Microwave Determination of the Probability of Collision of Slow Electrons in Gases*

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A microwave method is given for determining the collision probability for momentum transfer for electrons in thermal equilibrium with a gas. An integral equation is developed which gives the ratio of the resistive and reactive components of the microwave conductivity in terms of the collision frequency for electrons with the gas atoms. Two approximate solutions are given for the collision frequency as a function of electron energy. An experimental procedure is described in which the conductivity ratio is determined from the shift in the resonant frequency and the change in conductance of a resonant cavity. Measurements are made during the decay of a pulsed discharge after the electrons have cooled to the gas temperature. Preliminary results are given for several gases.

I. INTRODUCTION

EXPERIMENTAL determinations of the probability of elastic collision between electrons and gas atoms obtained by electron beam methods show wide discrepancies for electron energies below about one electron volt. Recently several attempts have been made to obtain a better understanding of the low energy region. Huxley and Zaazou[1] used measurements of the average energy and drift velocity of electrons in a dc field to calculate the electron mean free path for average energies down to about one-third of an electron volt. They carried out their calculations for the assumption of a constant collision probability, using both a Maxwell and a Druyvesteyn electron energy distribution function. Margenau and Adler[2] have used microwave measurements of the high frequency conductivity of the positive column in a mercury discharge to evaluate the mean free path for average electron energies between about 0.6 and 1.1 volts. These authors restricted their discussion to a maxwellian electron energy distribution function but treated both the cases of constant collision probability and of constant collision frequency. In this paper a microwave method is described for determining the probability of collision by measuring the conductivity of a decaying plasma after the electrons reach thermal equilibrium with the gas.

II. THEORY OF THE METHOD

Margenau[3] has given a general theory for the behavior of electrons in a gas under the action of a high frequency electric field when only elastic collisions need be considered. From his results we may write for the complex conductivity, σ_c:

$$\sigma_c=\sigma_r+j\sigma_i=\frac{J}{E}=-\frac{4\pi}{3}\frac{ne^2}{m\omega}\int_0^\infty \frac{[(\nu_c/\omega)-j]}{1+(\nu_c/\omega)^2}v^3\frac{\partial f_0}{\partial v}dv. \quad (1)$$

Here n is the electron density, e and m are the electronic charge and mass, ω is the radian frequency of the applied field, f_0 is the first term in the spherical harmonic expansion of the normalized electron velocity distribution function and ν_c is the collision frequency for momentum transfer for electrons of velocity v colliding with neutral atoms. The "probability of collision for momentum transfer," P_c, is related to the collision frequency by $\nu_c=v/l=vp_0P_c$, where l is the mean free path and p_0 is the pressure normalized to zero degrees centigrade. The momentum transfer collision probability[4] takes into account the fact that the effectiveness of collisions in resisting current flow increases as the scattering angle increases. The electron beam type of experiment determines a "total" collision probability,[5] since electrons are lost to the beam if they suffer any angular deflection greater than the angular aperture of the detector. The values of P_c calculated from measurements of the distribution in angle of the scattered electrons usually differ from the "total" collision probabilities by a few percent.

When the electron velocity distribution function is known, the collision frequency can be determined from the ratio of the real to the imaginary part of the conductivity. Thus

$$\frac{\sigma_r}{\sigma_i}=-\int_0^\infty \frac{(\nu_c/\omega)v^3df_0}{1+(\nu_c/\omega)^2}\bigg/\int_0^\infty \frac{v^3df_0}{1+(\nu_c/\omega)^2}, \quad (2)$$

is an equation in which the conductivity ratio, σ_r/σ_i, is a function of the parameters describing f_0 and ν_c/ω and is to be solved for ν_c as a function of the electron velocity. Note that by expressing the theory in terms of the conductivity ratio we have eliminated the necessity for knowing the electron density.

Margenau[3] showed that the steady-state distribution function for electrons in an atomic gas in the absence of inelastic collisions and large diffusion loss could be

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[1] L. G. H. Huxley and A. A. Zaazou, Proc. Roy. Soc. (London), **A196**, 402 (1949).

[2] H. Margenau and F. P. Adler, Phys. Rev. **79**, 970 (1950).

[3] H. Margenau, Phys. Rev. **69**, 508 (1946).

