On the Nature of the V-Particles

R. G. SACHS
University of Wisconsin, Madison, Wisconsin

(Received July 2, 1951)

Disintegration of a neutral V-particle into a proton and negative meson strongly suggests that the V-particle is an excited state of the neutron. In order to account for the very long lifetime of this state, it is suggested that the nucleon has a very complex structure involving many mesons in virtual states. The excitation is then distributed over many degrees of freedom, so the probability for formation of that state which leads directly to disintegration may be very small. If n_0 is the average number of pions of each charge in the nucleon and f is the number of states available to a single pion, the lifetime for pion emission is estimated to be $t=fn_0^{2f}\times 10^{-23}$ sec. For the reasonable choice $f=10$ and $n_0=4$ this agrees with the observed lifetime of the neutral V-particle.

1. INTRODUCTION

THE recent observation that the charged disintegration products of the neutral V-particle[1] are proton and meson[2] strongly suggests that this particle is simply an excited state of the neutron, with excitation energy of about 200 Mev, which disintegrates into a negative π-meson and a proton. However, the apparent lifetime for disintegration, about[3] 3×10^{-10} sec, provides a very serious difficulty for this interpretation. The order of magnitude of the lifetime against π-meson emission of an excited neutron would be expected to be

$$t_0 \approx \hbar/pc,$$

where p is the meson momentum. This is of the order of 10^{-23} sec.

This estimate of t_0 is obtained as follows: the meson current through the nucleon surface is of the order of $p/\mu V$, where μ is the meson mass and V the nucleon volume. The number emerging per unit time is then $p/\mu V^{\frac{1}{3}}$, so the lifetime is $t_0=\mu V^{\frac{1}{3}}/p$. The value given above is obtained by assuming that $V^{\frac{1}{3}}\approx \hbar/\mu c$, the Compton wavelength of the meson. An additional factor of $\hbar c/g^2$, where g is the meson-nucleon coupling constant, might be expected; but for the usual values of g the resulting increase in t_0 is not important. The estimated lifetime is also greater if a large transfer of angular momentum is involved in the transition. However, it seems unlikely that a low excited state of a nucleon would differ by many units of angular momentum from the ground state.

The purpose of this note is to suggest, tentatively, a description of the nucleon which could account for the long lifetime. It has often been proposed[4] that the coupling between nucleon and meson field is so strong that the weak-coupling approximation is inadequate. The usual values of the coupling constant suggest that the nucleon consists of a complex of particles, core particle (nucleore) plus virtual π-mesons. The probability for finding a fairly large number of pions in the nucleon should be rather large.[5]

We now take the extreme view that the nucleon is a very complex system for which the probabilities of occurrence of one, two, ten, or even twenty pions are comparable. Then the excited state of a nucleon has a similar description. The 200-Mev excited state, can, on energetic grounds, disintegrate in only the one way, by emission of a single pion. The excitation is distributed over the many degrees of freedom of the nucleon and the probability for formation of that state which leads directly to disintegration may be very small. Thus, the lifetime may be greatly increased by assuming a sufficiently complex nucleon structure.

This proposal is analogous to the explanation of the relatively long life of the unstable state of a complex system such as a compound nucleus. However, the analogy may be misleading, since in the compound nucleus, conservation of the number of particles limits the states over which the excitation can be distributed, while in the nucleon the virtual meson states are quite free of any such restriction.

2. DESCRIPTION OF THE NUCLEON

A more quantitative formulation may be made as follows: the neutron and proton wave functionals Ψ_N and Ψ_P may be expanded as a sum of products of non-interacting meson and nucleore functions. Denoting an independent particle functional for a nucleore with isotopic spin τ, m negative pions, n positive pions, and ν neutral pions by $\psi_\tau([+]^n, [-]^m, \nu)$ we can write

$$\Psi_N=\sum\{\psi_+([+]^n, [-]^n, \nu)(+, n, \nu|N) + \psi_-([+]^n, [-]^{n+1}, \nu)(-, n, \nu|N)\}, \quad (1)$$

[1] G. D. Rochester and C. C. Butler, Nature **160**, 855 (1947).

[2] Seriff, Leighton, Hsaio, Cowan, and Anderson, Phys. Rev. **78**, 290 (1950); V. D. Hopper and S. Biswas, Phys. Rev. **80**, 1099 (1950); Armenteros, Barker, Butler, Cachon, and Chapman, Nature **167**, 501 (1951); W. B. Fretter, Phys. Rev. **82**, 294(A) (1951); Leighton, Seriff, and Anderson, Phys. Rev. **83**, 896(A) (1951); Thompson, Cohn, and Flum, Phys. Rev. **83**, 197(A) (1951).

