The Energy Loss of Electrons Passing through Matter*†

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A discussion of the results of a study of the stopping properties for moderately energetic ($\lesssim$1.0-Mev) electrons is given, with special emphasis placed on the distribution in energy loss for a finite thickness of stopping material. Several theoretical expressions (Bethe-Bloch-Williams, Landau, Blunck-Leisegang) are compared with the data on the value of the most probable loss. A correction for polarization effects (e.g., in Be) can be made if the dependency of the mean excitation potential on velocity is known. The stopping properties of some materials used as, e.g., counter windows, have also been determined.

A. INTRODUCTION

ALTHOUGH the interaction of particulate radiation with matter has been one of the most studied of "atomic" phenomena, the theory having been developed[1] and fairly reliable measurements made[2] as early as 1912, there has never been a sufficiently accurate theory, nor precise enough measurements, to enable the worker who uses such radiation to apply these results to a general case where such interactions might obscure other and possibly more interesting effects. Although, therefore, the basic mechanism of what will be referred to as stopping of electrons is essentially understood, there are a number of side effects which make the interpretation rather difficult. In particular, the energy loss by ionization and excitation involves collisions whose frequency, with respect to the energy lost per collision, is distributed statistically. Since, however, an electron may lose all of its initial kinetic energy in a single collision with another electron, the distribution must give relatively more electrons emerging from a slab of material, with lower energy than would be predicted by a gaussian. An estimate of the most probable value of the energy loss can be obtained from a knowledge of the average value and by ignoring the infrequent but violent collisions that produce the familiar long "tail" on the actual distribution. For many purposes, as in dosimetry, for example, this is not satisfactory. What is needed, as pointed out by Wang,[3] is not the estimate of the most probable loss, but rather a measure of the local energy absorption. That is, the shape of the entire distribution function should be known. Besides the need for such a study felt by radiobiologists, there is further interest in the subject because of the recent popularity of the high energy accelerator. While much of the energy loss of the particles produced by these machines is due to radiative effects, still the results of this work, which will deal with the losses due to inoization using less than 1-Mev electrons, should apply to the nonradiative losses of the very energetic particles.

This article will discuss the results of some measurements of electron stopping in a number of metals selected to cover a wide range in atomic number Z, mica, and a number of organic compounds whose composition is analogous to biological tissue and which have been used in physics for, e.g., counter window materials. The sources of electrons were β-line spectra obtained from several isotopes with prominent internal conversion lines and the initial (prior to entering the foil of stopping material) and the final (on emerging from the foil) momentum spectra of the electrons were determined in a double thin lens magnetic spectrometer. A preliminary account of this work has already been published.[4]

B. THEORY

For convenience only a brief listing of some of the more pertinent results[5] of the theory of energy loss effects will be given. The average energy loss of electrons that have passed through a slab of material of thickness x, under the supposition that only a small fraction of the initial energy is lost is,[1] to first order in

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[1] See, for example, H. Bethe, *Handbuch der Physik* (1933), second edition, Vol. 24/1, 523, for the theory and a review of earlier work.

[2] Some of these pertinent to the present work are O. von Baeyer, Z. Physik **13**, 485 (1912); J. Danysz, Compt. rend. **154**, 1502 (1912) and J. phys. **3**, 949 (1913); A. Becker, Ann. Physik **78**, 209 (1925); E. Madgwick, Proc. Cambridge Phil. Soc. **23**, 970 (1927); P. White and G. Millington, Proc. Roy. Soc. (London) **A120**, 701 (1928). The last reference is the latest systematic study of the straggling curves.

[3] T. J. Wang, Nucleonics **7**, 55 (1950).

[4] S. D. Warshaw and J. J. L. Chen, Phys. Rev. **80**, 97 (1950).

[5] See N. Bohr, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **18**, 8 (1948) for a comprehensive qualitative review.

