On Bailey's Theory of Amplified Circularly Polarized Waves in an Ionized Medium*

R. Q. Twiss
Services Electronics Research Laboratory, Baldock, Herts, England

A critical analysis is given of Bailey's theory of propagation in an ionized medium. It is shown that the growing waves, which Bailey interprets as amplified waves, can only be excited by reflection and it is argued that this theory can explain neither the excess radiation observed from sunspots nor the excess noise observed in discharge tubes. However power amplification is possible in a drifting ionized medium under certain ideal conditions which are analyzed.

1. INTRODUCTION

In a series of recent papers, V. A. Bailey[1–3] has developed a theory for the amplification of circularly polarized waves in an ionized medium moving, with a drift velocity u_0, in a dc magnetic field; and he has applied this theory to explain the excess noise radiation observed in sunspots.[4]

In this paper we shall discuss in some detail the criticisms of this theory, which we have outlined elsewhere.[5] In particular, we shall show that the growing[6] electromagnetic wave which is interpreted as an amplified wave by Bailey is, in fact, a reflected wave which can be excited only at a surface of discontinuity in the medium.

Despite this it is ideally possible to find physically realizable boundary conditions such that the medium provides a definite power amplification over those frequency bands where this growing wave exists, and we have analyzed the physical mechanism by which the energy associated with this amplification is transferred to the electromagnetic field from the dc kinetic energy of the electron drift velocity. It is extremely improbable, however, that these conditions could be encountered in a sunspot, and we conclude that the explanation of the excess noise radiation must lie elsewhere.

In order to prove that a particular wave can be excited only by reflection, it is necessary to obtain a transient solution for the electromagnetic propagation in a moving ionized medium, which specifically takes into account the boundary conditions. Instead of attempting to do this by modifying Bailey's analysis, it seemed simpler to develop a direct solution with the aid of the Fourier-Laplace transform theory and then to compare the conclusions drawn from the steady-state and transient analysis with those obtained by Bailey.

The analysis in this paper has been developed under rather special limiting assumptions. However some of the conclusions are of more general application and, in order to develop these, we have discussed the physical nature of the propagation in some detail.

2. THE BOUNDARY AND INITIAL CONDITIONS

In his original paper,[2] Bailey considers the complex case of electromagnetic propagation at an arbitrary angle to the direction of motion of a two-beam plasma acted on by an external magnetic field. However, when applying this theory to the sunspot radiation, he deals only with the unidimensional case where the direction of propagation, of drift velocity and of magnetic field are all parallel, and we shall make the same restriction here.

It is customary, in the theory of propagation in a plasma or electron gas, to separate the disturbance in the medium into longitudinal and transverse oscillations, a procedure which is rigorously justifiable in the unidimensional case if, as we assume, the disturbance is so small that nonlinear effects are negligible. In this paper we consider only the transverse fields and ignore the longitudinal oscillations altogether.

The boundary conditions we shall use are illustrated graphically in Fig. 1. We assume that a plane electromagnetic wave of angular frequency Ω_0 is normally incident, at time $t=0$, upon the interface at $z=0$ between media I and II. Medium II contains a uniform electron stream, or plasma with infinitely massive positive ions, moving with velocity $\mathbf{u}_0=(0, 0, u_0)$ under the action of an external magnetic field of flux density $\mathbf{B}=(0, 0, B_0)$, which is bounded by surfaces of discontinuity at $z=0$ and $z=d$. In order to simplify the algebra we shall assume that any free charges in medium I are stationary. This restriction will not materially affect the nature of the solution, and does not apply to medium III which can contain free moving charge.

It is further necessary to make some assumptions about the initial transverse velocity modulation. A number of alternatives exist, but a complete discussion of these is beyond the scope of this paper. Instead we shall make the simplest choice that is physically plausible, and assume that this modulation is independent of the incident electromagnetic field over the surface at which the electrons enter medium II that is at $z=0$ for $u_0>0$ and at $z=d$ for $u_0<0$.

The boundary conditions described above are reasonably close to those characteristic of an idealized unidi-

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[1] V. A. Bailey, Nature **161**, 599 (1948).
[2] V. A. Bailey, J. Roy. Soc. N.S.W. **82**, 107 (1948).
[3] V. A. Bailey, Aust. S. Sci. Res. (A) **1**, 351 (1948).
[4] V. A. Bailey, Phys. Rev. **78**, 428 (1950).
[5] R. Q. Twiss, Phys. Rev. **80**, 767 (1950).
[6] By a growing wave we mean any wave the amplitude of which increases exponentially in the positive direction whether it be an amplified wave or a reflected wave attenuated in the negative direction.

mensional electron tube, where medium I might be a thermonic cathode emitting a stream of electrons with velocity u_0 while medium III might be an anode structure. They would also be applicable to the case where medium I was formed by a uniform stream of neutral gas molecules, which was partially ionized at the surface $z=0$, and in which partial recombination or further ionization took place at $z=d$. However they give at best a very idealized approximation to what we might expect to prevail in a gas discharge tube where the drift velocity u_0 is likely to be small compared with the root-mean-square thermal velocity, or in a sunspot where well-defined surfaces of discontinuity would hardly exist.

3. MATHEMATICAL THEORY

It is well known from the theory of the ionosphere that an arbitrary plane wave is split into two independent circularly polarized waves by an axial magnetic field. Accordingly we look for the transient response of the medium of Fig. 1 when a circularly polarized electromagnetic wave, with

$$\mathbf{E}=(E(\Omega_0)\cos\Omega_0 t,\ E(\Omega_0)\sin\Omega_0 t,\ 0) \tag{1}$$

is normally incident, at time $t=0$, upon the interface between media I and II at $z=0$.

When Ω_0 is negative we have a left-hand or counterclockwise wave, when Ω_0 is positive a right-hand or clockwise wave.

In the unidimensional case, which we are considering, the field variables $\mathbf{E}$, $\mathbf{H}$ and the ac transverse velocity $\mathbf{v}(z,t)$ are independent of the transverse co-ordinates while, since the longitudinal oscillations are not excited, we have for the time dependent fields

$$E_z(z,t)=H_z(z,t)=u_z(z,t)=\rho(z,t)=0. \tag{2}$$

In medium II where the dc magnetic flux density is $\mathbf{B}_0=(0,0,B_0)$ and the dc electron velocity is $\mathbf{u}_0=(0,0,u_0)$ the transverse fields satisfy the Lorentz force equation

$$\left(\frac{\partial}{\partial t}+u_0\frac{\partial}{\partial z}+\nu\right)m\mathbf{v}(z,t) = -e\mathbf{E}(z,t)-e\mu_0\mathbf{u}_0\times\mathbf{H}(z,t)-e\mathbf{v}(z,t)\times\mathbf{B}_0 \tag{3}$$

and the maxwell equations

$$\begin{aligned}\nabla\times\mathbf{E}(z,t)&=-\partial\mathbf{B}(z,t)/\partial t;\\ \nabla\times\mathbf{H}&=\rho_0\mathbf{v}(z,t)+\partial\mathbf{D}(z,t)/\partial t,\end{aligned} \tag{4}$$

where ρ_0 is the dc space charge density, ν is the collision frequency, μ_0 is the magnetic permeability of free space, $-e$ is the electron charge, and $m=m_0/(1-u_0^2/c^2)^{\frac{1}{2}}$ is the relativistic electron mass. The other maxwell equations and the charge conservation equation are automatically satisfied for the transverse unidimensional case.

