (\cos^2\theta + 0.2) \times 10^{-28}$ sterad/cm²	$6.1 \pm 0.6 \times 10^{-27}$ cm²
53 Mev	$21.5 (\cos^2\theta + 0.18) \times 10^{-28}$ sterad/cm²	$7.0 \pm 0.7 \times 10^{-27}$ cm²

[a] We include the result of the Berkeley group on the inverse reaction, inverted on the basis of the law of detailed balance and zero spin for the pion.

K = momentum space/(relative velocity of incoming nucleons × total meson energy).

The factor 1/total meson energy is not strictly a kinematical factor; it is due to the normalization of the meson wave and is included because it is omitted in the theoretical analysis which follows this letter.

$$k_{53}/k_{25} = 0.71.$$

The average square of the matrix element for the process therefore increases by the factor $2.3/0.71 = 3.25 \pm 45$ in the energy range 25–53 Mev. This is a considerable increase, especially since the effects of binding also decrease the cross section at higher relative to lower meson energy. This effect is approximately

$$|f(k_{f53})/f(k_{f25})|^2,$$

where $f(k_{f53})$ is the fourier amplitude of the deuteron or the diproton wave function for the momentum of one of the recoil protons resulting in the absorption of a 53-Mev meson. These fourier amplitudes decrease at least as $1/k^2$ and probably more nearly as $1/k^5$ for such large momenta. If this factor is also taken into account, then the meson-nucleon interaction must increase by a factor $\sqrt{4.5} - \sqrt{6}$ in the meson energy interval 25–53 Mev. The meson momentum increases by the factor 1.53 in this interval so that the meson nucleon interaction must increase approximately as the square of the momentum of the meson. This is of course a stronger momentum dependence than the linear dependence predicted in pseudoscalar theory with pseudovector coupling. Other theories predict an even weaker dependence and are also in conflict with other experiments. The steep energy dependence is probably of the same origin as the steep excitation function in neutral photomeson production[5,6] and may be due to some resonance effect, as has already been suggested in connection with the photomeson experiments.[7,8]

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A Theoretical Analysis of the Process $\pi^+ + d \rightleftharpoons p + p$

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THE experiment described in the preceeding letter[1] as well as those on the inverse reaction, the production of mesons in the collision of two protons, may be analyzed in the spirit of the impulse approximation.[2] It will be shown that the predominantly $\cos^2\theta$ angular distribution can be understood without great difficulty, and that the reaction may be used to obtain information on the high momentum components of the $n-p$ and $p-p$ interaction.

The matrix element for the transition is to be written

$$(\psi_f, T\psi_i), \tag{1}$$

where ψ_i is the internal wave function of the deuteron in its ground state and ψ_f the continuum wave function of a diproton with relative momentum, k_f. The operator T must have the form,

$$t_1 \exp(\tfrac{1}{2}i\mathbf{q}\cdot\mathbf{r})+t_2 \exp(-\tfrac{1}{2}i\mathbf{q}\cdot\mathbf{r}),$$

where $\mathbf{q}$ is the meson momentum, $\mathbf{r}$ is the relative nucleon coordinate and, if the meson is pseudoscalar, t_i must be some pseudoscalar operator. We assume that t_i is of the form,

$$t_i=\boldsymbol{\sigma}_i\cdot[a\boldsymbol{\nabla}_r+b\mathbf{q}]\boldsymbol{\tau}_i^+, \tag{2}$$

where $\boldsymbol{\sigma}_i$ and $\boldsymbol{\tau}_i$ are the nucleon spin and isotopic spin operators and a and b may be arbitrary scalar functions of q. The restricted form of the dependence of (2) on nucleon variables is motivated by meson theory.[3] Otherwise the formulation here is completely phenomenological.

The experiment to be analyzed involves sufficiently small meson energies that one may replace $\exp(\frac{1}{2}i\mathbf{q}\cdot\mathbf{r})$ by unity. The error incurred thereby is of the order $q^2/k_f^2\approx 0.1$. In this approximation the term in (2) proportional to a can lead only to final states of odd parity (triplet) and obviously yields an isotropic angular distribution.

The term proportional to b gives a contribution to the cross section which may be anisotropic if the deuteron is partly a D state. The final nucleon states here must be of even parity (singlet), and do not interfere with the $\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}$ terms. The contribution to the square of the matrix element, appropriately summed and averaged, is

$$\tfrac{1}{3}|b|^2\, q^2\{F_0^2+2^{\frac{1}{2}}\cos(\delta_0-\delta_2)\cdot\tfrac{1}{2}F_0F_2(3\cos^2\theta-1) +\tfrac{1}{8}F_2^2(3\cos^2\theta+1)\}, \tag{3}$$

where

$$F_0=\int_0^\infty u_f(r)u_i(r)dr,\quad F_2=\int_0^\infty w_f(r)w_i(r)dr,$$

if u_i and w_i are the radial parts of the deuteron S and D functions as defined by Rarita and Schwinger,[4] and u_f and w_f are the S and D radial functions for the continuum diproton system, normalized asymptotically to $\sin(kr+\delta_0)$ and $\sin(kr+\delta_2-\pi)$, respectively. The angle between the incident meson and one of the outgoing protons in the center-of-mass system is θ.