$\beta^2=v^2/c^2$,

$$\Delta E=\frac{x}{\beta^2}\log\frac{2\beta^2}{(1-\beta^2)\exp(\beta^2-1)}\frac{W_m}{I^2}. \quad (1)$$

Here, for convenience in notation, energy is measured in units of $mc^2=0.51$ Mev, and length in units of $t=(2\pi r_0^2 n)^{-1}$, where r_0 is the classical radius of the electron $=2.82\times10^{-13}$ cm and $n=L\rho Z/A$ is the density of electrons in the stopping material of atomic number Z, atomic weight A, and density ρ; L is Avogadro's number. (Note that thus t has the form of a mean free path for electron-electron collisions on a gas-kinetic model, indicating the origin of the stopping phenomenon.) In Eq. (1) W_m is the maximum allowed loss in a single collision and I is the mean excitation potential for the type of atom under consideration. For a correct application of this formula the velocity β is to be the instantaneous velocity at each collision. A first approximation to the instantaneous value can be obtained, in the case where $\Delta E\ll E_0$, the initial energy, by averaging the initial and final velocities; in the cases to be discussed this procedure produced only a 1 percent change in the energy loss. For electron-electron collisions the entire kinetic energy $E_0=(1-\beta^2)^{-\frac{1}{2}}-1$ may be transferred so that $W_m=E_0$ substituted in Eq. (1) gives the usual average energy loss formula.

However, as pointed out in the introduction, the average loss is not sufficient to describe the result, since, owing to the presence of a small number of violent collisions, the most probable energy loss will be smaller than the average value. Bohr[6] has discussed this qualitatively with the result that the distribution will be very nearly gaussian near the peak, and the tail will fall off like the inverse square of a linear function of T, the total energy transfer per incident electron. Landau's[7] treatment of the problem gives a result which bears out Bohr's discussion; his solution is given as a function $f(x, T)=\xi^{-1}\phi(\lambda)$, where fdT is the probability for a loss between T and $T+dT$ for an electron that has traversed the distance x; $\xi=x/\beta^2$ and $\phi(\lambda)$ is a function, given graphically, of the universal parameter

$$\lambda=\frac{T}{\xi}-\log\frac{2x}{(1-\beta^2)I^2\exp(\beta^2-0.42)}.$$

The maximum of f occurs at the value $\lambda=-0.05$, which results in the value of the most probable loss

$$T_0=\frac{x}{\beta^2}\log\frac{2x}{(1-\beta^2)I^2\exp(\beta^2-0.37)}. \quad (2)$$

Landau's theory is to the first order in the energy transfer from the distant collisions. A refinement of the theory due to Blunck and Leisegang[8] carries the calculation to second-order terms in the "resonance" energy transfer. These authors expand Landau's $\phi(\lambda)$ in a series of gaussian functions; the width of the distribution is increased by an amount controlled by the parameter $b^2=2\beta^4\langle E_r^2\rangle/x$, where $\langle E_r^2\rangle$ is the second moment of the energy transfer due to the resonance collisions. They show that for $b^2\ll3$ the effect of such collisions is negligible and the distribution reduces to Landau's. The effect on the value of T_0 is less than the experimental uncertainty, although the spread may be much greater.

A value for the most probable loss has also been obtained by Williams,[9] who suggested that the appropriate modification to the Bethe-Bloch formula, Eq. (1), would be simply to ignore those infrequent violent collisions which distort the distribution from a gaussian in the region of high loss; thus he first put $W_m=1.2\xi$ and later changed this value, to agree better with experiment, to 2.8ξ. That is, even though some electrons may lose a considerable fraction, or all, of their initial kinetic energy in a single collision, the frequency of such collisions is small enough to ignore those in which the transfer is, for example, greater than 6 kev for 1-Mev electrons passed through about 0.1 mm of aluminum.

The modified Bethe-Bloch expression is, then,

$$T_0=\frac{x}{\beta^2}\log\frac{1.4x}{(1-\beta^2)I^2\exp(\beta^2-1)}, \quad (1')$$

which is quite close to the Landau formula, both in form and numerically. (The difference, using 624-kev electrons, and 30 mg/cm^2 aluminum, is about 2 percent.)

The first point of investigation will be the characteristic nonlinear dependence of the probable loss expressions on the thickness of material traversed. This dependence would set a lower limit on the thickness of material to be studied, except that the thinnest sections for which multiple collisions may still occur[9] is thinner than the limits set by the region of validity of probable loss expressions, which are $I\ll\xi$ and $T_0\ll E_0$. Using aluminum ($I=155$ ev) and $E_0=624$ kev, this means that the probable loss formulas (and the distribution functions) are not valid for thickness below about 5 mg/cm^2 nor above a value that would give a loss much more than, say, 50 kev, or about 60 mg/cm^2.