The solution of Eqs. (3), (4) is much simplified if, following Stratton,[7] we use a complex algebra and write

[7] J. A. Stratton, *Electromagnetic Theory* (McGraw-Hill Book Company, Inc., New York, 1941), Sec. 5.16.

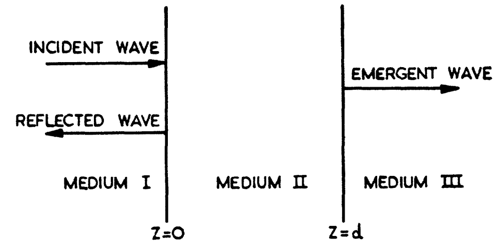

FIG. 1. Boundary conditions.

$$E(z,t)=E_x(z,t)+iE_y(z,t);\quad H(z,t)=H_x(z,t)+iH_y(z,t);$$
$$V(z,t)=v_x(z,t)+iv_y(z,t) \tag{5}$$

so that the incident wave at $z=0$ is

$$E_{\text{in}}(0,t)=E(\Omega_0)\cos\Omega_0 t+iE(\Omega_0)\sin\Omega_0 t =E(\Omega_0)\exp(i\Omega_0 t). \tag{6}$$

Equations (3), (4) may then be written

$$\left.\begin{aligned}&\left(\frac{\partial}{\partial t}+u_0\frac{\partial}{\partial z}+\nu\right)mV(z,t)\\ &\qquad=-eE(z,t)+ieB_0V(z,t)-ieu_0\mu_0H(z,t),\\ &\qquad\frac{\partial E(z,t)}{\partial z}-i\mu_0\frac{\partial}{\partial t}H(z,t)=0;\\ &\qquad i\frac{\partial}{\partial z}H(z,t)=\rho_0V(z,t)+\epsilon_0\frac{\partial}{\partial t}E(z,t).\end{aligned}\right\} \tag{7}$$

To solve these equations under the given initial conditions we take the fourier-laplace transforms first with respect to t, and then with respect to z, where

$$\left.\begin{aligned}&L_t\{f(z,t)\}=f^*(z,\omega)=\int_0^\infty f(z,t)\exp(-i\omega t)dt,\\ &L_t^{-1}\{f^*(z,\omega)\}=f(z,t)\\ &\qquad=(2\pi)^{-1}\int_{-\infty-i\gamma}^{+\infty-i\gamma}f^*(z,\omega)\exp(i\omega t)d\omega\end{aligned}\right\} \tag{8}$$

and

$$\left.\begin{aligned}&L_z\{f^*(z,\omega)\}=f^{*\dagger}(k,\omega)\\ &\qquad=\int_0^\infty f^*(z,\omega)\exp(-ikz)dz,\\ &L_z^{-1}\{f^{*\dagger}(k,\omega)\}=f^*(z,\omega)\\ &\qquad=(2\pi)^{-1}\int_{-\infty-i\gamma}^{+\infty-i\gamma}f^{*\dagger}(k,\omega)\exp(ikz)dk.\end{aligned}\right\} \tag{9}$$

γ is a positive real number such that all the singularities of $f^{*\dagger}(k,\omega)$ lie above the line $\text{Im}(k)+\gamma=0$ and all the singularities of $f^*(z,\omega)$ lie above the line $\text{Im}(\omega)+\gamma=0$.

If this is done we obtain a system of linear algebraic equations for $E^{*\dagger}(k,\omega)$, $H^{*\dagger}(k,\omega)$, and $V^{*\dagger}(k,\omega)$ which may be solved, in terms of $E^*(0,\omega)$, $H^*(0,\omega)$, and $V^*(0,\omega)$ to give

$$E^{*\dagger}(k,\omega)=\frac{\left[ik+\dfrac{u_0\omega_0^2}{c^2(\omega+u_0k-\omega_H-i\nu)}\right]E^*(0,\omega)-\omega\mu_0H^*(0,\omega)-\left[\dfrac{\omega\mu_0u_0\rho_0}{\omega+u_0k-\omega_u-i\nu}\right]V^*(0,\omega)}{-k^2+(\omega^2/c^2)-(\omega_0^2/c^2)(\omega+u_0k)/(\omega+u_0k-\omega_H-i\nu)},\tag{10}$$

$$H^{*\dagger}(k,\omega)=(-k/\omega\mu_0)iE^{*\dagger}(k,\omega)+[E^*(0,\omega)/\omega\mu_0],\tag{11}$$

$$V^{*\dagger}(k,\omega)=\frac{\omega+u_0k}{\omega+u_0k-\omega_H-i\nu}\,\frac{e}{\omega m}iE^{*\dagger}(k,\omega)+\frac{u_0V^*(0,\omega)-(u_0/\omega)H^*(0,\omega)}{\omega+u_0k-\omega_H-i\nu},\tag{12}$$

where

$$\omega_0=(-e\rho_0/\epsilon_0m)^{\frac{1}{2}}\tag{13}$$

is the plasma angular frequency, and

$$\omega_H=eB_0/m\tag{14}$$

is the cyclotron angular frequency.

The initial constants $E^*(0,\omega)$, $H^*(0,\omega)$, and $V^*(0,\omega)$ are to be determined by the boundary conditions which can be applied after inversion from the complex k-plane onto the real z-axis. In the present case, where $E^{*\dagger}(k,\omega)$ is an algebraic function of k which is $O(k^{-1})$ as $k\to\infty$, the inversion can be carried out immediately from Eq. (9) by aid of the theory of residues. We have

$$\left.\begin{aligned}E^*(z,\omega)&=\sum_{n=1}^{3}\exp(ik_nz)[a_n(\omega)E^*(0,\omega)\\&\qquad+b_n(\omega)iH^*(0,\omega)+c_n(\omega)V^*(0,\omega)],\\-iH^*(z,\omega)&=\sum_{n=1}^{3}\frac{1}{Z_n(\omega)}\exp(ik_nz)[a_n(\omega)E^*(0,\omega)\\&\qquad+b_n(\omega)iH^*(0,\omega)+c_n(\omega)V^*(0,\omega)],\end{aligned}\right\}\tag{15}$$