At very low meson energies the triplet transitions induced by the $\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}$ term in (2) dominate, but the experimental evidence shows that between 20 and 50 Mev the singlet transitions are more important. In the first place, the very small cross section for the reaction, $p+p\rightarrow\pi^0+2p$,[5] compared to that for $p+p\rightarrow\pi^++d$,[6] is easily explained[7] only if the $\boldsymbol{\sigma}\cdot\mathbf{q}$ term is dominant. Confirmation is given by the observed constancy of the angular distribution in the meson energy.[1] This constancy is not possible if singlet and triplet transitions are of comparable probability, since they have different energy dependences. It is therefore inconsistent with the experimental accuracy attained up to now in the energy region above 20 Mev to include the terms proportional to a in the analysis. They are omitted in the following.

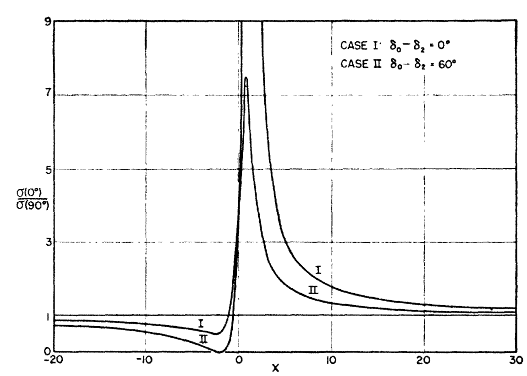

FIG. 1. Theoretical ratio of the cross section at 0° to that at 90° as a function of x.

Considering first the question of angular dependence, formula (3) may be written $\alpha+\beta\cos^2\theta$, where

$$\frac{\alpha+\beta}{\alpha}=\frac{\sigma(0°)}{\sigma(90°)}=\frac{x^2+4x\cos(\delta_0-\delta_2)+4}{x^2-2x\cos(\delta_0-\delta_2)+1}, \tag{4}$$

if $x=\sqrt{2}F_0/F_2$. The function (4) is plotted in Fig. 1 for two values of $\delta_0-\delta_2$. An attempted theoretical estimate of the constants F_0 and F_2, together with a discussion of terms of higher order in q^2/k_f^2, will be given later in a more complete report. It should be stated at once, however, that ignorance of the nuclear wave functions at short distances makes definite conclusions impossible. If the central part of the triplet $n-p$ force is equal to the singlet $p-p$ force, as suggested by the work of Pease and Feshbach,[8] then F_0 and x very nearly vanish. (The functions u_i and u_f are "almost" orthogonal.) Then $\sigma(0°)/\sigma(90°)=4$. On the other hand, nuclear force models designed only to fit scattering data have a sufficient number of free parameters that the sign and magnitude of x can be made almost anything. To fit the experimental value of $\sigma(0°)/\sigma(90°)=6$, x must be either 0 or 2-3, as seen from Fig. 1.

It will be shown in the more complete report that as the meson energy increases, the integrals F_0 and F_2 must decrease at least as fast as $1/k_f^2$ and probably as $1/k_f^4$. The observed increase in the absorption cross section then requires the parameter b in (2) to be an increasing function of meson energy. This is in definite disagreement with the weak coupling meson theory, indicating that the meson wave function is strongly perturbed in the region near the nucleon.

The authors wish to acknowledge great benefit from conversations with R. Serber. For comparison with other published phenomenological treatments of this problem, the following points of difference should be noted: Cheston,[9] Fujimoto and Yamaguchi,[10] and Brueckner[11] either omit or inadequately approximate both the diproton wave function and the deuteron D-state. Watson and Brueckner do not attempt to separate the meson-nucleon interaction from the interaction between the two nucleons and therefore their article is not an analysis in the sense of this work.

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Penetrating Showers Produced in Beryllium at Sea Level

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THE cloud-chamber equipment, previously reported,[1] has been set up at Purdue to investigate penetrating showers produced in Be, C, and Pb. Figure 1 shows the arrangement. All the counters have a diameter of one inch. The lengths of trays A, B, C and D (fivefold coincidence) are respectively 6, 3.5, 12, and 14 inches. Tray C counters are connected for shower detection of at least two particles. The chamber expands, when a shower is produced (by a charged primary particle, presumably a high energy proton) in the 10-inch C block above (later replaced by