[3] A. J. Seriff, *et al.*, reference 2.

[4] G. Wentzel, Helv. Phys. Acta **13**, 269 (1940); J. R. Oppenheimer and J. Schwinger, Phys. Rev. **60**, 150 (1941); E. Fermi, *Elementary Particles* (Yale University Press, New Haven, Connecticut, 1951), Sec. 25.

[5] Although strong coupling has been used here as a motivation for introducing large numbers of pions, it should be noted that strong coupling is not essential to the argument. The nucleon-meson coupling may involve operators which produce mesons multiply. A special case of such a coupling has recently been treated by R. J. Glauber (private communication, 1951).

$$\Psi_P=\sum\{\psi_-([+]^n,[-]^n,\nu)(-,n,\nu|P)+\psi_+([+]^n,[-]^{n-1},\nu)(+,n,\nu|P)\},\quad (2)$$

where the sum is taken over all possible numbers n, ν, and over all states of each pion for each n and ν. The important assumption here is that the coefficients $(\tau, n, \nu|N)$ are of comparable magnitude for values of n and ν running from 0 to n_0, where n_0 is some number larger than one. The excited state of the neutron is described by a functional

$$\Psi_N'=\sum\{\psi_+([+]^n,[-]^n,\nu)(+,n,\nu|N')+\psi_-([+]^n,[-]^{n+1},\nu)(-,n,\nu|N')\},\quad (3)$$

whose coefficiencients are of the same magnitude as in Eq. (1), but are phased in such a way that Ψ_N' is orthogonal to Ψ_N. Disintegration of this state occurs by formation of the state Ψ_P' which describes a proton plus an extra negative pion:

$$\Psi_P'=\sum{}'\{\psi_+([+]^n,[-]^n,\nu)(+,n,\nu|P)+\psi_-([+]^n,[-]^{n+1},\nu)(-,n,\nu|P)\},\quad (4)$$

where the coefficients have been taken to be the same as those in Eq. (2). The primed summation is meant to indicate that 1 is the minimum number of negative pions to be included in the sum.

The probability for the occurrence of Ψ_P' in the state Ψ_N' is $|(\Psi_P', \Psi_N')|^2$. When this state occurs, it presumably disintegrates with transition probability $1/t_0$; therefore, the lifetime of Ψ_N' is

$$t=t_0/|(\Psi_P', \Psi N')|^2. \quad (5)$$

Now it is necessary to make some more detailed assumption about the coefficients $(\tau, n, \nu|P)$ and $(\tau, n, \nu|N')$ in order to evaluate t. The total number of each of these coefficients, which we denote by N, is very large. Therefore, one possible assumption is that on the average the coefficients will be randomly phased with respect to one another. Then we have

$$|(\Psi_P', \Psi_N')|^2\approx 1/N, \quad (6)$$

so that

$$t\approx Nt_0. \quad (7)$$

Another possibility, which corresponds to a very high degree of correlation, is that the ground states of neutron and proton differ in structure essentially by the addition of a negative pion to the proton. Then we have

$$(\tau, n, \nu|P)=(\tau, n, \nu|N), \quad (8)$$

except for $n=0$, $\tau=+1$; and we find that

$$\Psi_P'=\Psi_N-\sum \psi_+([+]^0,[-]^0,\nu)(+,0,\nu|N). \quad (9)$$

The sum is extended over the totality, N^0, of neutral pion states. Since Ψ_N' is orthogonal to Ψ_N, we find

$$(\Psi_P', \Psi_N')=-\sum(+,0,\nu|N)^*(+,0,\nu|N'), \quad (10)$$

or

$$|(\Psi_P', \Psi_N')|\gtrsim N^0/N. \quad (11)$$

Thus we have

$$t\lesssim(N/N^0)^2t_0. \quad (12)$$

It will be found below that $(N/N^0)^2\gtrsim N$, so that this estimate leads to an even longer lifetime than Eq. (7). Therefore, the more conservative Eq. (7) will be applied henceforth.

3. EVALUATION OF N

Denote by f the number of accessible states of a single pion. Then, since the pions satisfy E.B. statistics, the number, N_n, of accessible states for n pions may be obtained from the relation[6]

$$(1-x)^{-f}=\sum_n N_n x^n \quad (13)$$

for any x. There are three kinds of particles to be considered, positive, negative, and neutral pions. The numbers of neutrals are presumed to be uncorrelated with the others, but the numbers of positive and negative particles are closely correlated. For a fixed number n, of negative pions, the number of positives is fixed within one unit. However, the momenta of the various pions can range independently over the allowed values.