where

$$\left.\begin{aligned}a_n(\omega)&=\left[k_n\left(k_n+\frac{\omega-\omega_H-i\nu}{u_0}\right)+\frac{\omega_0^2}{c^2}\right]\Big/\prod_{m\neq n=1}^{3}(k_n-k_m),\\b_n(\omega)&=\omega\mu_0\left(k_n+\frac{\omega-\omega_H-i\nu}{u_0}\right)\Big/\prod_{m\neq n=1}^{3}(k_n-k_m),\\c_n(\omega)&=\omega\mu_0\rho_0\Big/\prod_{m\neq n=1}^{3}(k_n-k_m),\end{aligned}\right\}\tag{16}$$

where k_n is one of the three roots of the characteristic equation

$$\left(k+\frac{\omega-\omega_H-i\nu}{u_0}\right)\left(k^2-\frac{\omega^2}{c^2}\right)+\frac{\omega_0^2}{c^2}\left(k+\frac{\omega}{u_0}\right)\equiv\prod_{n=1}^{3}(k-k_n)=0\tag{17}$$

and where

$$Z_n(\omega)=-\omega\mu_0/k_n\tag{18}$$

is the characteristic impedance of the wave with propagation constant k_n.

In the complex algebra of our present treatment, the impedance of a medium looking in the positive direction across a transverse plane may be written[8]

$$Z(\omega)=E^*(z,\omega)/-iH^*(z,\omega)\tag{19}$$

and the boundary condition on the electromagnetic field components is satisfied at a surface of discontinuity if there is an impedance match there.

Now from Eqs. (6) and (8) the fourier-laplace transform with respect to time of the incident electromagnetic wave is

$$E_{\text{in}}^*(\omega)=E(\Omega_0)/i(\omega-\Omega_0).\tag{20}$$

If in medium I there is just one wave reflected from the surface of discontinuity at $z=0$, then the condition for an impedance match at this surface is

$$-Z_{\text{I}}=\frac{E_{\text{in}}^*(\omega)-E^*(0,\omega)}{E_{\text{in}}^*(\omega)/Z_{\text{I}}+iH^*(0,\omega)},\tag{21}$$

where Z_{I} is the impedance of medium I, and $E_{\text{in}}^*(\omega)$ is given by Eq. (21). Similarly the boundary condition at $z=d$ gives the equation

$$Z_{\text{II}}=\frac{E^*(d,\omega)}{-iH^*(d,\omega)}=\frac{\sum_{n=1}^{3}\exp(ik_nd)[a_nE^*(0,\omega)+b_niH^*(0,\omega)+c_nV^*(0,\omega)]}{\sum_{n=1}^{3}\exp(ik_nd)[a_nE^*(0,\omega)+b_niH^*(0,\omega)+c_nV^*(0,\omega)]/Z_n(\omega)}.\tag{22}$$

[8] This definition is the natural one to use in the present case where we wish to utilize the connection of the impedance concept with the Poynting vector. Under other circumstances it would be useless, as the impedance so defined is dependent on the relative magnitude of the amplitudes of the electromagnetic and space charge velocity fields.

When $u_0>0$ we assume that $V^*(0,\omega)$ is independent of $E^*(0,\omega)$. It would be permissible to take it as zero, but in order to see what happens when the medium is excited by an initial velocity modulation rather than by an incident electromagnetic wave we shall take

$$V(0,t)=V(\Omega_0)\exp(i\Omega_0 t) \tag{23}$$

so that

$$V^*(0,\omega)=V(\Omega_0)/[i(\omega-\Omega_0)]. \tag{24}$$

When $u_0<0$ we assume that $V^*(d,\omega)=V(\Omega_0)/[i(\omega-\Omega_0)]$ is independent of $E(\Omega_0)$.

If we solve Eqs. (21), (22) for the unknowns $E^*(0,\omega)H^*(0,\omega)$ we find that

$$\left.\begin{aligned} E^*(z,\omega)&=\sum_{n=1}^{3}\frac{A_n(\omega)}{i(\omega-\Omega_0)}\exp(ik_nz),\\ -iH^*(z,\omega)&=\sum_{n=1}^{3}\frac{1}{Z_n(\omega)}\frac{A_n(\omega)}{i(\omega-\Omega_0)}\exp(ik_nz),\end{aligned}\right\} \tag{25}$$

where

$$A_n(\omega)=\frac{\sum_{m=1}^{3}(1-Z_{\mathrm{II}}/Z_m)[2(a_nb_m-a_mb_n)E(\Omega_0)+i\{c_n(b_n+Z_{\mathrm{I}}a_m)-c_m(b_n+Z_{\mathrm{I}}a_n)\}V(\Omega_0)]\exp(ik_md)}{\sum_{l=1}^{3}(1-Z_{\mathrm{II}}/Z_l)(b_l+Z_{\mathrm{I}}a_l)\exp(ik_ld)} \tag{26}$$

for $u_0>0$.

The solution for the reverse beam, where $u_0<0$, is of a similar although not identical form.

The steady-state solution, assuming that this exists, can immediately be written down from Eq. (25), and we have

$$\left.\begin{aligned} E(z,t)&\to\sum_{n=1}^{3}A_n(\Omega_0)\exp[i(k_n(\Omega_0)z+\Omega_0t)]\\ &\qquad\qquad\text{as } t\to\infty,\\ -iH(z,t)&\to\sum_{n=1}^{3}[A_n(\Omega_0)/Z_n(\Omega_0)]\\ &\qquad\times\exp[i(k_n(\Omega_0)z+\Omega_0t)]\ \text{as } t\to\infty,\end{aligned}\right\} \tag{27}$$

where $k_n(\Omega_0)$ are the roots of the characteristic Eq. (17) which may be written in the more familiar form

$$-k^2+\frac{\Omega_0^2}{c^2}-\frac{\omega_0^2}{c^2}\frac{\Omega_0+u_0k}{\Omega_0+u_0k-\omega_H-i\nu}=0. \tag{28}$$

Of course, the limiting expression of Eq. (27) is only valid if the moving plasma flow is stable. For the moment let us consider that this is so and analyze the steady state solution and Bailey's interpretation of it, before returning to the transient analysis.

4. THE STEADY-STATE ANALYSIS AND BAILEY'S THEORY

(i) The Propagation Constants

When $u_0=0$, the familiar ionosphere case, the characteristic equation, Eq. (28), reduces to

$$-k^2+\frac{\Omega_0^2}{c^2}-\frac{\omega_0^2}{c^2}\frac{\Omega_0}{\Omega_0-\omega_H}=0, \tag{29}$$

where we have also neglected the effects of scattering by taking $\nu=0$.

