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Elastic Scattering of Photons by Nuclei*

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The cross section for nuclear resonance scattering is estimated for the case when the photon energy is far from resonance with the excitation energy for any level of the scattering nucleus. For photon energy much less than 15 Mev we find that the cross section depends on the fourth power of the photon energy, and is very much smaller than the nuclear Thomson cross section for photon scattering by the nucleus as a whole.

I. INTRODUCTION

WILSON[1] has measured the elastic scattering of photons of 1.33 Mev (from Co^{60}) and 2.62 Mev (from ThC″). The purpose of his experiment is to find the Delbruck, or potential scattering, caused by the virtual production and annihilation of pairs in the coulomb field of the nucleus. The Delbruck scattering combines coherently with three other elastic scattering processes: Rayleigh scattering from the atomic electrons, Thomson scattering by the nucleus, and resonance scattering by the nucleus. It is necessary to know the amplitudes for the last three processes in order to interpret the experiments and find the value of the Delbruck scattering. The purpose of this note is to estimate the amplitude for nuclear resonance scattering.

II. NUCLEAR RESONANCE SCATTERING

There are two very different cases for nuclear resonance scattering of monoenergetic photons: (a) the photon energy is near a nuclear resonance energy, or (b) the photon energy is far from resonance. (By "far," we mean far enough from resonance so that the contribution of the nearest nuclear level is small compared to the contribution from all the other levels. 20 ev is about far enough.)

The first case, for nuclear resonance scattering by some particular level, has been studied by Guth,[2] and recently by Moon.[3] They find that the cross section for nuclear resonance scattering is of the order of 1 barn, if the photon energy is near resonance. At the moderate excitation energies used by Wilson the nuclear energy levels are widely spaced compared to 20 ev, so that it is unlikely that the photon energy is near a resonance. Further, nuclear resonance scattering would vary by several orders of magnitude, depending on whether a level of the scattering nucleus corresponded to the photon energy used. The other sources of scattering are smooth functions of atomic number and photon energy. Wilson has measured elastic scattering for several nuclei of similar atomic number (82 Pb, 83 Bi, 79 Au, 78 Pt), and has found only small variations of the scattering cross section with Z. This shows that in none of the cases measured is resonance scattering by some particular level of significance.

We therefore consider case (b), where the photon energy is far from resonance with any of the low-lying nuclear levels. Measurements[4] on photonuclear reactions indicate that nuclear levels which have large matrix elements for combination with the ground state lie mostly at an excitation energy of 15–20 Mev. (We are concerned in this note with nuclei of atomic number of at least 50, since these are the nuclei measured by Wilson.) The nuclear resonance scattering of a low energy gamma (such as 1.33 or 2.62 Mev) is mainly due to the "tails" of the resonance curves of the nuclear levels at 15 to 20 Mev.

The elastic scattering of photons by the charges bound in nuclei can be treated by the Kramers-Heisenberg dispersion formula.[5] The differential cross section is

$$d\sigma/d\Omega=(2\pi e\nu/c)^4\{\sum_n[(r_\sigma)_{0n}{}^*(r_\rho)_{n0}/(W-E_n) - (r_\rho)_{0n}{}^*(r_\sigma)_{n0}/(W+E_n)]\}^2. \quad (1)$$

Equation (1) applies to the special case of elastic scattering by dipole transitions to intermediate states. We have neglected the damping term in the resonance denominator since the photon energy is far from resonance. ν is the frequency of the photon and W its energy; $(r_\sigma)_{0n}{}^*$ and $(r_\rho)_{n0}$ are the matrix elements between the ground (0) and nth excited state of the coordinate components along the directions of polarization of the incident and scattered photon. (σ and ρ represent the two polarizations.) E_n is the excitation energy of the nth nuclear energy level. The scattering must be summed over the final polarizations and averaged over the initial polarizations; this gives the characteristic factor for dipole scattering of $\frac{1}{2}(1+\cos^2\theta)$, where θ is the scattering angle.

We rewrite Eq. (1) by combining the two fractions and introducing the oscillator strength as

$$f_{0n}=(2ME_n/\hbar^2)z_{0n}{}^*z_{n0}. \quad (2)$$

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[4] Johns, Katz, Douglas, and Haslam, Phys. Rev. **80**, 1062 (1950).