[4] E. H. Kennard, *Kinetic Theory of Gases* (McGraw-Hill Book Company, Inc., New York, 1942), Chapters III and IV.

[5] R. B. Brode, Revs. Modern Phys. **5**, 257 (1933).

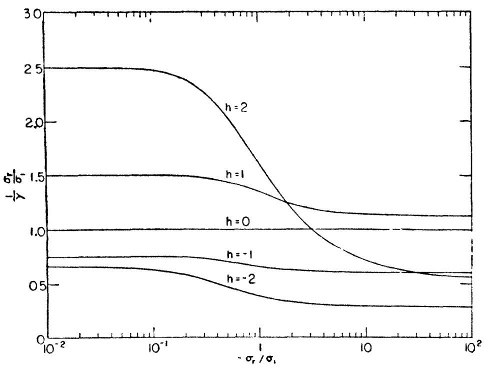

FIG. 1. Theoretical curves of $\sigma_r/\gamma\sigma_i$ as a function of σ_r/σ_i.

given as

$$\ln f_0 = -\int_0^u \{kT_g/e + MeE^2/(3m^2\omega^2[1+(\nu_c/\omega)^2])\}^{-1}du. \quad (3)$$

Here u is the electron energy expressed in volts, T_g is the temperature of the gas atoms of mass M, and E is the rms value of the applied electric field. For low pressures ($\nu_c^2 \ll \omega^2$) the distribution is nearly maxwellian with an energy $\langle u\rangle = (kT_g/e + MeE^2/3m^2\omega^2)$. At low fields $\langle u\rangle = kT_g/e$. Writing the maxwellian energy distribution as $f_0 = A\exp(-u/\langle u\rangle)$, Eq. (2) becomes

$$\frac{\sigma_r}{\sigma_i} = -\int_0^\infty \frac{(\nu_c/\omega)u^{1.5}\exp(-u/\langle u\rangle)du}{(\nu_c/\omega)^2+1} \Bigg/ \int_0^\infty \frac{u^{1.5}\exp(-u/\langle u\rangle)du}{(\nu_c/\omega)^2+1}, \quad (4)$$

We will present two approximate solutions of Eq. (4) for the collision frequency, ν_c. A low field approximation, restricted to electrons in thermal equilibrium with the gas and to a simple velocity dependence for ν_c, is valid over the whole range of ν_c/ω. A low pressure approximation is given in which the only restriction placed on ν_c is that $(\nu_c/\omega)^2 \ll 1$.

Low Field Approximation

To obtain $\nu_c(u)$ one can assume that ν_c is proportional to some power of the electron velocity and evaluate the proportionality constant and exponent by fitting experimental measurements of σ_r/σ_i to the theoretical curves calculated from Eq. (4). Thus if $\nu_c = ap_0v^h$, $P_c = \mathrm{a}v^{h-1}$ and Eq. (4) becomes

$$\frac{\sigma_r}{\sigma_i}(\gamma, h) = -\gamma\int_0^\infty \frac{y^{0.5H+1.5}\exp(-y)dy}{y^h+\gamma^{-2}} \Bigg/ \int_0^\infty \frac{y^{1.5}\exp(-y)dy}{y^h+\gamma^{-2}}, \quad (5)$$

where $\gamma = (2e\langle u\rangle/m)^{0.5h}ap_0/\omega = (2kT_g/m)^{0.5h}ap_0/\omega$ is the number of collisions per radian of the electric field for electrons having a velocity equal to the most probable velocity of the distribution function, and $y = (u/\langle u\rangle)$.

For the case of constant collision frequency, $h=0$ and Eq. (5) reduces to $\sigma_r/\sigma_i = -\nu_c/\omega$. The constant mean free path case, for which $h=+1$, has been evaluated by Margenau[3] in terms of error functions and exponential integrals. We have evaluated the integrals for $h=+2$ in terms of Lommel functions and for $h=-1$ and -2 in terms of the solutions for $h=+1$ and $+2$. Figure 1 shows the results of these calculations in the form of $\sigma_r/\gamma\sigma_i$ as a function of σ_r/σ_i for various values of "h".