In this case there are but two partial waves with the same polarization, one of which is interpreted as a wave reflected from the far side of the ionosphere, the plane $z=d$ of Fig. 1. Since there are two possible directions of polarization there are four partial waves in all. When there is no external magnetic field, so that $\omega_H=0$, the propagation constant is independent of the direction of polarization. However, when $\omega_H\neq0$ one distinguishes between the ordinary waves, the polarization of which rotates in the opposite sense to that of an electron moving in the external magnetic field, and the extraordinary waves, the polarization of which rotates in the same sense as an electron; the waves are extraordinary if $\Omega_0\omega_H>0$ and ordinary if $\Omega_0\omega_H<0$. From Eq. (1), the right-hand waves are ordinary if $\omega_H<0$, extraordinary if $\omega_H>0$ and the left-hand waves are extraordinary if $\omega_H<0$ ordinary if $\omega_H>0$.

From Eq. (29) the propagation constants may be written

$$k=\pm i\alpha=\pm i\{[\Omega_0/(\Omega_0-\omega_H)][\omega_0^2-\Omega_0(\Omega_0-\omega_H)]\}^{\frac{1}{2}}/c \tag{30}$$

hence, for the ordinary waves, where $\Omega_0\omega_H<0$, there is an attenuation band, where k is pure imaginary, in the range

$$0\leqslant|\Omega_0|<\tfrac{1}{2}[(\omega_H^2+4\omega_0^2)^{\frac{1}{2}}-|\omega_H|], \tag{31}$$

and a pass band, where k is real, in the range

$$\tfrac{1}{2}[(\omega_H^2+4\omega_0^2)^{\frac{1}{2}}-|\omega_H|]\leqslant|\Omega_0|<\infty; \tag{32}$$

while for the extraordinary waves, where $\Omega_0\omega_H>0$, there is an attenuation band

$$|\omega_H|<|\Omega_0|<\tfrac{1}{2}[(\omega_H^2+4\omega_0^2)^{\frac{1}{2}}+|\omega_H|], \tag{33}$$

and two pass bands

$$0<|\Omega_0|<|\omega_H|$$

and

$$\tfrac{1}{2}[(\omega_H^2+4\omega_0^2)^{\frac{1}{2}}+|\omega_H|]<|\Omega_0|<\infty. \tag{34}$$

When the electrons possess a drift velocity $u_0 \ll c$ the critical frequencies which define the edges of the pass and attenuation bands are slightly altered, as are the propagation constants of these fields waves. In addition there is now a third wave present with a propagation constant that is real for all frequencies. Except in the immediate neighborhood of the critical frequencies the propagation constant $k_2(\Omega_0)$, of this wave is given by

$$k_2(\Omega_0) = -(\Omega_0 - \omega_H)/u_0 \tag{35}$$

to the first order in u_0/c.

It is thus a space charge wave that propagates with a velocity very nearly equal to that of the electron stream. Its analog, in the ionosphere case, is a fixed spatial distribution of transverse velocity modulation, which would not appear as a solution.

Except near the critical frequencies, the propagation constants of the field waves can be found from Eq. (28) by a perturbation method such as Newton's rule, and if

$$k_1 = -i\alpha + \beta; \quad k_3 = i\alpha + \beta, \tag{36}$$

where $i\alpha$ is given by Eq. (30), then for $(u_0/c)^2 \ll 1$

$$\beta = u_0 \omega_H \omega_0^2 / [2c^2(\Omega_0 - \omega_H)^2]. \tag{37}$$

It will be noted that the characteristic roots are numbered in such a way that k_1 and k_3 are the propagation constants of the growing and decaying waves respectively, so that

$$\mathrm{Re}(ik_1) \geqslant \mathrm{Re}(ik_2) \geqslant \mathrm{Re}(ik_3).$$

From Eq. (37), we see that β is always real so that the effect of a small drift velocity of the medium is to turn the field waves in the attenuation band from pure evanescent into growing and decaying waves that propagate at a very high phase velocity $O(c^2/u_0)$. Except near the critical frequencies, the imaginary part of the propagation constant is unaltered to the first order in (u_0/c).

When we allow for the presence of scattering, so that ν in Eq. (28) is not zero, we have that

$$\left.\begin{aligned}
k_2(\Omega_0) &= (i\nu/u_0) - (\Omega_0 - \omega_H)/u_0, \\
k_1(\Omega_0) &= -i\alpha + \frac{\omega_0^2}{2c^2(\Omega_0-\omega_H)^2}\frac{\Omega_0\nu}{\alpha} + \frac{u_0\omega_H\omega_0^2}{2c^2(\Omega_0-\omega_H)^2}, \\
k_3(\Omega_0) &= +i\alpha - \frac{\omega_0^2}{2c^2(\Omega_0-\omega_H)^2}\frac{\Omega_0\nu}{\alpha} + \frac{u_0\omega_H\omega_0^2}{2c^2(\Omega_0-\omega_H)^2},
\end{aligned}\right\} \tag{38}$$

so that the space charge wave is attenuated as $\exp(-\nu z/u_0)$, while, in the pass band, the forward traveling field wave becomes a decaying wave and the backward traveling wave becomes a growing wave; on the other hand, in the attenuation band, the velocity of propagation of the growing wave is made more negative and that of the decaying wave more positive.

(ii) Bailey's Interpretation

All this is in accord with the conclusion that the growing wave is a reflected wave, but is opposed to Bailey's interpretation, based entirely upon the direction of propagation, that it is a directly excited amplified wave. From Eq. (38) we see that if

$$\nu < |u_0 \alpha \omega_H / \Omega_0| \tag{39}$$

and if

$$u_0 \omega_H / \Omega_0 < 0 \tag{40}$$

then the growing wave propagates in the positive direction. It is shown by Bailey that the direction of real energy flow associated with any individual wave is the same as its direction of propagation. Hence, when the inequalities (39) and (40) are satisfied, the energy flow associated with the growing wave is out of the medium at the surface of discontinuity at $z=d$; and Bailey concludes that the growing wave is not reflected but excited directly at the incident surface $z=0$, and that the energy can escape if the medium beyond $z=d$ is either amplifying or transparent at this frequency. Since $\omega_H \Omega_0 < 0$ for the ordinary waves, we see that, on this theory, they are amplified if

$$u_0 > 0$$

and that the extraordinary waves are amplified if

$$u_0 < 0.$$

It must be admitted that this argument from the direction of energy flow has been widely regarded as conclusive in determining whether a given partial wave is amplified or no; nevertheless it is insufficient. The proof of this, given in the next section, is based upon the transient analysis; but the importance of the question and its relevance to cases other than that considered by Bailey, may justify its consideration from the more familiar steady-state analysis as well.