One point of further interest is the effect of polarization of the medium[10] on the rate of energy loss. While, in the region that is considered here, this effect is generally small, in a metal like beryllium, with a rela-

[6] See reference 5, p. 53.
[7] L. Landau, J. Phys. (U.S.S.R.) **8**, 201 (1944).
[8] O. Blunck and S. Leisegang, Z. Physik **128**, 500 (1950). At the time of the original writing the authors were unfortunately unaware of this work; their attention was drawn to it by the referee's comment (for which he is thanked) with regard to R. D. Birkhoff's [Phys. Rev. **82**, 448 (1951)] attempt to verify this theory and Landau's earlier treatment.
[9] E. J. Williams, Proc. Roy. Soc. (London) **A125**, 470 (1929).
[10] Reference is made to the original work on this effect: E. Fermi, Phys. Rev. **57**, 485 (1940); O. Halpern and H. Hall, Phys. Rev. **73**, 477 (1948); and to the discussion given by A. Bohr, Kgl. Danske Videnskab. Selskab, Mat.-fys Medd. **24**, 19 (1948), for details.

tively large fraction of conduction electrons, there may be a very pronounced difference between the stopping power with, and without, an account being taken of polarization. In fact, only in beryllium was an unambiguous interpretation of the polarization effect obtained.

C. EXPERIMENTAL PROCEDURE

The method used here, for obtaining the initial and final energy spectra, has already been mentioned in the preliminary report.[4] That is, a thin source made of a β-active substance was mounted on a conventional source holder (Lucite) and introduced into the spectrometer. After the shape of the conversion line—taken to be the transmission curve of the spectrometer—was determined, the foil was mounted just in front of the source and the same line was studied with the foil in place. The effect of a variable foil thickness on the line most used in this study (K-conversion in Ba^{137}) is shown in the set of curves referred to.[4] It can be seen from this figure that the spread of the "line" with the foil in place is no more than two or three times the spread in the original transmission curve, the latter measured by the resolution of the instrument, which was set at 2.1 percent. Therefore, it is necessary to study in some detail just what will be the effect of such (relatively) poor resolution on the interpretation of the straggling curve. This section will consider this correction, and other possible sources of error.

1. Sources

The ideal source for the measurements would be a monoergic, well-collimated, intense beam from a high voltage accelerator with a variable voltage. In the absence of the generator, the ideal source was approximated by a series of conversion lines present in several radioactive isotopes and by a photoelectric gamma-ray converter. The approximation will be a good one (1) if the finite thickness of the source does not introduce a spectrum in the initial beam itself, and if the source mounting does not give too much back scatter at lower energy; (2) if the lines are isolated from any continuous spectrum that may also be present and also are distinct from any neighboring lines within the limits of resolution of the instrument. The source should, moreover, have a conveniently long half-life with a high specific activity. These conditions are more or less satisfied by all of the sources listed in Table I, with Cs^{137} being the best. Several lines from the photoelectric conversion in lead of the gamma-rays emitted by a radon seed were also used; these were obtained superposed on a Compton background, and absorption in the lead created a smeared out initial distribution; besides this, the gammas produced some secondary electrons in the heavier stopping substances, and for these reasons the data obtained with the radon source were weighted less than the other data.

TABLE I. Sources used for energy loss measurement.

Source	Type	Energy (kev)	Reference
Cs^{137}	K-conversion	623.9	a
Se^{75}	K-conversion	124.6 (K^6) 85.0 (K^4)	b
Ta^{182}	K-conversion	87.0	c
Rn	photoeffect Pb radiator	1032 591 517 336 262 206	d

[a] L. Langer and R. D. Moffat, Phys. Rev. **78**, 74 (1950).
[b] Cork, Rutledge, Branyan, Stoddard, and LeBlanc, Phys. Rev. **79**, 889 (1950).
[c] G. Wilkinson, Phys. Rev. **80**, 495 (1950).
[d] G. Gamow and E. Critchfield, *Atomic Nuclei and Nuclear Transformations* (Oxford University Press, London, 1949), third edition.