There are two experimentally convenient means for determining the two parameters "h" and "a" for this approximation. In one procedure the gas pressure p_0 is varied at constant frequency and temperature to obtain a curve of $\sigma_r/p_0\sigma_i$ *versus* σ_r/σ_i covering a range of values of σ_r/σ_i near unity. The value of γ/p_0 is adjusted to give the best fit between the experimental points and the theoretical curve for some value of "h". When ambiguity exists in the sign of "h", the gas temperature or the electric field can be used to raise the average electron energy and to determine whether σ_r/σ_i, and, therefore γ, increases or decreases. Another procedure is to make measurements of σ_r/σ_i as a function of pressure at two different gas temperatures. The pressure and temperature readings which give the same value of σ_r/σ_i, and therefore γ, fix a value of "h". The coefficient "a" can then be determined from the theoretical curves of σ_r/σ_i *versus* γ. The disadvantage of this approximation is that a single set of values for "a" and "h" may not fit the experimental data over a wide range of electron energies.

Low Pressure Approximation

Another means of determining $\nu_c(u)$ is to measure σ_r/σ_i at low enough pressures so that $(\nu_c/\omega)^2 \ll 1$ over the velocity range in which the distribution function, f_0, is significant. Equation (4) then reduces to

$$\sigma_r(\langle u\rangle)/\sigma_i = -[4/(3\pi^{\frac{1}{2}}\langle u\rangle^{2.5})] \times \int_0^\infty (\nu_c/\omega)u^{1.5}\exp(-u/\langle u\rangle)du. \quad (6)$$

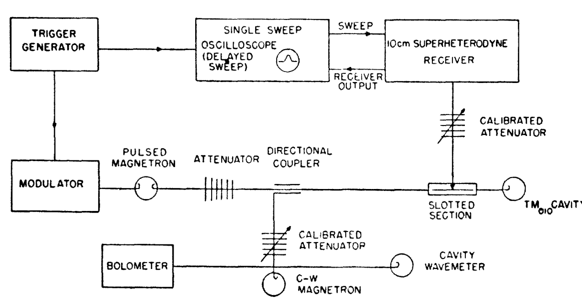

FIG. 2. Block diagram of apparatus used to measure the conductivity ratio.

This integral equation of the first kind can be solved approximately[6] by representing the experimental data for σ_r/σ_i by a polynomial of the form,

$$\sigma_r(\langle u\rangle)/\sigma_i = -\sum_{j\geq -3}^{l} a_j(\langle u\rangle^{0.5})^j p_0. \tag{7}$$

Multiplying Eqs. (6) and (7) by $\langle u\rangle^{2.5}$, and performing an inverse laplace transformation, we get

$$\nu_c(u)/p_0\omega = \sum_{j\geq -3}^{l} \frac{a_j(u^{0.5})^j}{[(j+3)/2]!}(3/2)! \tag{8}$$

and

$$P_c = \omega(m/2e)^{\frac{1}{2}} \sum_{j\geq -3}^{l} \frac{a_j(u^{0.5})^{j-1}}{[(j+3)/2]!}(3/2)!. \tag{9}$$

Thus the a_j's determined from the experimental curve of $\sigma_r/\sigma_i p_0$ can be used in Eqs. (8) and (9) to give curves of ν_c/p_0 and P_c as functions of electron energy.

III. EXPERIMENT

The normalized input impedance of a microwave resonant cavity can be written as[7,8]

$$z = z_s + \frac{1/\beta}{1/Q_a + j(\omega/\omega_a - \omega_a/\omega) + (1/\epsilon_0\omega)\int_V \mathbf{J}\cdot\mathbf{E}_a dv \Big/ \int_V \mathbf{E}\cdot\mathbf{E}_a dv}. \tag{10}$$

Here z_s is a series impedance including the losses in the coupling loops and the effects of resonant modes other than the dominant one. The coefficient β describes the degree of coupling between the transmission line and the fields of the dominant mode of the resonant cavity.