(iii) Criticisms of Bailey's Theory

The most striking conclusion from Bailey's theory is that the presence of even the smallest axial magnetic field in a drifting electron gas is sufficient to transform it from an attenuating into an amplifying medium where the rate of growth of a disturbance is nearly equal to the rate of decay in the absence of the magnetic field. It is true that, when scattering is taken into account, the magnetic field has to attain a finite value before the medium becomes amplifying, given, from inequality (39), by the inequality

$$|\omega_H| > |\Omega_0 \nu / \alpha u_0|. \tag{41}$$

But even so, an infinitesimal change in magnetic field transforms a reflected wave into an amplified wave.

A second criticism is suggested by relativity considerations: the dependence of the wave propagation on the uniform motion of the medium implies a special frame of reference. This can only be provided by the surfaces of discontinuity in the medium. For were these to move with the medium the propagation would be the same, except for a Lorentz transformation, as in the stationary ionosphere.

Accordingly the surfaces of discontinuity play a fundamental role in the excitation of these growing waves, which is in direct contrast to the familiar amplification process in a traveling wave tube or space charge wave amplifier where the nature of the initial and terminal impedance is comparatively unimportant.[9]

Finally we see from Eqs. (23)–(25) that the amplitudes of the three partial waves at the initial plane $z=0$ depend upon the boundary conditions at $z=d$, which strongly implies that one of the partial waves is reflected, while if

$$Z_{\mathrm{II}} \neq Z_{\mathrm{I}}(\Omega_0) \quad \text{and} \quad Z_{\mathrm{I}} \neq -(b_1/a_1) \tag{42}$$

then the amplitude $A_1(\Omega_0)$ of the growing wave is proportional to $\exp[-i(k_1-k_2)d]$, which is hardly compatible with the assumption that this wave is directly excited.

However, when the inequalities (42) are not satisfied, an electromagnetic wave incident at $z=0$ will be amplified as it crosses the medium; to show that this result is consistent with the claim that the growing wave is reflected it seems essential to use the transient analysis.

5. THE TRANSIENT ANALYSIS

There is no need to obtain a full transient solution to decide whether a given partial wave is reflected or not. Instead let us consider the term

$$[A_1(\omega)/i(\omega-\Omega_0)]\exp(ik_1z)$$

of $E^*(z,\omega)$ in Eq. (22), which corresponds to the growing wave, and ask how large t must be before this term contributes to $E(z,t)$. Now $E(z,t)$ is identically zero for $t<z/c$, since this is the minimum time taken by a signal, applied at $z=0$, $t=0$, to reach z. By the same token the necessary and sufficient condition that a given term of $E^*(z,\omega)$ should correspond to a reflected wave is that its contribution to $E(z,t)$ be identically zero, at least until a time $t \geqslant (2d-z)/c$ where d is the distance of the first reflecting surface from the initial plane $z=0$.

From the theory of the fourier-laplace transform, it is known that the inverse transform $f(t)$ of a meromorphic function $f^*(\omega)$ such that

$$f(\omega) \sim \exp\left(\frac{-i\omega\xi}{v}\right)\left[\frac{a_1}{\omega}+\frac{a_2}{\omega^2}+\cdots\right] \tag{43}$$

as $\omega \rightarrow \infty$, is identically zero for all $t<\xi/v$. Hence the question as to whether or not a particular partial wave is reflected is determined by its asymptotic behavior for large enough ω.

[9] In the traveling wave tube, a frame of reference is provided by the external helical structure, while in the space charge wave amplifier there is relative motion between the various electron beams.

TABLE I. Asymptotic approximations for the parameters of Eq. (26) as $\omega \rightarrow \infty$.

	$n=1$	$n=2$	$n=3$
k_n	$(\omega/c)(1-\omega_0^2/\omega^2)$	$-(\omega-\omega_H)/u_0$	$-(\omega/c)(1-\omega_0^2/\omega^2)$
Z_n	$-Z_0(1+\omega_0^2/\omega^2)$	u_0Z_0/c	$Z_0(1+\omega_0^2/\omega^2)$
A_n	$1/2$	$\omega_H\omega_0^2/\omega^3$	$1/2$
b_n	$Z_0/2$	$Z_0(u_0^3\omega_0^2\omega_H/c^3\omega^3)$	$-Z_0/2$

From Eqs. (6), (17) one can build up Table I, where $Z_0=(\mu_0/\epsilon_0)^{\frac{1}{2}}$ is the characteristic impedance of free space, and where we have ignored terms of $\mathrm{O}(u_0/c)$ and taken $\nu=0$.

With these approximations the denominator of $A_n(\omega)$ in Eq. (26) is given for large enough ω by

$$\left\{\frac{Z_0}{2}\left(1+\frac{Z_{\mathrm{I}}}{Z_0}\right)\left(1+\frac{Z_{\mathrm{II}}}{Z_0}\right)\exp\left(\frac{i\omega d}{c}\right) - Z_{\mathrm{II}}\frac{u_0^2\omega_0^2\omega_H}{c^2\omega^3}\exp\left[\frac{-i(\omega-\omega_0)d}{u_0}\right] - \frac{Z_0}{2}\left(1-\frac{Z_{\mathrm{I}}}{Z_0}\right)\left(1-\frac{Z_{\mathrm{II}}}{Z_0}\right)\exp\left(\frac{-i\omega d}{c}\right)\right\}^{-1}. \tag{44}$$

If Z_{I} and Z_{II} are physically realizable, they must have positive real parts, and therefore

$$\left|\frac{Z_0}{2}\left(1+\frac{Z_{\mathrm{I}}}{Z_0}\right)\left(1+\frac{Z_{\mathrm{II}}}{Z_0}\right)\right|>0.$$

If $\mathrm{Im}(\omega)$ is sufficiently negative, so that

$$\mathrm{Re}[\exp(i\omega d/c)]$$

is sufficiently large, one can expand this expression as a convergent binomial series of the form

$$\left[\frac{Z_0}{2}\left(1+\frac{Z_{\mathrm{I}}}{Z_0}\right)\left(1+\frac{Z_{\mathrm{II}}}{Z_0}\right)\right]^{-1}\exp\left(\frac{-i\omega d}{c}\right) + \text{terms of order } \exp\left[-i\left(\frac{n'\omega d}{c}+\frac{m'\omega d}{u_0}\right)\right],$$

where $n'+m' \geqslant 2$. Now the γ, in Eq. (9) which defines the contour for the inverse transforms, may assume any finite value; in particular it may be chosen so large that the above expansion is justified on and within the closed contour formed by the line $\mathrm{Im}(\omega)+\gamma=0$ and the lower infinite half-circle.

Hence we can express

$$[A_1(\omega)/i(\omega-\Omega_0)]\exp(ik_1z)$$

by an asymptotic series of the form

$$\sum_{n,m} a_{n,m}(\omega)\exp(i\omega/c)[z-(nd+mcd/u_0)], \tag{45}$$

where $a_{n,m}(\omega)$ is a power series in ω^{-1} and where

$n+m \geqslant 2$. A given term of this series contributes to $E(z,t)$ only if

$$t > (n/c + m/u_0)d - z/c, \quad \text{where } n+m \geqslant 2,$$

and therefore no term contributes until

$$t > (2d - z)/c,$$

which shows that the growing wave is indeed reflected.

