accounts for the change in exponential absorption of the star producing radiation with latitude; namely, $L_b(0°) > L_b(40°) > L_b(52°)$.

The above arguments show that the energy loss E' in a nucleon-nucleus collision producing a small star cannot be a homogeneous function of E'/E_p, where E_p is the primary particle energy. Thus, the expression $N_b = AV^{-2.5}$ for small burst production does *not* represent the primary energy spectrum. At much higher burst energies the measurements by Carmichael[12] and Montgomery[13] and the recent measurements in nuclear emulsions by Barton, George, and Jason[14] indicate that the large burst and star integral spectrum approaches the values for γ in the range 1.6 to 1.9.

[12] H. Carmichael, Phys. Rev. **74**, 1667 (1948).
[13] C. G. Montgomery and D. D. Montgomery, Phys. Rev. **76**, 1482 (1949).
[14] Barton, George, and Jason, Proc. Phys. Soc. (London) **A64**, 175 (1951).

The measurements described above show that the nuclear burst production and fast neutron production are substantially in equilibrium in the atmosphere below $\sim$200 g-cm^{-2}. Hence, the evidence given in reference 1 to show that the production of neutrons per primary nucleon is not strongly dependent on primary particle momentum above $\sim$4 Bev/c may be extended now to the production of small nuclear bursts in the atmosphere as a function of primary particle momentum.

The authors wish to thank Mr. L. Brodie and Mr. E. Hungerford for assistance with the measurements and to thank Mr. P. Fields and the Argonne National Laboratory for the preparation of thin polonium sources. The assistance of Dr. A. T. Biehl from the California Institute of Technology on some of the flights was of considerable aid to the authors. The cooperation of Major W. Gustavson and the officers and crew of the U. S. Air Force B-29 was greatly appreciated.

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The Absorption of Extensive Showers in Water*

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The absorption of the particles in extensive showers in water has been measured at 2765 meters elevation by detecting coincidences between trays of Geiger counters located under water. Coincidence rates have been measured with counter tray separations up to 5 meters and at seven depths from 0 to 10 meters of water. A theoretical calculation is shown to predict coincidence rates lower than those experimentally observed and further improvements in the calculations are suggested that might remove this discrepancy. If multiple cores exist in extensive showers, this experiment shows that they cannot be separated by more than 50 cm. The density spectrum and the spectrum of the number of particles in a shower have been calculated from the experimental results for elevations of 50 and 2765 meters.

I. INTRODUCTION

SINCE the discovery[1] of the extensive showers in cosmic rays by Geiger tube coincidences, many experiments have been performed to clarify the structure and composition of these showers. From the hypothesis of the cascade origin of these showers Molière[2] has calculated the electron distribution about the shower axis. These results have been shown to be in agreement with experiments performed with Geiger counters[3] and ionization chambers.[4]

A possible mechanism for the production of extensive showers[5] predicts the presence of multiple cores. However, the hypothesis of multiple cores has been shown to be inconsistent with the results from ionization chamber[4] and counter tray[3] experiments.

Fretter and Ise[6] have reported another type of experiment to detect the presence of multiple cores. With water as an absorber the low energy particles will be absorbed. Preliminary results indicated this approach to be promising. These experiments have been extended to greater depths of water in the research reported here in an attempt to study further the structure of the shower core. Experiments were carried out during the summer of 1950 at Lake Sabrina (elevation 2765 meters) near Bishop, California. Further data were obtained in the spring of 1951 near sea level at Berkeley, California.

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[1] Auger, Maze, and Grivet-Meyer, Compt. rend. **206**, 1721 (1938).
[2] G. Molière, *Cosmic Radiation*, edited by W. Heisenberg (Dover Publications, New York, 1946).
[3] Cocconi, Cocconi, Tongiorgi, and Greisen, Phys. Rev. **76**, 1020 (1949).
[4] R. W. Williams, Phys. Rev. **74**, 1689 (1948).
[5] Lewis, Oppenheimer, and Wouthuysen, Phys. Rev. **73**, 127 (1948).
[6] W. B. Fretter and J. Ise, Jr., Phys. Rev. **78**, 92 (1950).