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We obtain

$$d\sigma/d\Omega = R_0^2 W^4 [\sum_n f_{0n}/(E_n^2 - W^2)]^2 \tfrac{1}{2}(1+\cos^2\theta). \quad (3)$$

Here z is the component of displacement of the charge, M is the proton mass, and $R_0 = e^2/Mc^2$ is the classical proton radius.

We can estimate the value of this expression by writing for the denominator an appropriately weighted average value E_a of the nuclear excitation energy for dipole transitions. Present measurements[4] suggest that $E_a = 15$ Mev might be an appropriate guess. The summed oscillator strength for dipole transitions for a nucleus with Z protons and N neutrons, with an exchange potential providing a fraction x of the attractive neutron-proton potential has been calculated[6] as

$$\sum_n f_{0n} = (NZ/A)(1+0.8x). \quad (4)$$

This calculation is in reasonable agreement with experimental measurements of the integrated photonuclear cross section. Using these expressions for the denominator and numerator we find the differential cross section for nuclear resonance scattering as

$$d\sigma/d\Omega = R_0^2 (W/E_a)^4 [(NZ/A)(1+0.8x)]^2 \tfrac{1}{2}(1+\cos^2\theta). \quad (5)$$

We note that this has the same W^4 dependence on photon energy as that for the Rayleigh scattering of visible light. In both cases the photon energy is far below the energies of the significant intermediate states.

It is of interest to compare the cross section for nuclear resonance scattering with that for nuclear Thomson scattering, which is given by

$$(d\sigma/d\Omega)_{\mathrm{Th}} = (Z^2e^2/AMc^2)^2 \tfrac{1}{2}(1+\cos^2\theta) = R_0^2 (Z^2/A)^2 \tfrac{1}{2}(1+\cos^2\theta). \quad (6)$$

This expression is analogous to the well-known formula $(1/2) r_0^2 (1+\cos^2\theta)$ for Thomson scattering by free electrons. The electron charge e is replaced by Ze, and the electron mass m by the nuclear mass AM. The form factor for the nucleus is taken as unity since for the photon energies considered the photon wavelength is much greater than the nuclear radius.

The ratio of the cross section for nuclear resonance scattering to that for nuclear Thomson scattering is then

$$(d\sigma/d\Omega)/(d\sigma/d\Omega)_{\mathrm{Th}} = (W/E_a)^4 [(N/Z)(1+0.8x)]^2. \quad (7)$$

The cross section for nuclear resonance scattering is negligible for the photon energies used by Wilson (1.33 Mev and 2.62 Mev) for which the ratio $(W/E_a)^4$ is extremely small. However, since the amplitudes for nuclear resonance scattering and nuclear Thomson scattering combine coherently, we should rather compare their amplitudes (a_{NR} and a_{Th}). If we consider the hypothetical case where these are the only scattering processes (most nearly valid for large scattering angles), the scattering by the nucleus is proportional to

$$a^2 = (a_{\mathrm{NR}} + a_{\mathrm{Th}})^2 \cong a_{\mathrm{Th}}^2 [1 - 2(W/E_a)^2 (N/Z)(1+0.8x)]. \quad (8)$$

(The amplitude for nuclear resonance scattering has a sign opposite to that for nuclear Thomson scattering since for the former the charged particles are tightly bound, and for the latter they are free.) We have used $a_{\mathrm{NR}}/a_{\mathrm{Th}} \ll 1$. Using photon energy $W = 2.62$ Mev, nuclear excitation energy $E_a = 15$ Mev, fraction of exchange force $x = \frac{1}{2}$, and values of N and Z for lead we find

$$a^2 = a_{\mathrm{Th}}^2 (1 - 0.12) = 0.88 a_{\mathrm{Th}}^2. \quad (9)$$

The effect of nuclear resonance scattering is seen to be barely appreciable in Wilson's experiments at 2.62 Mev, and is negligible at 1.33-Mev photon energy.

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[6] J. S. Levinger and H. A. Bethe, Phys. Rev. **78**, 115 (1950).