A similar analysis can be carried through for the other two partial waves to show that the amplitude of the space charge wave is identically zero until a time $t > z/u_0$, while the decaying field wave is identically zero until a time $t > z/c$; this is what we would expect if these two waves were directly excited at the initial surface.

6. POWER AMPLIFICATION IN AN IONIZED MEDIUM

(i) The Ideal Termination

For the case of the forward waves, where $u_0 > 0$, we showed from Eqs. (24) and (27) that the steady-state amplitude of the growing wave was proportional, for large enough d, to

$$\exp(ik_1 z) \cdot \exp[i(k_2 - k_1)d] \sim \exp[-(\alpha + \nu/u_0)d] \cdot \exp(\alpha z)$$

if both the inequalities (42) are satisfied. Even at $z=d$ the amplitude of this wave is $\sim \exp(-\nu d/\mu_0)$ so appreciable amplification cannot be taking place. This is no longer the case however either if

$$Z_{\mathrm{II}} = Z_1(\Omega_0) \tag{46}$$

or if

$$Z_{\mathrm{I}} = -b_1(\Omega_0)/a_1(\Omega_0) \tag{47}$$

when, from Eqs. (24) and (27), the amplitude of the growing wave is proportional, for large enough d, to

$$A_1 \exp(ik_1 z) \approx \exp(ik_1 z) \exp(-ik_2 d) \sim \exp(-\nu d/u_0 + \alpha z). \tag{48}$$

If $\alpha > \nu/u_0$, which is normally the case, then the amplitude of this wave increases indefinitely with z, and power amplification is possible as we shall show.

Let us consider the first of these two alternatives that given by Eq. (46). The necessary and sufficient condition that this is physically realizable is that $Z_1(\Omega_0)$ have a positive real part, so that the ideal termination absorbs power from the medium.

From Eq. (18),

$$Z_1(\Omega_0) = -\Omega_0 \mu_0 / k_1(\Omega_0)$$

and since $k_1(\Omega_0) = \beta - i\alpha$ where β, α are given by Eq. (38), this condition requires that

$$\Omega_0 \beta = \frac{\Omega_0^2 \omega_0^2}{2c^2(\Omega_0 - \omega_H)^2} \left(\frac{u_0 \omega_H}{\Omega_0} + \frac{\nu}{\alpha \Omega_0^2} \right) < 0,$$

or that

$$\nu < |\Omega_0 \omega_H / \alpha| \quad \text{and} \quad u_0 \omega_H / \Omega_0 < 0,$$

which are just the conditions that the growing wave be amplified on Bailey's theory.[10]

That power amplification is indeed possible when the medium is thus ideally terminated follows from the fact that the rate at which real electromagnetic energy flows out of the medium at $z=d$ is given by

$$\operatorname{Re}[E(d,\Omega_0) \cdot -i\bar{H}(d,\Omega_0)] \sim \operatorname{Re}[Z_{\mathrm{II}}(\Omega_0)] \exp 2(\alpha - \nu/u_0)d, \tag{49}$$

where $\bar{H}(d,\Omega_0)$ is the complex conjugate of $H(d,\Omega_0)$. If $\alpha > \nu/u_0$ and $\operatorname{Re}[Z_{\mathrm{II}}(\Omega_0)] > 0$, this output power can be made arbitrarily large by choosing d large enough. However this is only possible when the terminating impedance possesses its ideal value, and no power amplification is possible unless the medium has nearly the right impedance. To show that this is so we will consider the normal case where the terminating impedance is non-ideal, which will throw additional light on the physical nature of the propagation.

(ii) Non-Ideal Termination

When the termination is so far from the ideal that the first term in the denominator of $A_n(\omega)$ is much larger than the others, that is when

$$|(1 - Z_{\mathrm{II}}/Z_1)(b_1 + Z_{\mathrm{I}} a_1)| \exp(\alpha d) \gg \left| \sum_{l=2}^{3} (1 - Z_{\mathrm{II}}/Z_l)(b_l + Z_{\mathrm{I}} a_l) \exp(ik_l d) \right|. \tag{50}$$

We have that

$$\left.\begin{aligned}
A_1(\Omega_0) &\approx E(\Omega_0) \frac{u_0}{c} \frac{\omega_0^2}{(\Omega_0 - \omega_H)^2} \times \frac{(Z_{\mathrm{II}}/Z_0) \exp[-(\alpha + \nu/u_0)d]}{(1 - i\alpha Z_{\mathrm{II}}/\Omega_0\mu_0)(1 - i\alpha Z_{\mathrm{I}}/\Omega_0\mu_0)}, \\
A_2(\Omega_0) &\approx E(\Omega_0) \frac{u_0^2}{c^2} \frac{2\Omega_0 \omega_0^2}{(\Omega_0 - \omega_H)^3} \frac{1}{(1 - i\alpha Z_{\mathrm{I}}/\Omega_0\mu_0)}, \\
A_3(\Omega_0) &\approx E(\Omega_0) \frac{2}{1 - i\alpha Z_{\mathrm{I}}/\Omega_0\mu_0},
\end{aligned}\right\} \tag{51}$$

where we have used the approximate values for a_n, b_n given in Table II, and have taken $V^*(0,\omega)$, the initial velocity modulation, equal to zero.

These results can be interpreted as follows. Let us suppose that medium I is transparent to the incident wave, frequency Ω_0 which carries a real power $P(\Omega_0)$. If $u_0 = 0$ all of this power is reflected if medium II is sufficiently wide; but if $u_0 \neq 0$ a fraction $\sim (u_0/c)P(\Omega_0)$ is transmitted by medium II. Of this transmitted power, the greater part is carried by the decaying wave

[10] The coincidence is trivial. The condition that $\operatorname{Re}[Z(\Omega^0)] > 0$ is identical with the requirement that the energy flow associated with the growing wave be in the positive direction.

TABLE II. Approximate values of the coefficients of Eq. (26) for $u_0/c \ll 1$ and $\nu/u_0 < \alpha$.