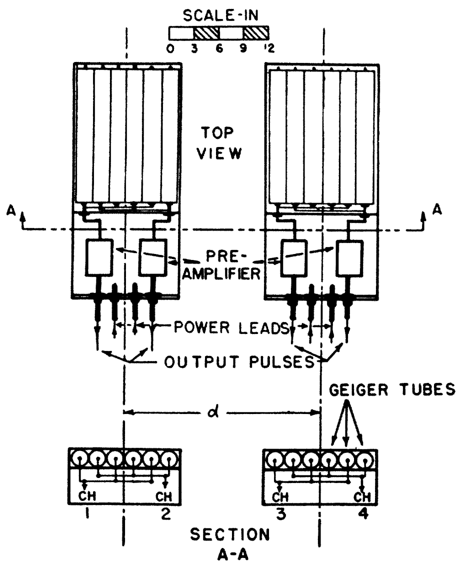

FIG. 1. Diagram of apparatus showing arrangement of counters.

II. EXPERIMENTAL ARRANGEMENT

The apparatus for the experiments consisted of two steel boxes each containing Geiger tubes alternately connected to two preamplifiers (see Fig. 1). The output pulses of the preamplifiers were fed through coaxial cables to the coincidence circuit. This circuit, along with the power supplies, counting circuits, etc., was located in a small wooden house. The output of the four preamplifiers (hereafter referred to as channels 1, 2, 3, and 4) were used in the following coincidence arrangements: (a) twofold coincidences when at least one Geiger tube was discharged in each box, i.e., channels 1 or 2, and 3 or 4; (b) fourfold coincidences when all four channels were discharged simultaneously. The recording of both twofold and fourfold coincidences in this way gives an indication of the density spectrum of the showers observed. The steel boxes and their connecting cables were watertight and were supported by ropes under water at various depths and separations.

The Geiger counters used in this experiment were of the all-metal type filled with an argon-ethylene mixture. They were all of 2-inch inside diameter and their effective length was measured to be 15.5 inches. The resolving time of the coincidence circuits was measured to be $2.0\pm0.1\times10^{-6}$ sec.

III. RESULTS AND CALCULATIONS

The data taken at Lake Sabrina are shown in Figs. 2 and 3. The data have been corrected for accidentals ($\tau=2\times10^{-6}$ sec) and the errors are standard deviations. These data have not been corrected for the barometric effect which is of the order of magnitude of 10 percent per cm of Hg. Because of the possible error due to the barometric effect, these curves can be interpreted as giving only the general form of the decoherence curves.

A theoretical calculation has been performed by Elihu Abrahams[7] to determine the twofold decoherence curve expected at various depths under water. His approach to the problem is briefly given below.

From the distribution function of electrons as calculated by Molière[2] the distribution of electrons with energy greater than E_l is calculated. E_l is the energy of an electron at the surface of the water with sufficient energy to have, on the average, at the depth l one electron remaining from its cascade shower in the water. The decoherence curves calculated from this distribution were found to be obtainable from the decoherence curve on the surface by a simple scale change. His results are given by

$$C_l(d/M_l, A)=(\alpha^\gamma/M_l^{2-\gamma})C_0(d, A), \tag{1}$$

where C_l is the counting rate at a depth l and counter separation d/M_l with counter tray area A, γ is the exponent in the integral spectrum of shower size, $M_l=E_l/38$ (E_l in Mev), $\alpha=2.24$, and C_0 the counting rate with no water absorber at a counter tray separation of d.