2. Scattering Effect

While the tentative limits on foil thickness have been set in Sec. B above, it will be seen that the maximum thickness is limited further by scattering due to the presence of the foil itself. Thus electrons, emitted by the source but not in a direction, or with an energy, that would enable them to be collected could, in the presence of the foil, be scattered into the beam, and some electrons in the beam could be removed by scattering. A full discussion of this[11] shows that the net effect of scattering is to cause more electrons of larger energy loss to appear, thereby both distorting the loss spectrum and shifting the most probable loss to a greater value. For thin enough foils this effect should be negligible.[12] A check was made on the scattered-in electrons by first running with about 20 mg/cm² of Al, and then repeating the run with the outer part of the foil (i.e., leaving an opening the size of the source) thickened to 450 mg/cm². Only a negligible effect (of the scattered-in electrons) was found on the peak shift, although the tail of the distribution was somewhat raised. Generally, unexpectedly high loss values were explainable by the scattering effects.

3. Resolution Correction

Besides a small correction for the obliquity of the foil to the beam[13] which has been applied to the data, the remaining source of ambiguity in the interpretation of the scattering curves is in the fact that the width of the transmission curve of the spectrometer is, for the loss distributions that could be studied under the limitations due to scattering and the requirement of small total loss, not more than a few times smaller

[11] P. White and G. Millington, see reference 2.
[12] For example S. Slawsky and H. Crane (Phys. Rev. **56**, 1203 (1939)) find that 68 mg/cm² of Al produced excessive scattering of 0.9-Mev electrons, while the scattering was negligible for 30 mg/cm².
[13] The extremes of the electron trajectories in the instrument used made angles of 7° and 11° to the normal to the foil; the effect was an average increase in foil thickness of 1.3 percent.

than the width of the observed straggling curve itself. Since the transmission curve of the instrument[14] is symmetrical in shape, the peak position of a symmetric emergent electron distribution will not be disturbed; however the loss distribution is known to be asymmetrical, and the effect of finite resolution will, in fact, make the most probable final position of the "line" appear at lower energies. Hence a correction must be applied, not only to the measured loss distribution, but, therefore, to the most probable loss.

The observed number of counts at any momentum p is given by

$$n_{\text{obs}}(p)=\int n_{\text{th}}(p')W(p-p')dp',$$

where $W(p-p')$ is the transmission curve of the instrument[14] and n_{th} is the theoretical distribution, which is to be verified. If a theoretical distribution is assumed, the effect of finite resolution can be determined by performing the integration (numerically) and comparing the appropriately normalized result with the actually observed curve. A procedure that has been used by some workers[15] is to calculate the expected n_{obs}, and then to use the ratio *expected n/observed n* as a correction factor to the empirical data, taking each measured point in turn. This procedure, of a *post hoc* nature, evidently weights the result rather strongly in favor of whatever theory has been assumed. Therefore, it was used only to apply a small correction to the peak shift, and small credence is placed in the fact that good agreement with the shape of the theoretical distribution can be obtained in this way. In Fig. 1 is shown a representative case: the passage of 624-kev electrons

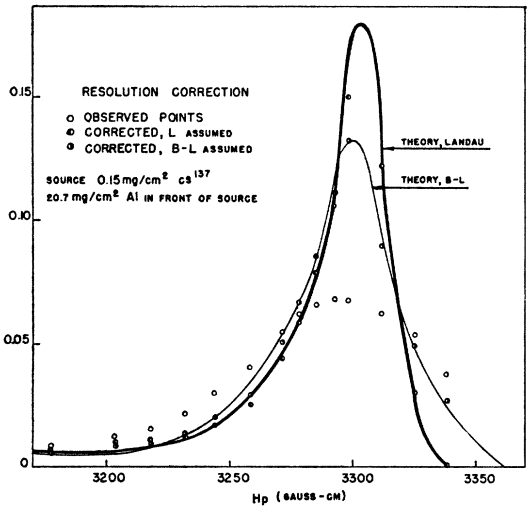

FIG. 1. Representative observed straggling curve compared with the Landau (L) and Blunck-Leisegang (B-L) theories. The open circles are the directly observed points while the half-filled-left circles are these points corrected by assuming the Landau theory, and the half-filled-right circles are the same points corrected by assuming the B-L theory.