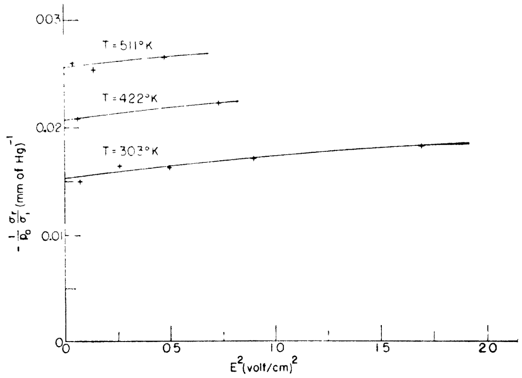

FIG. 3. Measured values of $\langle\sigma_r\rangle/p_0\langle\sigma_i\rangle$ as a function of the square of the electric field at three different gas temperatures for electrons in nitrogen.

[6] R. D. Crout, J. Math. Phys. **19**, 34 (1940).

[7] J. C. Slater, *Microwave Electronics* (D. Van Nostrand Company, New York, 1950), Chapter V.

[8] S. C. Brown *et al.*, "Methods of measuring properties of ionized gases at microwave frequencies," Technical Report No. 66, Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts.

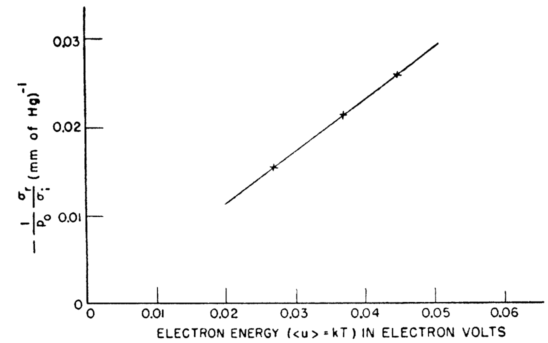

FIG. 4. Zero field value of the conductivity ratio, σ_r/σ_i, as a function of the electron energy.

Q_a and ω_a are the Q and resonant frequency of the cavity, ϵ_0 the permittivity of free space, $\mathbf{J}$ and $\mathbf{E}$ are the electron current and perturbed electric field in the cavity, and $\mathbf{E}_a$ is the electric field associated with the dominant mode. The magnitude of the electron current, J, is assumed small compared to the magnitude of the displacement current, $\omega\epsilon_0 E$, so that the electric field, $\mathbf{E}$, is very nearly equal to that of the dominant mode, $\mathbf{E}_a$.

Equation (10) shows that the effect of an electron current can be determined by measuring the change in resonant frequency and the change in conductance of a cavity at resonance. Substituting in Eq. (10) the definition of $\mathbf{J} = (\sigma_r + j\sigma_i)\mathbf{E}$, the change in resonant frequency is given by

$$\frac{2\Delta\omega}{\omega_a} = -\frac{1}{\epsilon_0\omega}\int_V \sigma_i E^2 dv \Big/ \int_V E^2 dv.$$

The change in cavity conductance from that of the empty cavity is given by

$$\Delta g \equiv \Delta(\beta/Q_a) = (\beta/\epsilon_0\omega)\int_V \sigma_r E^2 dv \Big/ \int_V E^2 dv,$$

so that we may write

$$\omega\Delta g/2\beta\Delta\omega = \int_V \sigma_r E^2 dv \Big/ \int_V \sigma_i E^2 dv = \langle\sigma_r\rangle/\langle\sigma_i\rangle. \tag{11}$$

In general σ_r and σ_i are functions of the electric field. Complications in the solution arising from non-uniformity in the electric field of the cavity are avoided by restricting the discussion to the zero-field values of σ_r/σ_i. These are obtained in practice by extrapolating the measured values of $\langle\sigma_r\rangle/\langle\sigma_i\rangle$ to zero electric field.

Measurement of the change in cavity conductance rather than the width of the resonance curve avoids the problem of changes in the electric field and in conductivity ratio occurring as the cavity impedance changes with the frequency. The electric field is determined in essentially the same manner as for breakdown measurements.[8] The coupling coefficient β is measured on the empty cavity.

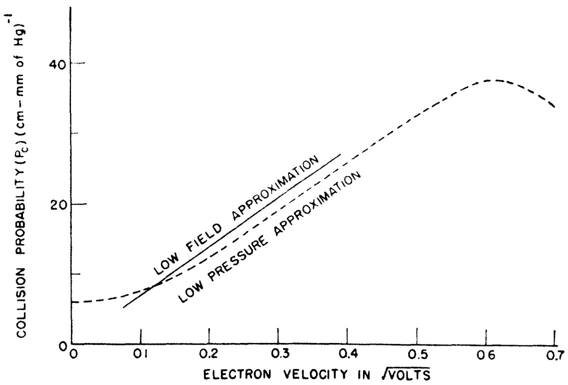

FIG. 5. Calculated values of the probability of collision for themal electrons in nitrogen.