Considerable difficulty arises from the determination of E_l, and consequently M_l. The most recent cascade theory calculations of Bernstein[8] give considerable uncertainty as to the depth at which on the average one electron remains from the shower. Figure 4 shows a plot of the average number of electrons $N(t)$ induced by a single incident electron *vs* the absorber thickness t (in radiation units) for different initiating electron energies

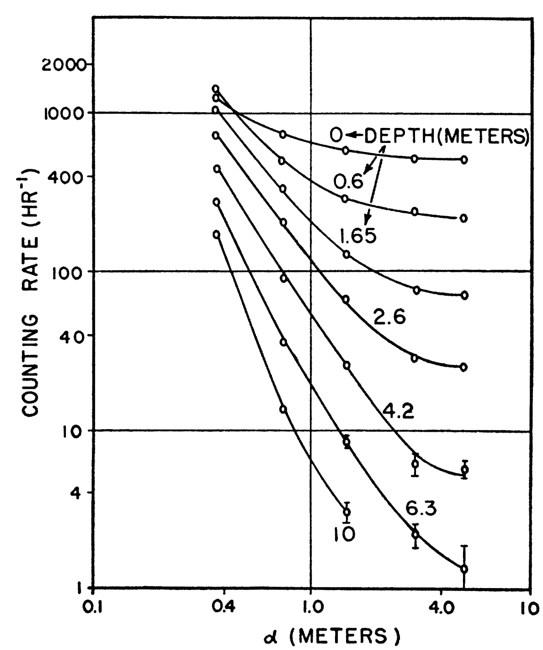

FIG. 2. Twofold decoherence curves at different depths of water (2765 meters elevation).

[7] E. Abrahams (to be published).
[8] I. B. Bernstein, Phys. Rev. 80, 995 (1950).

E_0 (the curve for an initiating γ-ray is given for $E_0=17$ Bev).

The desirability of obtaining coincidence rates at large separation was not known at the time the data were taken at Lake Sabrina and $C_0(d, A)$ was obtained only for d up to 17 ft (5.2 meters). To obtain $C_0(d, A)$ for the required values of d up to several hundred meters, calculations were made from the following integral:

$$C_0(d, A)=K\int_{N=2}^{\infty}\int_S[1-e^{-\rho(r_1)NA}]\times[1-e^{-\rho(r_2)NA}]N^{-(\gamma+1)}dSdN. \quad (2)$$

The differential spectrum of shower size is assumed to be given by

$$f(N)dN=KN^{-(\gamma+1)}dNhr^{-1}m^{-2}. \quad (3)$$

The electron density distribution function $\rho(r)$ is obtained from Molière[2] and r_1 and r_2 are the distances from the shower core to the two counter trays. The surface integral is over the horizontal plane.

The integration of (2) has been performed numerically for $\gamma=1.3$, 1.4 and 1.5 for the values of $d=5$, 50 and 200 meters. The results are shown in Fig. 5 along with points calculated by Molière[2] for Pic du Midi (2870 meters). These calculations are normalized to the experimental point at 5 meters (506 hr^{-1}). The curve for $\gamma=1.4$ agrees well with the experimental results of Auger[9] beyond 20 meters. The experimental value of C_0 ($5m$, $0.117m^2$)$=506$ hr^{-1} yields $K=3.2\times10^5$.

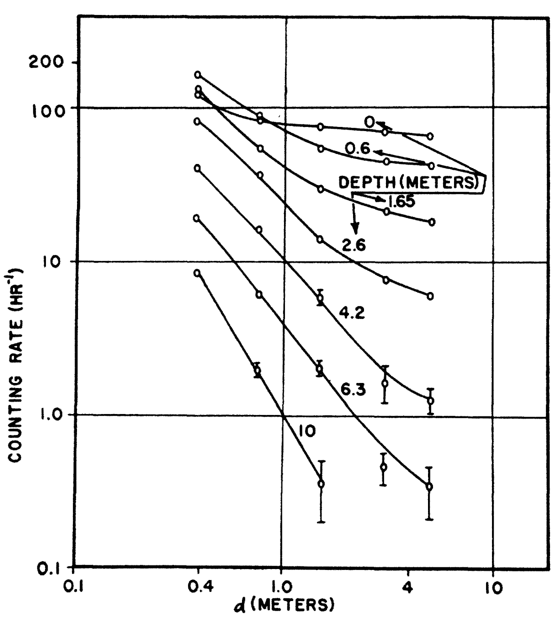

FIG. 3. Fourfold decoherence curves at different depths of water (2765 meters elevation).