[14] Van Atta, Warshaw, Chen, and Taimuty, Rev. Sci. Instr. **21**, 985 (1950).

[15] Hughes, Eggler, and Alburger, Phys. Rev. **77**, 726 (1950).

through 20.7 mg/cm² of Al. The experimental points lie on a curve which bears only a superficial resemblance to either the Landau distribution, or the Blunck-Leisegang[8] distribution, although the observed peak is within 1.5 percent of either theory, that is, within the experimental error. The resolution correction procedure, applied in turn to each of the two distributions, results in the experimental data agreeing well with both the Landau distribution and the Blunck-Leisegang, although agreement with the latter seems better. Evidently, there is no point in attempting to decide which distribution is correct with the use of such poor resolution. The parameter b^2, defined above for the Blunck-Leisegang theory, has the value 4.1 for the case shown in the figure, so that there is not as great a difference between the two theories as one might wish. For larger b^2 (smaller x), however, the theoretical curve is much narrower than the transmission curve, so that no conclusion, based on the correction procedure described, can be safely obtained. If, however, Birkhoff's[8] published data, for which $b^2\doteq 6.6$, is corrected, rather good agreement with the Blunck-Leisegang theory is obtained. (His resolution was 0.6 percent, still a considerable fraction of the observed width.) In any event, the correction to the observed most probable loss was small in every case, and nearly independent of which theory was used. Therefore, it seemed justifiable to use simply the more easily calculable Landau function for the correction purpose. It should be stressed that when the source itself was made very thick (1 mg/cm² of CsCl), there were significant deviations of the corrected curve from the theoretical in the tail region, although the peak position was not appreciably affected. As can be seen from Fig. 1, the correction to the peak shift is small. Other discrepancies between theory and observation were of a random nature, e.g., due to errors in measuring the foil thickness, and in the measurement of the spectrometer coil current. These are the basis for the size of the vertical dashed lines through the experimental points shown in the graphs in the next section.

It should also be pointed out that a correction for bremsstrahlung need not be applied for this work. While, in the case of the heavy elements, the radiative process may account for several percent of the observed average loss, since in bremsstrahlung the loss per electron occurs in a few large transfers, the actual contribution to the energy spread curve will come only in the tail region. Hence, while the average loss will be affected, the most probable loss, as observed, will not. Since, in fact, a good study of the tail region is not possible with the sources that were used here, because of the presence of the continuous spectrum, no contribution from bremsstrahlung has been observed at all.

D. RESULTS

Two forms of presentation of the experimental results are given; each consists of a comparison of the

TABLE II. Loss in heavy elements. Incident energy 624 kev.

Material	I(ev)[a]	x(mg/cm²)	T_{obs/mc^2} [b]	T_{theory/mc^2} [c]	Discrepancy (%)
Pt	718	27.1	0.0377	0.0380	+ 0.8
		54.3	0.0963	0 0818	−21.3
Au	730	7.23	0.0085	0.0086	+ 1.2
		16.88	0.0229	0.0225	− 1.8
Pb	737	23.6	0.0353	0.0337	− 6.4
		47.0	0.0937	0.0708	−27.9

[a] Bakker and Segrè, see reference 17.
[b] Measurement error is about ±0.001.
[c] Equation (2).

line peak shifts (corrected for resolution) with the energy loss predicted by the formulas given in Sec. B, above. The first form is a verification of the thickness dependence of the probable loss. This, in turn, will serve two purposes: first it serves to verify Eq. (1′) or (2) directly. The validity of the two theoretical distributions (used to obtain the resolution correction) has already been mentioned in C above. Second, this presentation makes it possible to fix the properties with respect to energy loss of certain compound materials of "practical" interest, and, at least in the case of beryllium, makes it easy to determine the effect of polarization.

Figure 2 shows a graph of the probable energy loss in a number of metals as a function of the thickness of metal used, at the energy of the Ba^{137} conversion line (624 kev). The experimental points have been corrected according to the discussion in C; on the same graph is included, for reference, a straight line that corresponds to the Bethe-Bloch formula for aluminum (Eq. (1)) with W_m arbitrarily set at 10 kev. While this choice of W_m has no theoretical significance, other than being of the correct order of magnitude for experiments in which the thickness of the aluminum is not too small, it is a procedure that has found some popularity[16] in the past.