	$n=1$	$n=2$	$n=3$
k_n	$-i\alpha+(u_0/c^2)\omega_H\omega_0^2/[2(\Omega_0-\omega_H)^2]$	$-[(\Omega_0-\omega_H)/u_0]+i\nu/u_0$	$i\alpha+(u_0/c^2)\omega_H\omega_0^2/[2(\Omega_0-\omega_H)^2]$
a_n	$1/2$	$(u_0/c)^2\cdot\Omega_0\omega_0^2/(\Omega_0-\omega_H)^3$	$1/2$
b_n	$-\Omega_0\mu_0/2i\alpha$	$-Z_0(u_0/c)^3\Omega_0\omega_0^2\omega_H/(\Omega_0-\omega_H)^4$	$\Omega_0\mu_0/2i\alpha$
Z_n	$\Omega_0\mu_0/i\alpha$	$Z_0(u_0/c)\Omega_0/(\Omega_0-\omega_H)$	$-\Omega_0\mu_0/i\alpha$

of amplitude $A_3(\Omega_0)$, but a fraction $(u_0^2/c^2)(u_0/c)P(\Omega_0)$ is carried by the space charge wave. If the medium is sufficiently wide and $\alpha > \nu/u_0$ the space charge wave is much larger in amplitude than the decaying wave[11] at $z=d$, and the power in the reflected wave is therefore of the same order of magnitude at $z=d$ as the power in the space charge wave. This power is very much smaller than the incident power so that under these circumstances power amplification is out of the question.

The conditions for power amplification are thus extremely critical, as they rely on what is essentially resonant reflection; moreover when these conditions are met they are more likely to lead to instability around the feedback path formed by the space charge wave and the reflected field wave than to amplification. It may be noted, however, that once the reflected wave has been resonantly excited, there exists a physical mechanism by which it can be maintained.

(iii) The Physical Mechanism of Power Amplification

We shall now show how the ac kinetic and electromagnetic energy associated with the growing wave is obtained from the dc kinetic energy of the electron beam once the wave has been established.

The work done by the electromagnetic field on the moving charge in unit time and unit volume is

$$W = -e\mathbf{E}\cdot\rho\mathbf{v}.$$

In the present case, where $\mathbf{E}$ has no longitudinal component and where ρ is time-independent

$$W = -e\mathbf{E}_\tau\cdot\rho_0\mathbf{v}_\tau$$

where the subscript τ refers to the transverse components.

If we use the complex algebra of Sec. 3 we may write this as

$$W = -e\rho_0 \operatorname{Re}[E(z,\Omega_0)\cdot\bar{V}(z,\Omega_0)],$$

where

$$\bar{V}(z,\Omega_0) = v_x(z,\Omega_0) - iv_y(z,\Omega_0).$$

Hence, if the electromagnetic field is to gain energy at the expense of the moving charge, we must have

$$W = -e\rho_0 \operatorname{Re}[E\cdot\bar{V}] < 0.$$

[11] The power in the space charge wave is dissipated by scattering and decays at a rate $\sim\exp(-2\nu z/u_0)$, while the power in the decaying wave is transferred by radiation pressure to increase the dc kinetic energy of the moving electrons and decays at a rate $\exp(-2\alpha z)$.

Now this energy is gained at the expense of the space charge ac kinetic energy which is itself continually increasing. The common source for both these increasing ac energies is the dc kinetic energy of the electrons, and it is made available by the interaction of the electron drift velocity with the ac magnetic field. The effect of this interaction is continually to turn the electrons at right angles both to the ac transverse magnetic vector and to the dc longitudinal drift velocity, in just the right sense to supply the required transverse ac energy.

We can derive the mathematical conditions for this energy transfer from the transverse force Eq. (12) which may be written

$$\left(\frac{\partial}{\partial t}+u_0\frac{\partial}{\partial z}\right)mV = -eE - ieu_0\mu_0 H + iem\omega_H V.$$

If we multiply through by $\bar{V}$, the complex conjugate of V, and equate the real parts of both sides we get

$$\operatorname{Re}\left[\frac{d}{dt}\frac{1}{2}mV\bar{V}\right] = \operatorname{Re}[-eE\cdot\bar{V} - ieu_0\mu_0 H\cdot\bar{V}]$$

and if the ac velocity field is to increase exponentially we must have

$$\operatorname{Re}[E\cdot\bar{V} + iu_0\mu_0 H\cdot\bar{V}] < 0.$$

From Eqs. (10)–(12) it can be shown that these conditions will be met, in the steady state, for the growing wave if

$$\nu < |\Omega_0\omega_H/\alpha| \quad \text{and} \quad u_0\omega_H/\Omega_0 < 0$$

which are just the conditions, given by Bailey, of Eqs. (39), (40).

From some points of view this is rather surprising. We have an electron stream moving at a very slow speed interacting with an electromagnetic field propagating with a very high phase velocity; just the condition that normally justifies neglect of the interaction between the space charge and the magnetic components of the field. However, when the field waves have almost pure imaginary propagation constants, $\mathbf{H}$ is nearly parallel to $\mathbf{E}$. Hence, although $\mathbf{H}\times\mathbf{u}_0$ is small it is nearly parallel to $\mathbf{v}$, while $\mathbf{E}\cdot\mathbf{v}$ is small since $\mathbf{E}$ is nearly perpendicular to $\mathbf{v}$. The effect of the magnetic field components on the moving charge is thus comparable with, and indeed greater than the effect of the electric field components, in an attenuating medium.

(iv) Conditions in Sunspots and Discharge Tubes

What we have said above applies directly only to the forward waves when terminated ideally at the surface $z=d$; other special cases arise when Eq. (47) is satisfied, and for the reverse waves when $u_0<0$. Furthermore the medium might be excited by an initial velocity modulation rather than by an external electromagnetic wave. In all of these, however, amplification is only possible, if at all, under very critical conditions, which would hardly be encountered in a sunspot or indeed anywhere in a star. If the parameters of the medium such as the magnetic field or space charge density do not change rapidly with wavelength, then the impedance looking in any direction in a region where the field waves have a complex propagation constant is nearly equal to the characteristic impedance of the decaying wave and not, as is essential for amplification, to that of the growing wave. Even with the most generous allowance for the effects of turbulence, it seems incredible that the impedance discontinuity could be so large as to involve a complete reversal of sign. Furthermore this critical impedance would have to be just right for an appreciable time since the steady state in which power amplification is possible has to be built up by a resonant reflection which, in the language of the electronics engineer, has a very high Q. Finally it may be noted that even if the electromagnetic energy could be generated by freak conditions it could not escape, since it would occur at a level where the characteristic impedance of the medium was largely reactive and therefore attenuating.