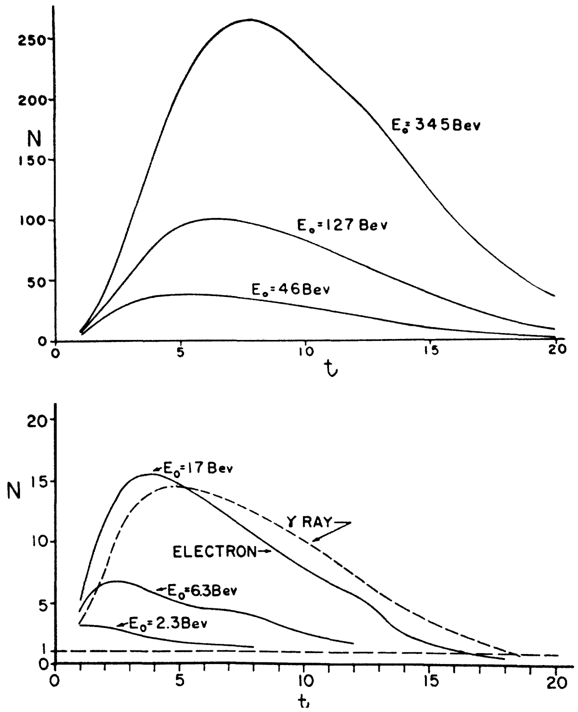

FIG. 4. $N(t)$, the average number of electrons induced by a single incident electron of energy E_0, *vs* the depth of water in radiation units. A curve is also shown for an incident γ-ray of 17 Bev.

With the values of $C_0(d, A)$ of Fig. 5 an attempt has been made to obtain the values of $C_l(d/M_l, A)$ from (1). The value of $\gamma=1.4$ was used in this calculation and values of E_l were obtained from Fig. 4. The calculated values of $C_l(d/M_l, A)$ from (1) are shown in Fig. 6 with the experimental curves. Values of C_l for 10 meters depth were below 1.0 hr^{-1}.

The discrepancy between the calculated and observed coincidence rates is quite serious as to both magnitude and over-all shape of the decoherence curve. The experimentally observed leveling off of the underwater decoherence curves between 3- and 5-meter separations is definitely not predicted by the theoretical results. Efforts to adjust different values of α, γ, and M_l to predict one decoherence curve from another did not produce a consistent method of prediction.

Before discussing Abrahams' calculation further, it would be well to consider from a different viewpoint whether the coincidences occurring at 10 meters depth could be produced by the electron-photon component of the showers. The mean-square lateral displacement of electrons and photons from the axis of a shower is given by Roberg and Nordheim[10] as

$$\langle r^2(E)\rangle_{Av}=0.64(E_s/E)^2X_0^2, \quad (4)$$

$$\langle r^2(W)\rangle_{Av}=1.13(E_s/W)^2X_0^2, \quad (5)$$

where E is the energy of the electrons and W is the en-

[9] Auger, Maze, and Robley, Compt. rend. **208**, 1641 (1938).

[10] J. Roberg and L. W. Nordheim, Phys. Rev. **75**, 444 (1949).