From the graph it can be seen that the theoretical expression [Eq. (2)] fits the data quite well, both in regard to magnitude, and to the predicted curvature. Several points measured on mica are also included on this figure (crosses). Evidently, while there is a small difference, for most practical purposes the stopping properties of mica are close enough to those of aluminum to ignore the difference. Table II shows the same type of data for some heavier metals (platinum, gold, lead) and attention is called to the rather large deviations from the theory when the thickness of these metals becomes large. This discrepancy is not surprising in view of the scattering effect mentioned in C above, and also in view of the near failure of the condition $\xi \gg I$ for all of these foils. However, the scattering would seem to cause most of the deviation, since the discrepancy becomes worse even when (as for the thick foils) the condition is better fulfilled. The condition $T_0 \ll E_0$ is always met. The values of I, the mean excitation potential, for the heavy elements are taken from the data of Bakker and Segrè.[17] Although these values are considerably lower than Mano's,[18] the values that are customarily used, and also brought the experimental points in closer agreement with the theory, the difference was so small, inasmuch as the loss is not sensitive to changes in I, that it was not considered worthwhile to press the point.

For the two elements, carbon and beryllium, there is (e.g., Fig. 3 gives the data on beryllium) a relatively large discrepancy between the prediction and the observation, and in a direction opposite to what might be described as instrumental errors. This can only be interpreted as an indication of the phenomenon of polarization of the medium. Both Eqs. (1′) and (2) with $I=91$ ev fit the data for beryllium rather well, while the same expressions with $I=64$ ev[19] or $I=60.4$ ev[17] give a poor fit. The choice, $I=91$ ev was made as the result of a calculation by Christy and Thomas[20] that gives the variation of the excitation potential in

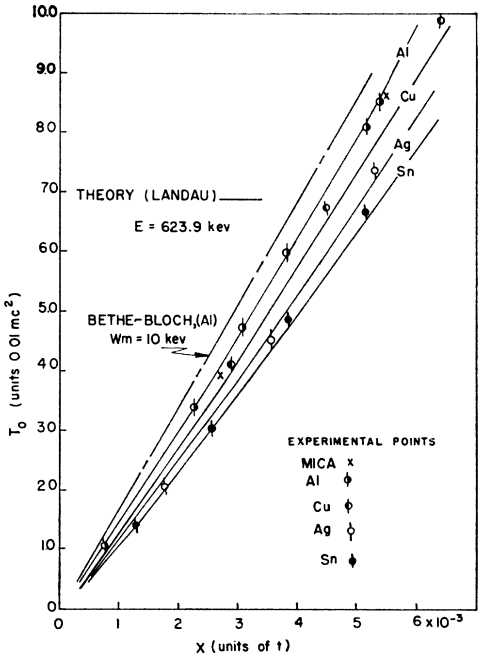

FIG. 2. Thickness dependence of the most probable energy loss in four metals. The units of thickness are 3.94-cm (Al), 8.14-cm (Cu), 7.22-cm (Ag), 10.71-cm (Sn). Solid curves are theoretical (Eq. (2)). The two points for mica are based on $Z/A=0.522$ for $H_2(K \text{ or } Na)Al_3Si_3O_{12}$.

[16] See, for example, the footnote on p. 702 in F. K. Richtmyer and E. H. Kennard, *Introduction to Modern Physics* (McGraw-Hill Book Company, New York, 1947), fourth edition; and also L. Voybodic, Can. J. Research **A28**, 315 (1950).

[17] C. J. Bakker and E. Segrè, Phys. Rev. **81**, 489 (1951).
[18] M. G. Mano, Compt. rend. **197**, 319 (1933).
[19] C. Madsen and P. Venkateswarlu, Phys. Rev. **74**, 648 (1948).
[20] R. F. Christy and R. Thomas, unpublished.