To take account of the reduced probability for finding large values of n, we introduce a weight function $P(n)$ which decreases with increasing n. Then the total number of states for the neutrals is

$$N^0=\sum_n N_nP(n), \quad (14)$$

while the number of states available to both positively and negatively charged particles is

$$N^\pm=\sum_n N_n(N_n+N_{n+1})P(n). \quad (15)$$

The total number of accessible staes is

$$N=N^0N^\pm. \quad (16)$$

Now, to be definite, we take

$$P(n)=\exp(-n/n_0). \quad (17)$$

Then, according to Eq. (13), we have

$$N^0=[1-\exp(-1/n_0)]^{-f}. \quad (18)$$

Since N_n increases with increasing n, we can set

$$N^\pm>N_1\sum_n N_nP(n)=N_1[1-\exp(-1/n_0)]^{-f}. \quad (19)$$

Now we have

$$N_1=f, \quad (20)$$

so that

$$N^\pm>f[1-\exp(-1/n_0)]^{-f},$$

and, according to Eqs. (16) and (18)

$$N>f[1-\exp(-1/n_0)]^{-2f}. \quad (21)$$

[6] R. H. Fowler, *Statistical Mechanics* (Cambridge University Press, London, 1936), p. 43ff.

For large n_0 this can be rewritten as

$$N > f n_0^{2f}. \quad (22)$$

An estimate of f can be obtained by treating the pion as a free particle constrained to a volume V centered on the nucleon. Then we assume that all pion states are equally probable up to some maximum momentum, P_0, and that states of higher momentum do not occur at all. Thus we have

$$f = (4\pi/3)[VP_0^3/(2\pi\hbar)^3].$$

On the assumption that the upper limit of the momentum is primarily determined by the nucleon recoil, we take

$$P_0 = Mc,$$

where M is the nucleon mass. The volume V should probably be taken to be of the order of $(\hbar/\mu c)^3$, where μ is the pion mass. Then we have

$$f \approx (M/\mu)^3/6\pi^2 \approx 5.$$

A considerably larger value of f is obtained if, instead of a sharp cutoff, states of higher momenta are assumed to occur with diminishing weight. For example, a distribution of the form $\exp(-p/P_0)$ leads to the value

$$f \approx (M/\mu)^3/\pi^2 \approx 30.$$

As a conservative compromise we may take[7]

$$f = 10. \quad (23)$$

Application of this result to Eqs. (7) and (22) leads to

$$t/t_0 > 10 n_0^{20}. \quad (24)$$

Thus, for a rather small value of n_0, the lifetime may be increased by a factor of 10^{10} or so.

The above enumeration of states is actually somewhat optimistic, since no account has been taken of the conservation of angular momentum. The total orbital angular momentum, L, of the pions can be assumed fixed without any serious loss of generality. The orbital angular momenta of all but one of the pions are completely independent. If they combine to form angular momentum L', then the orbital angular momentum of the last pion must take on one of the values $L'+L$, $L'+L-1$, $\cdots |L'-L|$. For $L' \geq L$ there are $2L+1$ values, otherwise $2L'+1$. However, out of the $(2L+1)$ m_L values only one is to be considered, so in effect there is about one state available to the last pion for each specific configuration of the other pions. This correlation will reduce the number of states by a smaller factor than that which was lost in going from Eq. (15) to Eq. (19). Therefore, an underestimate of t is obtained if Eq. (22) is replaced by

$$N \approx f n_0^{2f}. \quad (25)$$

The estimated lifetime is then

$$t \approx f n_0^{2f} t_0. \quad (26)$$

For $n_0 = 4$, this yields

$$t \approx 10^{13} t_0 \approx 10^{-10} \text{ sec}, \quad (27)$$

which is in reasonable agreement with the observed lifetime of the V-particle.

4. CONCLUSION

The most definite result of this discussion is that the number of parameters required to describe a nucleon state increases extremely rapidly as the number of virtual pions increases. This increase in complexity leads to an increase in lifetime of an excited state of a neutron under either one of two oppositely extreme assumptions: (1) there is very little correlation between the parameters describing the neutron excited state and those describing the proton ground state, (2) there is an extreme correlation of a very special type.

It therefore seems possible that the V-particle is a manifestation of the excited state of such a nucleon. This state may disintegrate by either pion or muon emission. The arguments presented here lead to such a long lifetime for pion emission that decay into muon and neutrino by means of a beta-type interaction could compete with pion decay.

[7] Note that f also increases rapidly with the radius of the nucleon which may easily be underestimated here.