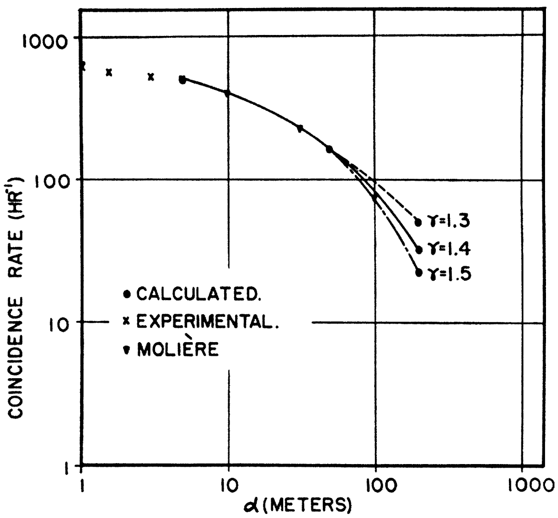

FIG. 5. Experimental and calculated decoherence curves for 2765 meters elevation. A few points from Molière's calculated curve are also shown. All curves are normalized to the experimental coincidence rate of 506 hr^{-1} at $d=5$ meters.

ergy of the photons ($E_s=21$ Mev and $X_0=493$ meters). Table I shows the values of the root mean square lateral displacement of photons and electrons capable of penetrating to the three largest depths. The values of W were obtained from curves, similar to those in Fig. 4, for the case of an initiating photon and are slightly less than the corresponding values of E. When d has the value 0.75 meter, the separation of the two closest Geiger tubes is 0.46 meter, which is about the mean diameter of a shower penetrating 10 meters of water. Thus it is seen that the observed coincidences at small separations are compatible with the electron-photon distribution calculated from cascade theory and coulomb scattering.

An indication of why there is the large difference in the experimental and theoretical results might be gained from the following considerations. Unshielded counters and ionization chambers measure the electron distribution in the shower; however, when shielding is placed over the counters, photons produce pairs in the shielding and the photon distribution is superimposed on the electron distribution. Cascade theory predicts that at the maximum of a shower the number of photons is 9/7 the number of electrons. In accounting for the contribution of the photons, Abrahams assumed that they had the same distribution as the electrons and inserted the factor $(1+9/7)$ at the appropriate place to allow for their contribution to the particle density. This factor appears in the constant α in Eq. (1). Recently Molière[11] has calculated the lateral photon distribution in extensive showers and found that it differs greatly from that of the electrons by its more rapid singularity at $r=0$; varying as r^{-1} for photons as compared with $r^{-\frac{1}{2}}$ for electrons. This means a much stronger concentration of photons within the core and a more rapid variation of photons in its vicinity than expected from the corresponding behavior of electrons. The somewhat larger rms spread of photons is almost entirely due to the tail of the distribution and does not, therefore, contradict the above results.

There is a serious discrepancy between Abrahams' calculation and the experimental results. Abrahams has indicated a theoretical approach to the problem and has shown how a solution in the form of a scale change results. However, the problem appears much more complex than was assumed in these calculations. An extension of these calculations should allow for the difference between the electrons and photons both as to their distribution about the shower core and as to the range of their cascade in water. Also, the complex problem of the fluctuation in the number of particles about the average number of particles in a shower would have to be included in any rigorous solution of the problem.

A possible mechanism for the production of extensive showers predicts[5] the presence of multiple cores. These cores could be separated as much as ten meters. No evidence from other experiments has indicated the presence of these cores, but the possibility that core separations less than 1 meter exist has not been eliminated. If one assumes that multiple cores exist, and that they are distributed over a region of diameter q, the following results would be expected for this experiment: (1) At depths of water where only the high energy particles near the core remain (e.g., 10 meters depth) the decoherence curve would be nearly flat for separations less than q (i.e., $d<q$); (2) As the cores would be very dense clusters of particles, nearly every twofold coincidence would be accompanied by a fourfold and the result would be a ratio of twofold to fourfold counting rates of about unity. As neither of these effects is observed, it may be concluded that if multiple cores exist, they are located within a region of 50 cm. Because of mesons and their knock-on electrons the results for $d=0.38$ meters cannot be unambiguously interpreted as showing the absence of multiple cores.