In a discharge tube definite surfaces of discontinuity exist at cathode and anode, and it is conceivable that at certain frequencies the right conditions might exist for power amplification. Even granting this, however, it seems hardly possible to explain the experimental results of Bailey and Landecker[12] on this theory, because this amplification is too small. Using glass discharge tubes and a longitudinal magnetic field of a few hundred gauss, these workers measured, at low frequencies, noise power outputs corresponding to a blackbody temperature of 10^7 to 10^{13} degrees Kelvin. At low frequencies for which $\Omega_0<\omega_H$ it is only the ordinary waves that are amplified. From Eq. (30) it follows that the maximum power amplification attainable with the ordinary waves is $\exp(2\omega_0 z/c)$ which works out at 20 db/meter when, as in the experiments of Bailey and Landecker the plasma frequency is about 100 Mc/sec. The observed noise power corresponded to a blackbody temperature which at some frequencies was as high as 10^{13} degrees Kelvin. As the electron temperature is only 3×10^4 degrees an amplification of 85 decibels would be needed if the observed phenomena were thus to be explained. The discrepancy is even more serious when the effects of magnetic field are taken into account. At a frequency of 1 Mc/sec the amplification of the ordinary waves with an axial magnetic field of 200 gauss is only 2 db/meter while the observed noise power corresponded to blackbody temperature of 10^{12} degrees Kelvin.

[12] V. A. Bailey and K. Landecker, Nature **166**, 259 (1950).

7. CONCLUSIONS

We have shown that the growing waves which Bailey interpreted as amplified waves can be excited only by reflection. If the medium is ideally terminated in the characteristic impedance of this growing wave, the wave amplitude can be built up to a large amplitude by a process of resonant reflection. But this termination is very critical and it is most unlikely that the required conditions would be found in a sunspot. Even so if amplification did take place the radiation would be generated at a level from which it could not escape. The explanation of the excess noise radiation from sunspots must be sought elsewhere.

Under special circumstances, it is conceivable that amplification of the transverse waves could take place in a discharge tube. However the maximum amplification attainable seems too small to account for the experimental results of Bailey and Landecker,[12] even under the most favorable circumstances.

From relativity considerations, we argued that any amplification in an infinite ionized medium drifting with a monochromatic velocity parallel to a magnetic field must depend entirely on conditions at the transverse surfaces of discontinuity, since these alone provide a frame of reference with respect to which the medium is moving.

The same argument would also apply to a cylindrical beam of finite cross section moving in empty space, or bounded by a perfect conductor, or with any transverse boundary condition the qualitative form of which was unaltered by a longitudinal Lorentz transformation.

A general criterion as to the conditions that a growing wave be a true amplified wave cannot be obtained from a unidimensional analysis. From the transient analysis for this special cases however, it appears that a growing field[13] wave is probably a reflected and not an amplified wave though this is not the case for the traveling wave tube. On the other hand, if the associated energy flow is in the positive direction, a growing space charge wave is almost certainly a true amplified wave, at least as long as all the electrons are moving in the same direction.

Whether or not a given wave is excited by reflection cannot be decided from its direction of propagation and energy flow at a finite frequency. We have shown that only the behavior at arbitrarily large frequencies is relevant.

[13] By a field wave we mean a wave which propagates, if at all, at a speed comparable to c and which exists even when the space charge density is zero. By a space charge wave we mean a wave which propagates at a speed comparable to that of the moving space charge and which vanishes when the space charge density goes to zero.

It has usually been assumed in the past that one can neglect the interaction between an electron beam, moving at a speed small compared with c, and the magnetic components of an electromagnetic wave propagating with a phase velocity near to or greater than c. We have shown that this is not always justified and that in some circumstances such as propagation in an attenuating medium, the interaction with the magnetic components is more important than the interaction with the electric components in some particulars at least.

This analysis has served once again to stress the importance of taking initial and boundary conditions into account in a discussion of propagation in an active medium.

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A High Energy Proton-Proton Collision with Associated Events

V. D. Hopper, S. Biswas, and J. F. Darby
Physics Department, University of Melbourne, Melbourne, Australia
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An event observed in a nuclear emulsion is interpreted as an incident proton of energy 1000 Bev colliding with a hydrogen nucleus and producing a cone of six charged mesons. The presence of three neutral mesons is deduced from two electron pairs and one triplet in the region of the cone. The triplet is due to an electron pair produced by a gamma-ray of energy 24 Bev in the field of an electron. An additional electron pair is produced by bremsstrahlung. The kinetic energies of several particles are deduced from scattering measurements and these range from 800 Mev to 17 Bev. The total energy of the shower is estimated as 100 Bev. In the center-of-mass system the mesons have energies of the order of 400 Mev and an almost isotropic distribution. It is concluded that most of the energy is carried by two neutrons, and only one-tenth of the energy of the primary particle goes to meson production. The results are compared with the theories of multiple meson production.

I. INTRODUCTION

Most of the data published on very high energy collisions with nuclei deal with the interaction of protons, neutrons, or heavier nuclei with the light (C, N, O) and heavy (Ag, Br) groups of nuclei present in photographic emulsions, and in these cases as many as thirty minimum ionization tracks have been produced, which have been shown[1] to be mainly mesons. Some of these mesons arise from the interaction of the primary particle with a single nucleon (multiple production), but if the energy of the primary particle is distributed over a part of the nucleus plural production of mesons by interactions between many pairs of nucleons is possible. It is clear that it is of fundamental importance to the theory of meson production to study collisions of protons or neutrons with hydrogen nuclei—events which are likely to be missed in nuclear emulsions as all tracks are at minimum ionization. Very few examples of these collisions have been published. An event described by Camerini *et al.*[2] occurred near the edge of the plate, so no detailed measurements could be made. Pickup and Voyvodic[3] have found four such events, but the energies of only a few particles could be measured because the lengths of tracks in the emulsion were short. A collision of a very energetic singly charged particle with a nucleus heavier than hydrogen has been observed by Lord, Fainberg, and Schein,[4] in which, however, the mesons are most probably produced by a single nucleon-nucleon encounter.

This paper describes an event in which a singly-charged particle with an estimated energy of the order of 1000 Bev collides with a nucleus to produce a shower of seven particles at minimum ionization one of which, however, is an electron. The event was observed in an Ilford G-5 emulsion, 400μ thick, which was exposed to cosmic radiation at 70,000 ft. Since no tracks of ionization above the minimum were observed, it is most likely that the atom struck was hydrogen. The event fortunately occurred near the center of the plate, and the primary particle was traveling almost parallel to the emulsion surface, so very long tracks of most of the secondary particles are visible, allowing detailed study. The region around the event was carefully scanned, and three electron pairs and one triplet associated with the event were found. The triplet is due to production of an electron pair in the field of an electron by a photon of energy about 24 Bev.

II. DESCRIPTION OF THE EVENT

The primary collision consists of a singly-charged particle at minimum ionization producing seven secondary particles, also at minimum ionization, within a cone of half-angle about 11°. One of the secondaries is an electron of energy 60 Mev, and is probably a knock-on

[1] P. H. Fowler, Phil. Mag. 41, 169 (1950).
[2] Camerini, Fowler, Lock, and Muirhead, Phil. Mag. 41, 413 (1950).
[3] E. Pickup and L. Voyvodic, Phys. Rev. 82, 265 (1951).
[4] Lord, Fainberg, and Schein, Phys. Rev. 80, 970 (1950).