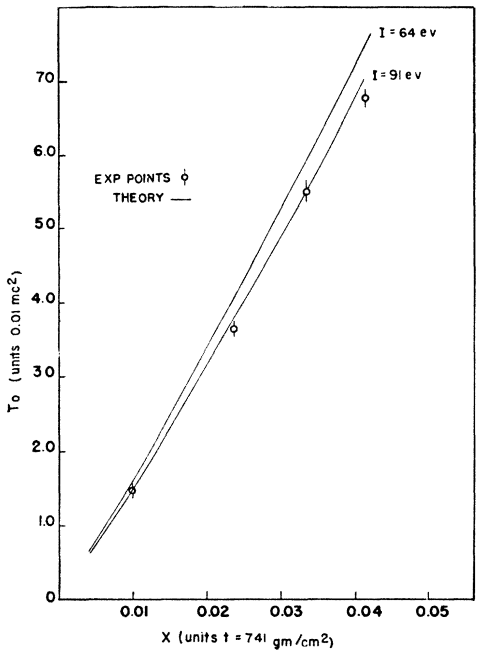

FIG. 3. Thickness dependence of the most probable energy loss in Be. The value $I=91$ ev is a theoretical result that is designed to account for polarization.

beryllium as a function of the particle velocity with an account taken of polarization. (The same phenomenon was not observed by Bakker and Segrè, of course, since they used protons, and the excitation potential depends only on the incident particle velocity.) Measurements of the thickness dependence were also made in carbon, with, however, not as satisfactory results as those for Be. The straggling curves gave relatively more electrons with small loss than in the curves for the same total average loss in, say, Al, making the resolution correction procedure unjustified. There was also a large discrepancy (the data is not shown here) between the measured most probable loss and the theoretical value, with $I=76$ ev.[17]

In the case of the compound materials, there are several difficulties in the way of an accurate calculation of their stopping properties. First, while their composition is known, so that an estimate of the mean excitation potential might be made, their densities fluctuate considerably from one sample to another, depending on the manner of preparation. Thus, only the mass of material presented to the beam is of significance. Because of the uncertainty attached to the excitation potential only an approximate primary loss distribution could be calculated; since the correction for resolution obtained by use of such a distribution was small, the procedure seemed justified. Since the purpose of this measurement is for purely practical application, the result is given in the form of a table of values for a hypothetical element with $I=60$ ev and an appropriate Z/A for each compound. The close agreement between the true Z/A and the Z/A given in Table III, as effective for stopping purposes indicates that the stopping effects for the individual atomic species in the compound materials are additive. For the calculation of radiological dose in biological material, it is suggested that $I=60$ ev and $Z/A=0.56$ be used.

For the energy dependence of the probable loss, a study was made using a variety of β-line spectra in the region from 0.08 to 1.03 Mev with aluminum foil of several thicknesses as the stopping material. It is convenient now to define a "reduced stopping power," since the actual loss rate depends on the thickness and, since it is desirable to keep the measured total loss at various energies a fairly constant value, the thickness must be varied. This quantity which should, according to theory, be independent of thickness, is denoted S_r and is defined by

$$S_r=(T_0/x)-(1/\beta^2)\log x,$$

where T_0 is the observed most probable loss. To convert S_r to the more usual units Mev/g/cm^2, multiply by $mc^2/\rho l=0.154Z/A$. The results are given in Fig. 4 together with the results of earlier workers. (Each measured point is an average value taken on two separate foils; the two numbers so obtained differed by less than experimental uncertainty in every case.) Note that the points corresponding to sources composed of γ-line-plus-Pb-radiator are less precise than those taken with internal conversion lines because of the unavoidable Compton background and high source scattering. No resolution correction was applied to these points, since both the primary and displaced lines were quite unsymmetrical. The points that are due to the other workers agree fairly well with the theoretical loss [solid curve, Eq. (2)] in spite of the fact that the appropriate corrections to the data were apparently ignored. White and Millington's data for mica, if plotted on the same graph, would show a scatter about the theoretical curve for aluminum, although with a tendency toward greater loss at all energies. It is evident that the experimental points (this work) lie very close to the predicted curve [Eq. (2)].

SUMMARY

A study has been made of the characteristics of the energy loss distribution for electrons passing through a

TABLE III. Effective Z/A for organic compounds.