The presence of mesons in extensive showers has been shown and their density has been measured to be about 2 percent of the electron density.[12] Their distribution in the shower is thought to be similar to the distribution of electrons, although this has not been checked within about 5 meters of the core. If the assumption is made that mesons have the same density

TABLE I. Root-mean-square distance from shower core of electrons and photons having on the average one electron remaining in its cascade at a depth l of water.

l	$[r^2(W)_{Av}]^{\frac{1}{2}}$	$[r^2(E)_{Av}]^{\frac{1}{2}}$
4.2m	4.4m	—
6.3m	1.4m	0.8m
10.0m	0.21m	0.12m

[11] G. Molière, Phys. Rev. **77**, 715 (1950).

[12] J. Ise, Jr., and W. B. Fretter, Phys. Rev. **76**, 933 (1949).

distribution function as electrons but only 2 percent of their intensity, the contribution of mesons to the counting rates in this experiment can be calculated. Substitute $N\rho_m(r)=0.02N\rho(r)$ for the meson density in Eq. (2) and the coincidence rate for mesons is given by

$$C_m(d,A)=K\int_{N=2}^{\infty}\int_S\left[1-e^{-0.02\rho(r_1)NA}\right] \times\left[1-e^{-0.02\rho(r_2)NA}\right]N^{-(\gamma+1)}dSdN. \quad (6)$$

With a change of variable this can be shown to be

$$C_m(d,A)=(0.02)^{\gamma}C_0(d,A). \quad (7)$$

Thus the coincidence rate of just the mesons would be 0.4 percent that due to electrons. If all these mesons could penetrate to 10 meters of water (this would require a meson energy of 2 Bev),[13] their contribution to the coincidence rate would be about 2 hr^{-1} at that depth. A coincidence rate of about this magnitude is observed at $d=1.5$ meters (10 meters depth), but for larger separations any true coincidence could not be differentiated from an accidental coincidence. The decoherence curve for mesons as calculated from (7) is shown in Fig. 6.

The concept of shower density and density spectrum is useful in explaining experimental results. It is customary to express the differential density spectrum in the form

$$H(\Delta)d\Delta=B\Delta^{-(\gamma+1)}d\Delta \text{ hr}^{-1}, \quad (8)$$

where Δ is the average particle density. From ordinary statistical considerations the coincidence rate of n counters, each of effective area A, can be shown to be

$$C_n(A)=B\int_0^{\infty}(1-e^{-\Delta A})^n\Delta^{-(\gamma+1)}d\Delta. \quad (9)$$

Substituting $\Delta A=x$ this becomes

$$C_n(A)=BA^{\gamma}\int_0^{\infty}(1-e^{-x})^n x^{-(\gamma+1)}dx. \quad (10)$$

This integral can be done formally.[14] With $C_2=506$ hr^{-1}, $A=0.117m^2$, and $\gamma=1.4$ the value of B was computed to be 5150 (Δ expressed in m^{-2}).

In this experiment twofold and fourfold coincidences were measured (see Sec. II) having respective counter tray areas of $0.117m^2$ and $0.0585m^2$. The ratio of the twofold and fourfold counting rates may be shown from (10) to be

$$\frac{C_2(A)}{C_4(A/2)}=\frac{2^{\gamma}(2^{\gamma}-2)}{4^{\gamma}-4\times3^{\gamma}+6\times2^{\gamma}-4}=R. \quad (11)$$

[13] Cocconi, Tongiorgi, and Greisen (see reference 3) report from absorption data a mean energy of mesons in extensive showers of at least 2 Bev.

[14] G. Cocconi and V. Cocconi Tongiorgi, Phys. Rev. 75, 1058 (1949).