The time required for the electrons to cool from the high average energies during the discharge to the temperature of the gas by loss of energy through elastic collisions can be estimated from average electron considerations. The rate of this energy loss is equal to the product of the fractional energy loss per collision, which is assumed to be $2m/M$ for monatomic gases; the average excess energy, $u-kT_g/e$; and the collision frequency,

$$du/dt = -(2m/M)(u-kT_g/e)\nu_c. \tag{12}$$

If the collision frequency is independent of the electron velocity, the time constant for the energy decay is $M/2m\nu_c$. As an example of the times involved, the time required for electrons in helium to cool to within 10 percent of thermal energies is about $90/p_0$ microseconds. Conductivity measurements were made at post-discharge times varying from about 0.2 millisecond in H_2 to 10 milliseconds in He.

A block diagram of the apparatus used to measure the conductivity ratio is shown in Fig. 2. A 10-cm pulsed magnetron is used to break down the gas periodically and provide a plasma of electrons whose conductivity is measured. The gas is contained in a resonant cavity operating in the TM_{010} mode. The change in the cavity impedance due to the electrons is measured by using a continuous wave tunable magnetron and a standing wave detector which is sensitive only for a period of a few microseconds during the afterglow. The transient standing wave detector[8] consists of a calibrated wave guide-beyond-cut-off attenuator followed by a superheterodyne receiver whose local oscillator frequency is controlled by a delayed sweep from a single sweep oscilloscope. The output of the receiver is observed on the single sweep oscilloscope and the standing wave ratio is determined by adjusting the calibrated attenuator to maintain constant amplitude at the output of the receiver. The delay circuit for the single sweep oscilloscope and the modulator for the pulsed magnetron are synchronized by a trigger generator operating at 60 cps. The output of the continuous wave magnetron is monitored with a cavity wavemeter and a bolometer. The power incident on the cavity is controlled by a calibrated variable attenuator.

IV. CONCLUSION

To illustrate the methods already discussed, we give preliminary results obtained from the microwave measurements of the conductivity of thermal electrons in nitrogen. Figure 3 shows measured values of $\langle\sigma_r\rangle/p_0\langle\sigma_i\rangle$ as a function of the square of the electric field in a cavity at three different gas temperatures. The zero field values of $\sigma_r/p_0\sigma_i$ from Fig. 3 are plotted as a function of the electron energy, $\langle u\rangle$, in Fig. 4. Since the conductivity ratios for these data were of the order of -0.1, $(\nu_c/\omega)^2\sim 0.01$ and the data may be analyzed using either the low pressure approximation or the low field approximation. The experimental points lie on a straight line through the origin and, over the range of velocities in which the distribution function is significant, both solutions give the same values of P_c to within the experimental error. These results are shown in Fig. 5 where the low field approximation gives $h=2$ and $P_c=70\sqrt{u}$. The low pressure approximation gives $P_c=6.0+160u-220u^2$ for $j=1$, 3, and 5.

Preliminary microwave measurements of the collision probability for electrons having a mean energy of 0.039 ev yield the following results for P_c at 1 mm of Hg pressure in units of cm^2/cm^3: He, 19; Ne, 3.3; A, 2.1; Kr, 54; Xe, 180; H_2, 46; and N_2, 15. The P_c values obtained for He, Ne and A are in good agreement with the theoretical calculations by Allis and Morse.[9]

The importance of the present method for determining collision probabilities lies in the fact that the microwave field may be used as a probe for measuring the conductivity ratio under conditions approaching thermal equilibrium between electrons and gas atoms. The advantage of using microwaves is that the radian frequency of the measuring field can be made appreciably larger than the average collision frequency of the electrons at pressures such that the mean free path of the electrons is small compared to the dimensions of a practical container.

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[9] W. P. Allis and P. M. Morse, Z. Physik **70**, 567 (1931).