Material	Composition[a]	$\Sigma Z/\Sigma A$	(Z/A) effective
Nylon	$C_{12}H_{26}O_4N_2$	0.550	0.54±0.01
Cellophane	$C_6H_{10}O_5$	0.531	0.53±0.01
Paper	$C_6H_{10}O_5$	0.531	0.53±0.01
Tenite	$C_{24}H_{32}O_{16}$	0.527	0.52±0.01
Vinylite	$C_4H_6O_2$	0.534	0.49±0.01
Biol. tissue	$C_5H_{40}O_{18}N$	0.559	(0.56)[b]

[a] J. B. Conant and A. H. Blatt, *The Chemistry of Organic Compounds* (Macmillan Company, New York, 1947), third edition; P. Karrar, *Organic Chemistry* (Elsevier, Amsterdam, 1947), third English edition.
[b] Suggested for use.

number of substances, with particular attention paid to the dependency of the loss on the energy of the incident electrons and the thickness of the material. The finding is that the most probable loss differs considerably from the average value (given by Eq. (1) with $W_m=E_0$) and is given accurately—within the experimental uncertainty of the order of 1–2 percent—by the Landau theory, Eq. (2) or by a modification of this due to Blunck and Leisegang;[8] for all comparisons with theory, the values of the excitation potential used were those determined by Bakker and Segrè for protons; while these values differ from those given by Mano, the difference has a small effect on the energy loss. The Williams modification of the Bethe-Bloch formula, Eq. (1′), gives values of T_0 about 2 percent or more greater than the value from Eq. (2); this is just barely outside the experimental error, but the observed curves were, when scattering did not affect the result, consistently below those calculated from Eq. (1′).

In the case of beryllium, the value of I used to bring the data into agreement with the theory was that calculated[20] to account for polarization effects. It should be pointed out that the tacit assumption that the only effect of polarization appears in an effective (velocity dependent) excitation potential may be incorrect and that the shape of the distribution function itself should possibly be changed.

For dosimetry, the most accurate calculation would be actually to use the distribution function, for a finite thickness of stopping material, to obtain the total energy absorbed by the material. It is easy to show that this is proportional to

$$S=\int_{T_{\min}}^{E_0} T\partial f(T,\xi)\delta T/\partial\xi,$$

where, in the dimensionless notation, S is analogous to the logarithmic factor in the usual rate of energy loss expression: $SB=dE/dx$, with $B=mc^2/t\beta^2=2\pi e^4 n/mv^2$, and $T_{\min}$ is the least energy[1] that may be transferred in a single collision. The Bethe-Bloch formula can be obtained from this in the limit as $x\to 0$, and thus appears as the initial rate of loss. However, numerical integration of the above expression shows that, using either the Landau function, or the Blunck-Leisegang, and 1-Mev electrons in Al, S changes by less than 1.5 percent as x is varied from zero to a value near the limit of validity of the theory. It would therefore appear that the Bethe-Bloch formula for the initial rate of loss is sufficiently accurate for most purposes.

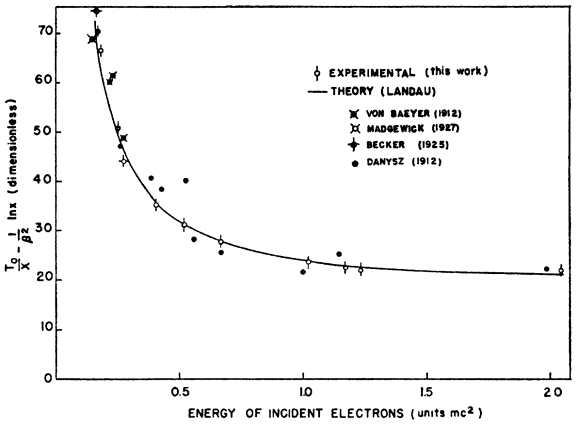

FIG. 4. Energy dependence of the "reduced stopping power," S_r, in aluminum.

While, because of poor resolution, it was not possible to actually verify the theoretical loss distributions in this work, but only to use them to furnish a correction to the peak position, the indication is that the principal result of the Landau-Blunck-Leisegang theory, namely the value of the most probable loss, has been verified quite well. It would be interesting to study the two distributions with an instrument of high resolving power.

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