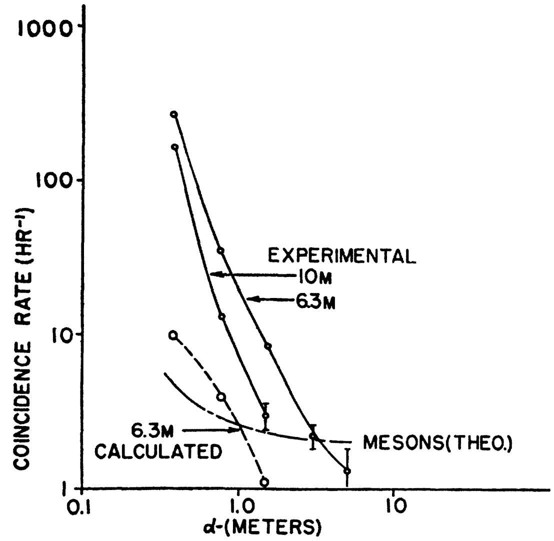

FIG. 6. Comparison of experimental and theoretical underwater decoherence curves. The calculated contribution of mesons to the coincidence rate is shown.

The average value of R for $d=1.5$, 3, and 5 meters from the surface data at Lake Sabrina is $R=7.3\pm0.3$, which yields from (11) $\gamma=1.30\pm0.02$. This value is less than the value $\gamma=1.4$ which is obtained from experiments designed for the accurate determination of γ.[12,15] Equation (11) indicates that an increase in the effective γ will yield a larger value of R, i.e., relatively fewer large showers.

The conditions upon which Eq. (9) was derived are not satisfied for values of $d<2$ meters, as showers very poor in particles (generated by knock-on electrons

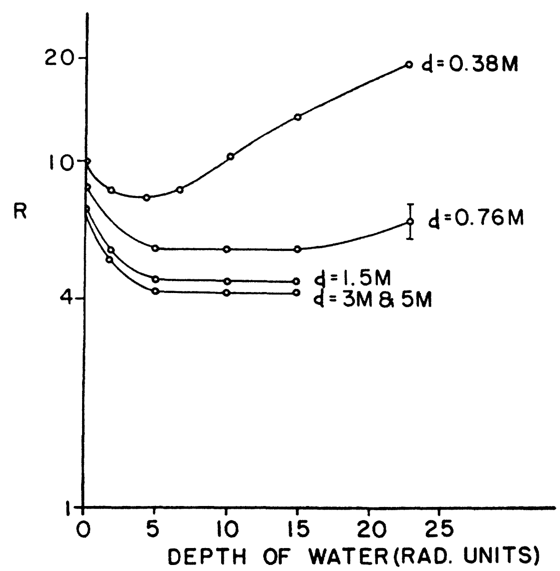

FIG. 7. The ratio of twofold to fourfold coincidence rates, R, *vs* depth of water in radiation units.

[15] Broadbent, Kellermann, and Hakeem, Proc. Phys. Soc. (London) A63, 864 (1950).

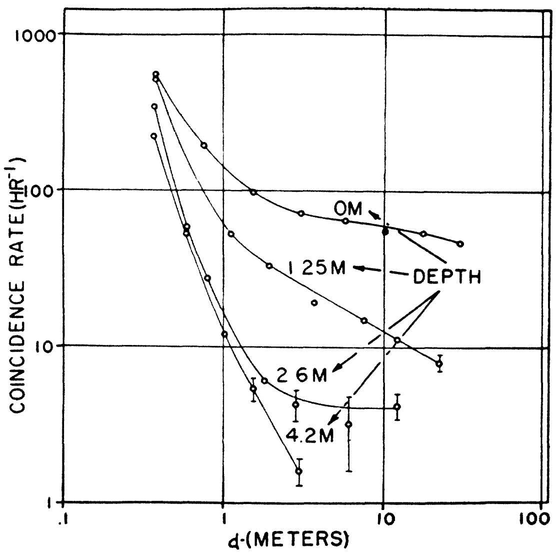

FIG. 8. Twofold decoherence curves at different depths of water (50 meters elevation).

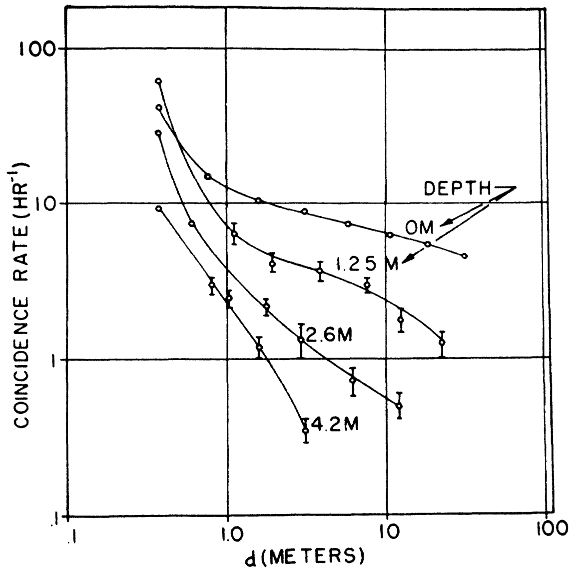

FIG. 9 Fourfold decoherence curves at different depths of water (50 meters elevation).

of the mesons, or locally generated penetrating showers) can be recorded in large percentage besides extensive showers. This is borne out by the experiment as the twofold to fourfold ratio (R) becomes ten for the two trays when contiguous ($d=0.38$ meter) indicating narrow, low density showers.

The meaning of R for the underwater measurements is still about the same, i.e., an indication of the density spectrum; however, the spreading out of the cascade showers produced in the water will increase the fourfold rate relative to the twofold and thus decrease R. The cascade shower produced in the water by an electron or photon incident on the surface reaches its maximum in about 5 radiation units (2.1 meters) and keeps a constant diameter (though decreasing in density) of about 17 cm (6.7 in.). The result of this can be seen in the plot of R *vs* l (depth of water) for the different values of d (counter separation) in Fig. 7. When several values agreed within their statistical uncertainty, their average value was plotted. The decrease in R down to 5 radiation units is shown. The curve associated with $d=0.38$ meter is seen to increase after its initial decrease. This can be interpreted as a nearly constant twofold coincidence rate owing to mesons and their knock-on electrons being superimposed upon the coincidence rates caused by particles associated with extensive showers.

Experiments similar to those at Lake Sabrina were carried on near sea level at Berkeley, California (about 50 meters elevation). The results corrected for accidentals are shown in Figs. 8 and 9. There is more uncertainty in the twofold 2.6-meter depth curve beyond $d=2$ meters than the statistical uncertainty shown because of the large correction made for the accidental coincidence rate. These measurements were made to determine if the leveling off of the underwater decoherence curves beyond 3 meters which was observed at 2765 meters elevation exists also at sea level. This leveling off effect does not seem to be characteristic of the underwater decoherence curves at sea level.

From the unshielded coincidence rates at Berkeley the density spectrum and the spectrum of shower sizes were calculated to be

$$H(\Delta)d\Delta = 710\Delta^{-2.4}d\Delta \text{ hr}^{-1},$$
$$f(N)dN = 4.4\times 10^{4}N^{-2.4}dN \text{ hr}^{-1}\text{m}^{-2}.$$

The writer wishes to express his gratitude to Professor R. B. Brode for making possible the carrying out of these experiments at Lake Sabrina, and to Professor W. B. Fretter, who suggested the experiment and who has guided its progress. The many discussions with Mr. E. Abrahams have been most helpful. Invaluable aid in the development and checking of the electronics was given by Mr. Bruce Harris. Appreciation is also expressed to the East Bay Regional Park District and the California Electric Power Company for the cooperation of their employees and the use of their facilities. Permission to install the equipment at Lake Sabrina was given by the U. S. Forest Service.